Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.4-Cotangent/109-4.4.10-c+d-x-^m-a+b-cot-^n

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September 27, 2022 Compiled on September 27, 2022 at 4:49pm

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [61]. This is test number [109].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (61)	0.00 (0)
Mathematica	100.00 (61)	0.00 (0)
Fricas	100.00 (61)	0.00 (0)
Maple	95.08 (58)	4.92 (3)
Maxima	80.33 (49)	19.67 (12)
Giac	57.38 (35)	42.62 (26)
Mupad	45.90 (28)	54.10 (33)
Sympy	45.90 (28)	54.10 (33)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

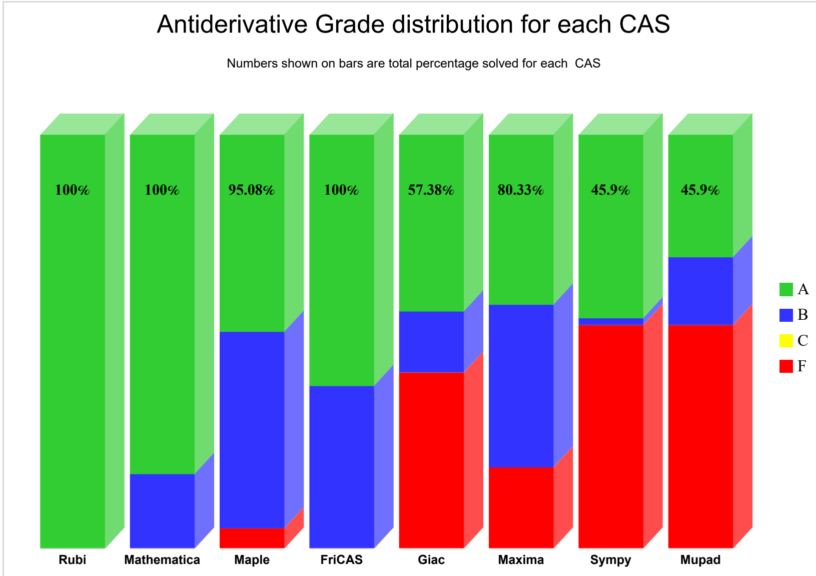
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

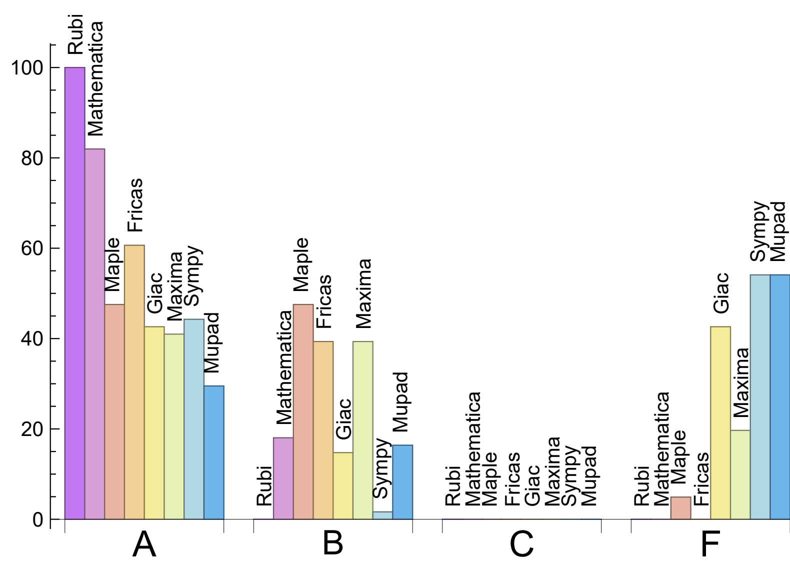
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	81.97	18.03	0.00	0.00
Fricas	60.66	39.34	0.00	0.00
Maple	47.54	47.54	0.00	4.92
Sympy	44.26	1.64	0.00	54.10
Giac	42.62	14.75	0.00	42.62
Maxima	40.98	39.34	0.00	19.67
Mupad	N/A	16.39	0.00	54.10

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	3	100.00 %	0.00 %	0.00 %
Fricas	0	0.00 %	0.00 %	0.00 %
Giac	26	100.00 %	0.00 %	0.00 %
Maxima	12	25.00 %	0.00 %	75.00 %
Sympy	33	93.94 %	6.06 %	0.00 %
Mupad	33	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

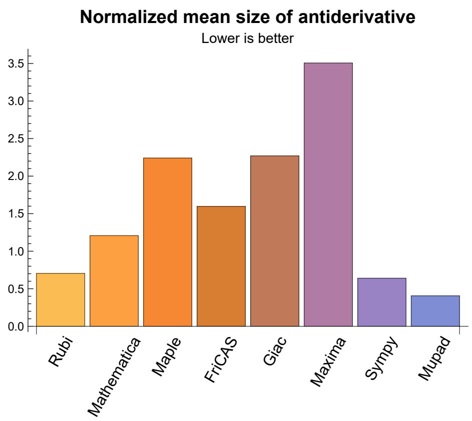
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

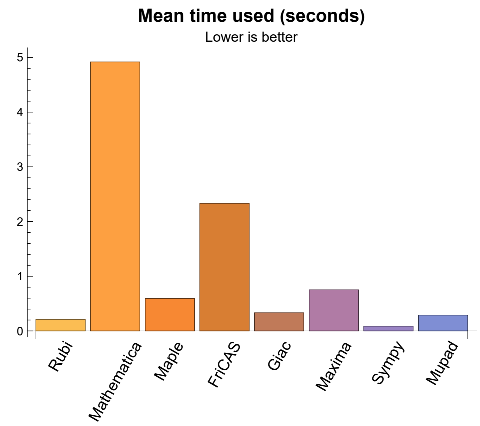
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.21	169.20	0.70	126.00	1.00
Mathematica	4.92	306.51	1.21	165.00	1.09
Maple	0.59	600.14	2.24	214.50	1.73
Maxima	0.75	914.80	3.51	225.00	0.78
Fricas	2.33	377.25	1.60	146.00	0.77
Sympy	0.09	133.79	0.64	0.00	0.00
Giac	0.33	429.46	2.27	0.00	0.00
Mupad	0.29	79.00	0.41	-1.00	-0.04

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
5
4
3
2
1
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{4, 5, 9, 10, 14, 15, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{13, 42, 43, 47, 48, 57, 59\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

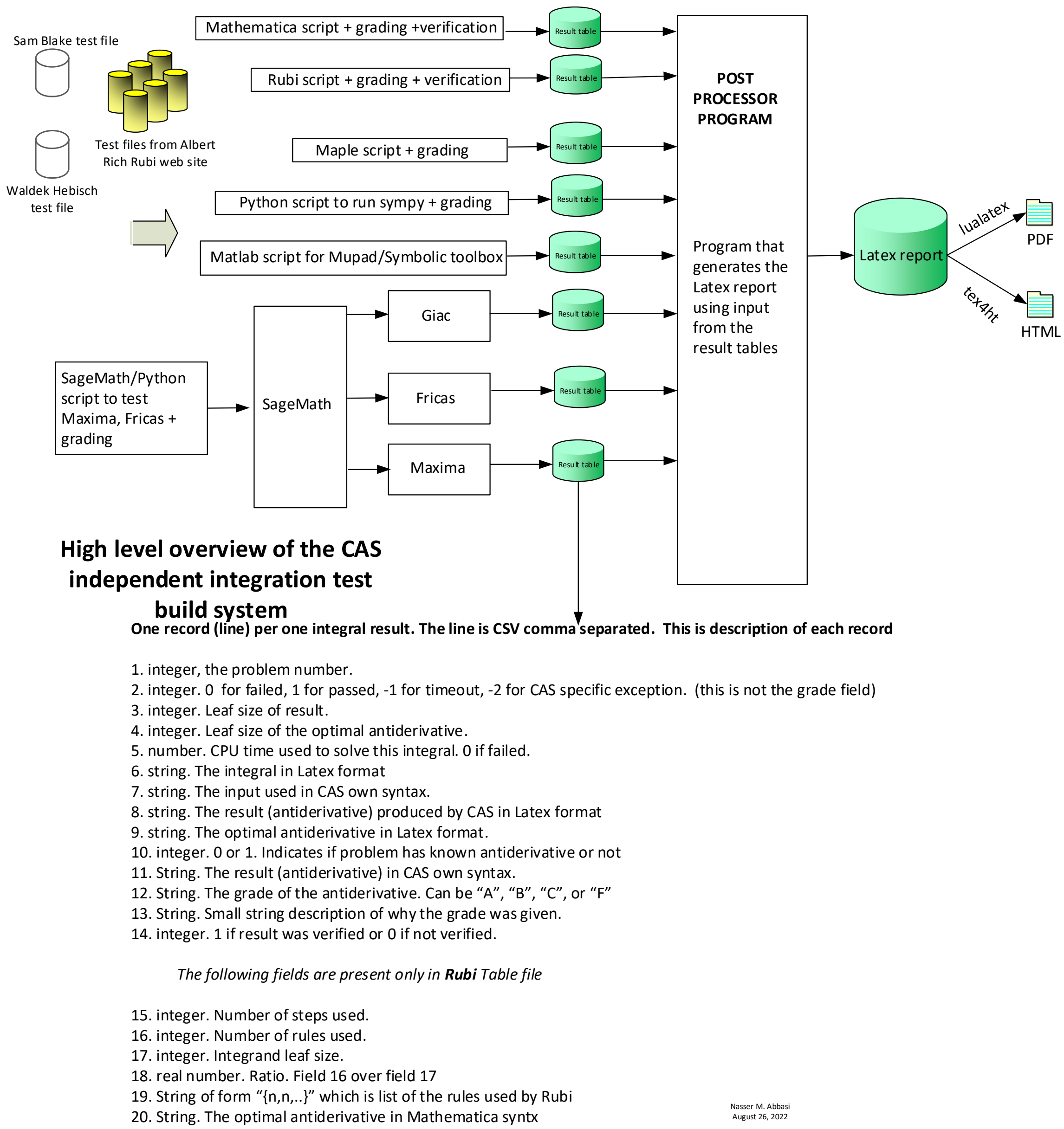
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 58, 60, 61 }

B grade: { 3, 7, 37, 38, 39, 42, 43, 47, 48, 57, 59 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 4, 5, 8, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 25, 26, 30, 31, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61 }

B grade: { 1, 2, 3, 6, 7, 11, 12, 13, 22, 23, 24, 27, 28, 29, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

C grade: { }

F grade: { 34, 35, 36 }

2.1.4 Maxima

A grade: { 4, 5, 9, 10, 14, 15, 19, 20, 21, 25, 26, 30, 31, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61 }

B grade: { 1, 2, 3, 6, 7, 8, 11, 12, 13, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

C grade: { }

F grade: { 16, 17, 18, 22, 23, 24, 27, 28, 29, 34, 35, 36 }

2.1.5 FriCAS

A grade: { 4, 5, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61 }

B grade: { 1, 2, 3, 6, 7, 8, 11, 12, 13, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

C grade: { }

F grade: { }

2.1.6 Sympy

A grade: { 4, 5, 9, 10, 14, 15, 16, 17, 18, 22, 23, 24, 27, 28, 29, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61 }

B grade: { 8 }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 11, 12, 13, 19, 20, 21, 25, 26, 30, 31, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

2.1.7 Giac

A grade: { 4, 5, 9, 10, 14, 15, 16, 17, 18, 23, 24, 27, 28, 29, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61 }

B grade: { 8, 19, 20, 21, 22, 25, 26, 30, 31 }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 11, 12, 13, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

2.1.8 Mupad

A grade: { 4, 5, 9, 10, 14, 15, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61 }

B grade: { 8, 16, 17, 18, 22, 23, 24, 27, 28, 29 }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 11, 12, 13, 19, 20, 21, 25, 26, 30, 31, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	98	240	391	306	0	0	-1
N.S.	1	1.00	0.97	2.38	3.87	3.03	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.546	0.302	0.360	3.525	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	75	198	257	244	0	0	-1
N.S.	1	1.00	1.01	2.68	3.47	3.30	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.396	0.240	0.336	3.287	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	135	150	140	174	0	0	-1
N.S.	1	1.00	2.55	2.83	2.64	3.28	0.00	0.00	-0.02
time (sec)	N/A	0.057	3.936	0.231	0.345	4.124	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.012	1.862	0.106	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.012	3.277	0.101	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	120	231	952	372	0	0	-1
N.S.	1	1.00	1.24	2.38	9.81	3.84	0.00	0.00	-0.01
time (sec)	N/A	0.117	1.257	0.376	0.595	3.262	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	181	183	386	281	0	0	-1
N.S.	1	1.00	2.45	2.47	5.22	3.80	0.00	0.00	-0.01
time (sec)	N/A	0.079	6.164	0.316	0.575	3.453	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	44	40	269	75	65	1250	54
N.S.	1	1.00	1.42	1.29	8.68	2.42	2.10	40.32	1.74
time (sec)	N/A	0.017	0.242	0.313	0.500	3.682	0.225	0.839	0.452

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.019	3.291	0.184	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.020	3.409	0.142	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	349	444	1960	569	0	0	-1
N.S.	1	1.00	1.73	2.20	9.70	2.82	0.00	0.00	-0.00
time (sec)	N/A	0.214	6.754	0.386	0.479	4.154	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	154	293	1208	425	0	0	-1
N.S.	1	1.00	1.22	2.33	9.59	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.135	2.684	0.338	0.439	4.063	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	179	197	586	291	0	0	-1
N.S.	1	1.00	1.97	2.16	6.44	3.20	0.00	0.00	-0.01
time (sec)	N/A	0.078	4.416	0.291	0.386	3.817	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.020	4.827	0.168	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.020	5.286	0.213	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	246	170	0	155	314	233	423
N.S.	1	1.00	1.30	0.90	0.00	0.82	1.66	1.23	2.24
time (sec)	N/A	0.146	0.441	0.796	0.000	3.947	0.186	0.414	1.236

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	149	108	0	97	194	137	241
N.S.	1	1.00	1.09	0.79	0.00	0.71	1.42	1.00	1.76
time (sec)	N/A	0.089	0.277	0.763	0.000	2.789	0.148	0.415	0.769

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	107	50	0	49	100	64	105
N.S.	1	1.00	1.27	0.60	0.00	0.58	1.19	0.76	1.25
time (sec)	N/A	0.040	0.205	0.693	0.000	4.997	0.109	0.445	0.382

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	77	67	119	54	0	351	-1
N.S.	1	1.00	0.48	0.42	0.74	0.34	0.00	2.18	-0.01
time (sec)	N/A	0.216	0.260	0.662	0.319	2.715	0.000	0.431	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	215	105	129	76	0	357	-1
N.S.	1	1.00	1.30	0.63	0.78	0.46	0.00	2.15	-0.01
time (sec)	N/A	0.172	1.271	0.635	0.347	2.096	0.000	1.415	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	283	143	169	122	0	1558	-1
N.S.	1	1.00	1.25	0.63	0.74	0.54	0.00	6.86	-0.00
time (sec)	N/A	0.229	1.613	0.688	0.362	2.984	0.000	0.432	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	362	2277	0	259	651	413	294
N.S.	1	1.00	1.34	8.43	0.00	0.96	2.41	1.53	1.09
time (sec)	N/A	0.195	1.418	1.141	0.000	4.572	0.324	0.452	1.119

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	255	1081	0	156	405	235	186
N.S.	1	1.00	1.26	5.35	0.00	0.77	2.00	1.16	0.92
time (sec)	N/A	0.140	0.548	0.945	0.000	3.866	0.247	0.460	0.579

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	165	396	0	70	212	102	102
N.S.	1	1.00	1.09	2.62	0.00	0.46	1.40	0.68	0.68
time (sec)	N/A	0.108	0.501	0.793	0.000	3.717	0.182	0.473	0.338

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	136	378	208	89	0	939	-1
N.S.	1	1.00	0.45	1.24	0.68	0.29	0.00	3.08	-0.00
time (sec)	N/A	0.550	0.555	0.753	0.357	3.272	0.000	0.458	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	203	537	225	136	0	2249	-1
N.S.	1	1.00	0.47	1.24	0.52	0.31	0.00	5.18	-0.00
time (sec)	N/A	0.496	0.524	0.780	0.427	2.023	0.000	1.566	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	664	4079	0	363	932	593	418
N.S.	1	1.00	1.68	10.30	0.00	0.92	2.35	1.50	1.06
time (sec)	N/A	0.261	2.743	1.072	0.000	2.177	0.455	0.568	1.624

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	369	1879	0	214	575	333	263
N.S.	1	1.00	1.26	6.39	0.00	0.73	1.96	1.13	0.89
time (sec)	N/A	0.179	0.676	0.882	0.000	4.119	0.344	0.520	1.001

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	244	665	0	96	298	142	144
N.S.	1	1.00	1.17	3.18	0.00	0.46	1.43	0.68	0.69
time (sec)	N/A	0.165	0.561	0.631	0.000	3.398	0.247	0.516	0.625

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	197	531	295	122	0	1791	-1
N.S.	1	1.00	0.44	1.18	0.66	0.27	0.00	3.99	-0.00
time (sec)	N/A	1.255	0.538	0.588	0.363	4.028	0.000	0.543	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	712	712	292	755	319	194	0	4284	-1
N.S.	1	1.00	0.41	1.06	0.45	0.27	0.00	6.02	-0.00
time (sec)	N/A	1.202	0.617	0.628	0.438	3.183	0.000	1.666	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	8.838	0.265	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.019	5.713	0.327	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	190	0	0	86	0	0	-1
N.S.	1	1.00	1.94	0.00	0.00	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.083	1.250	0.480	0.000	0.495	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	155	0	0	146	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.130	9.647	0.377	0.000	0.723	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	238	0	0	201	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.80	0.00	0.00	-0.00
time (sec)	N/A	0.170	31.601	0.992	0.000	0.463	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	524	876	1037	644	0	0	-1
N.S.	1	1.00	3.56	5.96	7.05	4.38	0.00	0.00	-0.01
time (sec)	N/A	0.180	7.141	0.536	0.393	4.349	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	361	535	570	421	0	0	-1
N.S.	1	1.00	3.22	4.78	5.09	3.76	0.00	0.00	-0.01
time (sec)	N/A	0.143	6.442	0.500	0.372	3.406	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	204	240	223	240	0	0	-1
N.S.	1	1.00	2.46	2.89	2.69	2.89	0.00	0.00	-0.01
time (sec)	N/A	0.088	5.341	0.392	0.362	3.551	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.022	2.017	0.264	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.022	8.000	0.298	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	1313	1604	4022	1175	0	0	-1
N.S.	1	1.00	4.45	5.44	13.63	3.98	0.00	0.00	-0.00
time (sec)	N/A	0.378	7.011	0.856	1.502	3.290	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	635	932	2031	740	0	0	-1
N.S.	1	1.00	2.80	4.11	8.95	3.26	0.00	0.00	-0.00
time (sec)	N/A	0.261	6.970	0.682	0.610	2.791	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	200	365	828	403	0	0	-1
N.S.	1	1.00	1.46	2.66	6.04	2.94	0.00	0.00	-0.01
time (sec)	N/A	0.123	2.109	0.577	0.396	3.959	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	15.949	0.325	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	12.970	0.378	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	603	603	2539	3161	10500	2746	0	0	-1
N.S.	1	1.00	4.21	5.24	17.41	4.55	0.00	0.00	-0.00
time (sec)	N/A	0.656	7.937	1.043	15.692	2.802	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	1825	1788	5334	1579	0	0	-1
N.S.	1	1.00	4.21	4.13	12.32	3.65	0.00	0.00	-0.00
time (sec)	N/A	0.448	7.444	0.871	3.140	5.478	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	276	745	2149	750	0	0	-1
N.S.	1	1.00	0.99	2.68	7.73	2.70	0.00	0.00	-0.00
time (sec)	N/A	0.226	3.379	0.774	0.776	2.614	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	9.251	0.431	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	12.190	0.467	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	419	1402	1045	1047	0	0	-1
N.S.	1	1.00	1.73	5.79	4.32	4.33	0.00	0.00	-0.00
time (sec)	N/A	0.251	2.179	1.053	0.733	3.378	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	243	897	760	748	0	0	-1
N.S.	1	1.00	1.34	4.96	4.20	4.13	0.00	0.00	-0.01
time (sec)	N/A	0.205	1.635	0.917	0.653	2.952	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	113	445	428	491	0	0	-1
N.S.	1	1.00	0.90	3.53	3.40	3.90	0.00	0.00	-0.01
time (sec)	N/A	0.115	1.008	0.846	0.619	3.801	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	1.678	0.563	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	3.535	0.558	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	839	839	2879	3470	4518	3413	0	0	-1
N.S.	1	1.00	3.43	4.14	5.38	4.07	0.00	0.00	-0.00
time (sec)	N/A	1.383	10.693	1.364	2.755	3.233	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	650	554	2161	2560	2092	0	0	-1
N.S.	1	1.00	0.85	3.32	3.94	3.22	0.00	0.00	-0.00
time (sec)	N/A	1.065	8.071	1.256	1.295	4.317	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	730	990	1207	1101	0	0	-1
N.S.	1	1.00	3.43	4.65	5.67	5.17	0.00	0.00	-0.00
time (sec)	N/A	0.214	6.839	1.125	0.874	3.492	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	16.449	0.766	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	15.525	0.741	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [16] had the largest ratio of [23]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	6	1.00	10	0.600
2	A	5	5	1.00	10	0.500
3	A	4	4	1.00	8	0.500
4	A	0	0	0.00	0	0.000
5	A	0	0	0.00	0	0.000
6	A	7	7	1.00	12	0.583
7	A	6	6	1.00	12	0.500
8	A	3	3	1.00	10	0.300
9	A	0	0	0.00	0	0.000
10	A	0	0	0.00	0	0.000
11	A	13	10	1.00	12	0.833
12	A	9	8	1.00	12	0.667
13	A	7	7	1.00	10	0.700
14	A	0	0	0.00	0	0.000
15	A	0	0	0.00	0	0.000
16	A	5	3	1.00	23	0.130
17	A	4	3	1.00	23	0.130
18	A	3	3	1.00	21	0.143
19	A	7	4	1.00	23	0.174
20	A	7	4	1.00	23	0.174
21	A	8	5	1.00	23	0.217
22	A	10	3	1.00	23	0.130
23	A	8	3	1.00	23	0.130
24	A	7	3	1.00	21	0.143
25	A	21	5	1.00	23	0.217

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	24	7	1.00	23	0.304
27	A	14	3	1.00	23	0.130
28	A	11	3	1.00	23	0.130
29	A	11	3	1.00	21	0.143
30	A	53	7	1.00	23	0.304
31	A	60	9	1.00	23	0.391
32	A	0	0	0.00	0	0.000
33	A	0	0	0.00	0	0.000
34	A	2	2	1.00	23	0.087
35	A	4	2	1.00	23	0.087
36	A	5	2	1.00	23	0.087
37	A	8	7	1.00	18	0.389
38	A	7	6	1.00	18	0.333
39	A	6	5	1.00	16	0.312
40	A	0	0	0.00	0	0.000
41	A	0	0	0.00	0	0.000
42	A	15	9	1.00	20	0.450
43	A	13	10	1.00	20	0.500
44	A	9	7	1.00	18	0.389
45	A	0	0	0.00	0	0.000
46	A	0	0	0.00	0	0.000
47	A	28	11	1.00	20	0.550
48	A	22	11	1.00	20	0.550
49	A	16	9	1.00	18	0.500
50	A	0	0	0.00	0	0.000
51	A	0	0	0.00	0	0.000
52	A	6	6	1.00	20	0.300
53	A	5	5	1.00	20	0.250
54	A	4	4	1.00	18	0.222
55	A	0	0	0.00	0	0.000
56	A	0	0	0.00	0	0.000
57	A	21	9	1.00	20	0.450
58	A	18	10	1.00	20	0.500
59	A	5	5	1.00	18	0.278
60	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^3 \cot(a+bx)\,dx$

Optimal. Leaf size=101

$$-\frac{ix^4}{4}+\frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{3ix^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{3x\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3i\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}$$

```
[Out] -1/4*I*x^4+x^3*ln(1-exp(2*I*(b*x+a)))/b-3/2*I*x^2*polylog(2,exp(2*I*(b*x+a)
))/b^2+3/2*x*polylog(3,exp(2*I*(b*x+a)))/b^3+3/4*I*polylog(4,exp(2*I*(b*x+a
)))/b^4
```

Rubi [A]
time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3798, 2221, 2611, 6744, 2320, 6724}

$$\frac{3i\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}+\frac{3x\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}-\frac{3ix^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}+\frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{ix^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cot[a + b*x],x]
```

```
[Out] (-1/4*I)*x^4 + (x^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*x^2*PolyLo
g[2, E^((2*I)*(a + b*x))])/b^2 + (3*x*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b
^3) + (((3*I)/4)*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3\cot(a+bx)\,dx &= -\frac{ix^4}{4}-2i\int\frac{e^{2i(a+bx)}x^3}{1-e^{2i(a+bx)}}\,dx\\
&= -\frac{ix^4}{4}+\frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{3\int x^2\log\left(1-e^{2i(a+bx)}\right)\,dx}{b}\\
&= -\frac{ix^4}{4}+\frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{3ix^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}+\frac{(3i)\int x\text{Li}_2\left(e^{2i(a+bx)}\right)\,dx}{b^2}\\
&= -\frac{ix^4}{4}+\frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{3ix^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}+\frac{3x\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}-\frac{3\int \text{Li}_3\left(e^{2i(a+\right.}{2b^3}\\
&= -\frac{ix^4}{4}+\frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{3ix^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}+\frac{3x\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{(3i)\text{Subst}\left(\int\right.}{}\\
&= -\frac{ix^4}{4}+\frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{3ix^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}+\frac{3x\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{3i\text{Li}_4\left(e^{2i(a+bx\right.}{4b^4}
\end{aligned}$$

Mathematica [A]

time = 0.55, size = 98, normalized size = 0.97

$$-\frac{i\left(b^4x^4+4ib^3x^3\log\left(1-e^{2i(a+bx)}\right)+6b^2x^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)+6ibx\text{PolyLog}\left(3,e^{2i(a+bx)}\right)-3\text{PolyLog}\left(4,e^{2i(a+bx)}\right)\right)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cot[a + b*x],x]
```

```
[Out] ((-1/4*I)*(b^4*x^4 + (4*I)*b^3*x^3*Log[1 - E^((2*I)*(a + b*x))] + 6*b^2*x^2
*PolyLog[2, E^((2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[3, E^((2*I)*(a + b*x))
] - 3*PolyLog[4, E^((2*I)*(a + b*x))]))/b^4
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(82) = 164$.
time = 0.30, size = 240, normalized size = 2.38

method	result
risch	$-\frac{i x^4}{4}-\frac{2 i a^3 x}{b^3}-\frac{3 i \operatorname{polylog}\left(2, \mathrm{e}^{i(b x+a)}\right) x^2}{b^2}-\frac{a^3 \ln \left(\mathrm{e}^{i(b x+a)}-1\right)}{b^4}+\frac{2 a^3 \ln \left(\mathrm{e}^{i(b x+a)}\right)}{b^4}+\frac{6 \operatorname{polylog}\left(3, \mathrm{e}^{i(b x+a)}\right) x}{b^3}+\frac{6 \operatorname{polylog}(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*x^4-2*I/b^3*a^3*x-3*I/b^2*polylog(2,exp(I*(b*x+a)))*x^2-1/b^4*a^3*ln
(exp(I*(b*x+a))-1)+2/b^4*a^3*ln(exp(I*(b*x+a)))+6/b^3*polylog(3,exp(I*(b*x+
a)))*x+6/b^3*polylog(3,-exp(I*(b*x+a)))*x-3/2*I/b^4*a^4+6*I/b^4*polylog(4,-
exp(I*(b*x+a)))+6*I/b^4*polylog(4,exp(I*(b*x+a)))+1/b*ln(1-exp(I*(b*x+a)))*
x^3-3*I/b^2*polylog(2,-exp(I*(b*x+a)))*x^2+1/b^4*ln(1-exp(I*(b*x+a)))*a^3+1
/b*ln(exp(I*(b*x+a))+1)*x^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(78) = 156$.
time = 0.36, size = 391, normalized size = 3.87

$$-\frac{i (bx+a)^4 - 4i (bx+a)^3 a + 6i (bx+a)^2 a^2 + 4 a^3 \log(\sin(bx+a)) - 24 bx \mathrm{Li}_3(-e^{(ibx+ia)}) - 24 bx \mathrm{Li}_3(e^{(ibx+ia)}) + 4(-i(bx+a)^3 + 3i(bx+a)^2 a - 3i(bx+a)a^2) \arctan(\sin(bx+a), \cos(bx+a)+1) + 4(i(bx+a)^3 - 3i(bx+a)^2 a + 3i(bx+a)a^2) \arctan(\sin(bx+a), -\cos(bx+a)+1) + 12(i(bx+a)^2 - 2i(bx+a)a + ia^2)\mathrm{Li}_2(-e^{(ibx+ia)}) + 12(i(bx+a)^2 - 2i(bx+a)a + ia^2)\mathrm{Li}_2(e^{(ibx+ia)}) - 2((bx+a)^3 - 3(bx+a)^2 a + 3(bx+a)a^2)\log(\cos(bx+a)^2 + \sin(bx+a)^2 + 2\cos(bx+a)+1) - 2((bx+a)^3 - 3(bx+a)^2 a + 3(bx+a)a^2)\log(\cos(bx+a)^2 + \sin(bx+a)^2 - 2\cos(bx+a)+1) - 24i \mathrm{Li}_4(-e^{(ibx+ia)}) - 24i \mathrm{Li}_4(e^{(ibx+ia)})}{4 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*(I*(b*x + a)^4 - 4*I*(b*x + a)^3*a + 6*I*(b*x + a)^2*a^2 + 4*a^3*log(s
in(b*x + a)) - 24*b*x*polylog(3, -e^(I*b*x + I*a)) - 24*b*x*polylog(3, e^(I
*b*x + I*a)) + 4*(-I*(b*x + a)^3 + 3*I*(b*x + a)^2*a - 3*I*(b*x + a)*a^2)*a
rctan2(sin(b*x + a), cos(b*x + a) + 1) + 4*(I*(b*x + a)^3 - 3*I*(b*x + a)^2
*a + 3*I*(b*x + a)*a^2)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 12*(I*(b
*x + a)^2 - 2*I*(b*x + a)*a + I*a^2)*dilog(-e^(I*b*x + I*a)) + 12*(I*(b*x +
 a)^2 - 2*I*(b*x + a)*a + I*a^2)*dilog(e^(I*b*x + I*a)) - 2*((b*x + a)^3 -
3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*
cos(b*x + a) + 1) - 2*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*log
(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 24*I*polylog(4, -e
^(I*b*x + I*a)) - 24*I*polylog(4, e^(I*b*x + I*a)))/b^4
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(78) = 156$.
time = 3.52, size = 306, normalized size = 3.03

$$\frac{-6i\,b^2x^2\mathrm{Li}_2(\cos(2\,bx+2\,a)+i\,\sin(2\,bx+2\,a))+6i\,b^2x^2\mathrm{Li}_2(\cos(2\,bx+2\,a)-i\,\sin(2\,bx+2\,a))-4\,a^3\log\left(-\frac{1}{2}\,\cos(2\,bx+2\,a)+\frac{1}{2}i\,\sin(2\,bx+2\,a)+\frac{1}{2}\right)-4\,a^3\log\left(-\frac{1}{2}\,\cos(2\,bx+2\,a)-\frac{1}{2}i\,\sin(2\,bx+2\,a)+\frac{1}{2}\right)+6\,bx\mathrm{polylog}(3,\cos(2\,bx+2\,a)+i\,\sin(2\,bx+2\,a))+6\,bx\mathrm{polylog}(3,\cos(2\,bx+2\,a)-i\,\sin(2\,bx+2\,a))+4\,(b^3x^3+a^3)\log(-\cos(2\,bx+2\,a)+i\,\sin(2\,bx+2\,a)+1)+4\,(b^3x^3+a^3)\log(-\cos(2\,bx+2\,a)-i\,\sin(2\,bx+2\,a)+1)+3i\,\mathrm{polylog}(4,\cos(2\,bx+2\,a)+i\,\sin(2\,bx+2\,a))-3i\,\mathrm{polylog}(4,\cos(2\,bx+2\,a)-i\,\sin(2\,bx+2\,a))}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(-6*I*b^2*x^2*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + 6*I*b^2*x^
2*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) - 4*a^3*log(-1/2*cos(2*b*x +
 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) - 4*a^3*log(-1/2*cos(2*b*x + 2*a) - 1
/2*I*sin(2*b*x + 2*a) + 1/2) + 6*b*x*polylog(3, cos(2*b*x + 2*a) + I*sin(2*
b*x + 2*a)) + 6*b*x*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 4*(
b^3*x^3 + a^3)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 4*(b^3*x^3
 + a^3)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 3*I*polylog(4, co
s(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - 3*I*polylog(4, cos(2*b*x + 2*a) - I*
sin(2*b*x + 2*a)))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cot(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cot(b*x+a),x)
```

```
[Out] Integral(x**3*cot(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*cot(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{cot}\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(a + b*x),x)
```

```
[Out] int(x^3*cot(a + b*x), x)
```

3.2 $\int x^2 \cot(a+bx)\,dx$

Optimal. Leaf size=74

$$-\frac{ix^3}{3}+\frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{ix\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}+\frac{\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}$$

```
[Out] -1/3*I*x^3+x^2*ln(1-exp(2*I*(b*x+a)))/b-I*x*polylog(2,exp(2*I*(b*x+a)))/b^2
+1/2*polylog(3,exp(2*I*(b*x+a)))/b^3
```

Rubi [A]
time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3798, 2221, 2611, 2320, 6724}

$$\frac{\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}-\frac{ix\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{ix^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cot[a + b*x],x]
```

```
[Out] (-1/3*I)*x^3 + (x^2*Log[1 - E^((2*I)*(a + b*x))])/b - (I*x*PolyLog[2, E^((2
*I)*(a + b*x))])/b^2 + PolyLog[3, E^((2*I)*(a + b*x))]/(2*b^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \cot(a+bx)\,dx &= -\frac{ix^3}{3} - 2i\int \frac{e^{2i(a+bx)}x^2}{1-e^{2i(a+bx)}}\,dx \\
&= -\frac{ix^3}{3} + \frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2\int x\log\left(1-e^{2i(a+bx)}\right)\,dx}{b} \\
&= -\frac{ix^3}{3} + \frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{ix\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{i\int \text{Li}_2\left(e^{2i(a+bx)}\right)\,dx}{b^2} \\
&= -\frac{ix^3}{3} + \frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{ix\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x}\,dx, x, e^{2i(a+bx)}\right)}{2b^3} \\
&= -\frac{ix^3}{3} + \frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{ix\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}
\end{aligned}
$$

Mathematica [**A**]
time = 0.40, size = 75, normalized size = 1.01

$$\frac{2b^2x^2\left(-ibx+3\log\left(1-e^{2i(a+bx)}\right)\right)-6ibx\text{PolyLog}\left(2,e^{2i(a+bx)}\right)+3\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cot[a + b*x],x]
```

```
[Out] (2*b^2*x^2*((-I)*b*x + 3*Log[1 - E^((2*I)*(a + b*x))]) - (6*I)*b*x*PolyLog[
2, E^((2*I)*(a + b*x))] + 3*PolyLog[3, E^((2*I)*(a + b*x))])/(6*b^3)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(62) = 124.
time = 0.24, size = 198, normalized size = 2.68

method	result
risch	$-\frac{i x^3}{3}+\frac{2 i a^2 x}{b^2}-\frac{\ln\left(1-e^{i(bx+a)}\right) a^2}{b^3}+\frac{\ln\left(1-e^{i(bx+a)}\right) x^2}{b}-\frac{2 i \operatorname{polylog}\left(2, e^{i(bx+a)}\right) x}{b^2}+\frac{2 \operatorname{polylog}\left(3, e^{i(bx+a)}\right)}{b^3}+\frac{\ln\left(e^{i(bx+a)}\ldots\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I*x^3+2*I/b^2*a^2*x-1/b^3*ln(1-exp(I*(b*x+a)))*a^2+1/b*ln(1-exp(I*(b*x
+a)))*x^2-2*I/b^2*polylog(2,exp(I*(b*x+a)))*x+2/b^3*polylog(3,exp(I*(b*x+a)
))+1/b*ln(exp(I*(b*x+a))+1)*x^2-2*I/b^2*polylog(2,-exp(I*(b*x+a)))*x+2/b^3*
polylog(3,-exp(I*(b*x+a)))+1/b^3*a^2*ln(exp(I*(b*x+a))-1)-2/b^3*a^2*ln(exp(
I*(b*x+a)))+4/3*I/b^3*a^3
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(59) = 118$.
time = 0.34, size = 257, normalized size = 3.47

$$-\frac{2i\,(bx+a)^3-6i\,(bx+a)^2a+12i\,bx\mathrm{Li}_2\left(-e^{(ibx+ia)}\right)+12i\,bx\mathrm{Li}_2\left(e^{(ibx+ia)}\right)-6\,a^2\log\left(\sin\left(bx+a\right)\right)+6\left(-i\,(bx+a)^2+2i\,(bx+a)a\right)\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)+1\right)+6\left(i\,(bx+a)^2-2i\,(bx+a)a\right)\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right)-3\left((bx+a)^2-2\,(bx+a)a\right)\log\left(\cos\left(bx+a\right)^2+\sin\left(bx+a\right)^2+2\,\cos\left(bx+a\right)+1\right)-3\left((bx+a)^2-2\,(bx+a)a\right)\log\left(\cos\left(bx+a\right)^2+\sin\left(bx+a\right)^2-2\,\cos\left(bx+a\right)+1\right)-12\,\mathrm{Li}_3\left(-e^{(ibx+ia)}\right)-12\,\mathrm{Li}_3\left(e^{(ibx+ia)}\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(2*I*(b*x + a)^3 - 6*I*(b*x + a)^2*a + 12*I*b*x*dilog(-e^(I*b*x + I*a)
) + 12*I*b*x*dilog(e^(I*b*x + I*a)) - 6*a^2*log(sin(b*x + a)) + 6*(-I*(b*x
+ a)^2 + 2*I*(b*x + a)*a)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 6*(I*(b
*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 3*(
(b*x + a)^2 - 2*(b*x + a)*a)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*
x + a) + 1) - 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 - 2*cos(b*x + a) + 1) - 12*polylog(3, -e^(I*b*x + I*a)) - 12*polylog
(3, e^(I*b*x + I*a)))/b^3
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(59) = 118$.
time = 3.29, size = 244, normalized size = 3.30

$$\frac{-2i\,bx\mathrm{Li}_2(\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right))+2i\,bx\mathrm{Li}_2(\cos\left(2\,bx+2\,a\right)-i\,\sin\left(2\,bx+2\,a\right))+2\,a^2\log\left(-\frac{1}{2}\,\cos\left(2\,bx+2\,a\right)+\frac{1}{2}i\,\sin\left(2\,bx+2\,a\right)+\frac{1}{2}\right)+2\,a^2\log\left(-\frac{1}{2}\,\cos\left(2\,bx+2\,a\right)-\frac{1}{2}i\,\sin\left(2\,bx+2\,a\right)+\frac{1}{2}\right)+2\,(b^2x^2-a^2)\log\left(-\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right)+1\right)+2\,(b^2x^2-a^2)\log\left(-\cos\left(2\,bx+2\,a\right)-i\,\sin\left(2\,bx+2\,a\right)+1\right)+\mathrm{polylog}(3,\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right))+\mathrm{polylog}(3,\cos\left(2\,bx+2\,a\right)-i\,\sin\left(2\,bx+2\,a\right))}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(-2*I*b*x*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + 2*I*b*x*dilog(
cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 2*a^2*log(-1/2*cos(2*b*x + 2*a) +
1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*a^2*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin
(2*b*x + 2*a) + 1/2) + 2*(b^2*x^2 - a^2)*log(-cos(2*b*x + 2*a) + I*sin(2*b*
```

```
x + 2*a) + 1) + 2*(b^2*x^2 - a^2)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a
) + 1) + polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + polylog(3, cos
(2*b*x + 2*a) - I*sin(2*b*x + 2*a)))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cot{(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cot(b*x+a),x)
```

```
[Out] Integral(x**2*cot(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*cot(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{cot}\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(a + b*x),x)
```

```
[Out] int(x^2*cot(a + b*x), x)
```

3.3 $\int x \cot(a+bx)\,dx$

Optimal. Leaf size=53

$$-\frac{ix^2}{2}+\frac{x\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}$$

```
[Out] -1/2*I*x^2+x*ln(1-exp(2*I*(b*x+a)))/b-1/2*I*polylog(2,exp(2*I*(b*x+a)))/b^2
```

Rubi [A]
time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3798, 2221, 2317, 2438}

$$-\frac{i\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}+\frac{x\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{ix^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cot[a + b*x],x]
```

```
[Out] (-1/2*I)*x^2 + (x*Log[1 - E^((2*I)*(a + b*x))])/b - ((I/2)*PolyLog[2, E^((2
*I)*(a + b*x))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
```

```
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\cot(a+bx)\,dx &= -\frac{ix^2}{2} - 2i\int \frac{e^{2i(a+bx)}x}{1-e^{2i(a+bx)}}\,dx \\
&= -\frac{ix^2}{2} + \frac{x\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{\int \log\left(1-e^{2i(a+bx)}\right)\,dx}{b} \\
&= -\frac{ix^2}{2} + \frac{x\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i(a+bx)}\right)}{2b^2} \\
&= -\frac{ix^2}{2} + \frac{x\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{i\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 135 vs. $2(53) = 106$.
time = 3.94, size = 135, normalized size = 2.55

$$
\frac{1}{2}\left(x^2\cot(a) - \frac{-ibx(\pi - 2\text{ArcTan}(\tan(a))) - \pi\log\left(1+e^{-2ibx}\right) - 2(bx+\text{ArcTan}(\tan(a)))\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right) + \pi\log(\cos(bx)) + 2\text{ArcTan}(\tan(a))\log(\sin(bx+\text{ArcTan}(\tan(a)))) + i\text{PolyLog}\left(2, e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)}{b^2} - e^{i\text{ArcTan}(\tan(a))}x^2\cot(a)\sqrt{\sec^2(a)}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cot[a + b*x],x]
```

```
[Out] (x^2*Cot[a] - ((-I)*b*x*(Pi - 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)
] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi
*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLo
g[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])/b^2 - E^(I*ArcTan[Tan[a]])*x^2*Cot[
a]*Sqrt[Sec[a]^2])/2
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(43) = 86$.
time = 0.23, size = 150, normalized size = 2.83

method	result
risch	$-\frac{ix^2}{2} - \frac{2ixa}{b} - \frac{ia^2}{b^2} + \frac{\ln\left(1-e^{i(bx+a)}\right)x}{b} + \frac{\ln\left(1-e^{i(bx+a)}\right)a}{b^2} - \frac{i\,\text{polylog}\left(2, e^{i(bx+a)}\right)}{b^2} + \frac{\ln\left(e^{i(bx+a)}+1\right)x}{b} - \frac{i\,\text{polylog}(2, -e^{i(}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*x^2-2*I/b*x*a-I/b^2*a^2+1/b*ln(1-exp(I*(b*x+a)))*x+1/b^2*ln(1-exp(I*
(b*x+a)))*a-I/b^2*polylog(2,exp(I*(b*x+a)))+1/b*ln(exp(I*(b*x+a))+1)*x-I/b^
```

```
2*polylog(2,-exp(I*(b*x+a)))-1/b^2*a*ln(exp(I*(b*x+a))-1)+2/b^2*a*ln(exp(I*
(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(40) = 80$.
time = 0.34, size = 140, normalized size = 2.64

$$\frac{-i\,b^2x^2 + 2i\,bx\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)+1\right) - 2i\,bx\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right) + bx\log\left(\cos\left(bx+a\right)^2+\sin\left(bx+a\right)^2+2\,\cos\left(bx+a\right)+1\right) + bx\log\left(\cos\left(bx+a\right)^2+\sin\left(bx+a\right)^2-2\,\cos\left(bx+a\right)+1\right) - 2i\,\mathrm{Li}_2\left(-e^{(i\,bx+i\,a)}\right) - 2i\,\mathrm{Li}_2\left(e^{(i\,bx+i\,a)}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*b^2*x^2 + 2*I*b*x*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*I*b*x
*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + b*x*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) + b*x*log(cos(b*x + a)^2 + sin(b*x + a)^2 -
2*cos(b*x + a) + 1) - 2*I*dilog(-e^(I*b*x + I*a)) - 2*I*dilog(e^(I*b*x + I*
a)))/b^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(40) = 80$.
time = 4.12, size = 174, normalized size = 3.28

$$-\frac{2\,a\log\left(-\frac{1}{2}\,\cos\left(2\,bx+2\,a\right)+\frac{1}{2}i\,\sin\left(2\,bx+2\,a\right)+\frac{1}{2}\right) + 2\,a\log\left(-\frac{1}{2}\,\cos\left(2\,bx+2\,a\right)-\frac{1}{2}i\,\sin\left(2\,bx+2\,a\right)+\frac{1}{2}\right) - 2\left(bx+a\right)\log\left(-\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right)+1\right) - 2\left(bx+a\right)\log\left(-\cos\left(2\,bx+2\,a\right)-i\,\sin\left(2\,bx+2\,a\right)+1\right) + i\,\mathrm{Li}_2(\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right)) - i\,\mathrm{Li}_2(\cos\left(2\,bx+2\,a\right)-i\,\sin\left(2\,bx+2\,a\right))}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*a*l
og(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) - 2*(b*x + a)*log(
-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) - 2*(b*x + a)*log(-cos(2*b*x +
2*a) - I*sin(2*b*x + 2*a) + 1) + I*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2
*a)) - I*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\cot\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a),x)
```

```
[Out] Integral(x*cot(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cot(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\cot(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(a + b*x),x)
```

```
[Out] int(x*cot(a + b*x), x)
```

3.4 $\int \frac{\cot(a+bx)}{x}\,dx$

Optimal. Leaf size=13

$$\operatorname{Int}\left(\frac{\cot(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)/x,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot(a+bx)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[a + b*x]/x,x]
[Out] Defer[Int][Cot[a + b*x]/x, x]
```

Rubi steps

$$\int \frac{\cot(a+bx)}{x}\,dx = \int \frac{\cot(a+bx)}{x}\,dx$$

Mathematica [A]
time = 1.86, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[a + b*x]/x,x]
[Out] Integrate[Cot[a + b*x]/x, x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(bx+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)/x,x)
```

```
[Out] int(cot(b*x+a)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(cot(b*x + a)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(a + bx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x,x)
```

```
[Out] Integral(cot(a + b*x)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\cot(a + b\,x)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)/x,x)
```

```
[Out] int(cot(a + b*x)/x, x)
```

3.5 $\int \frac{\cot(a+bx)}{x^2}\,dx$

Optimal. Leaf size=13

$$\operatorname{Int}\left(\frac{\cot(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)/x^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot(a+bx)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[a + b*x]/x^2,x]
```

```
[Out] Defer[Int][Cot[a + b*x]/x^2, x]
```

Rubi steps

$$\int \frac{\cot(a+bx)}{x^2}\,dx = \int \frac{\cot(a+bx)}{x^2}\,dx$$

Mathematica [A]
time = 3.28, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[a + b*x]/x^2,x]
```

```
[Out] Integrate[Cot[a + b*x]/x^2, x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(bx+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)/x^2,x)
```

```
[Out] int(cot(b*x+a)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(cot(b*x + a)/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(a + bx)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x**2,x)
```

```
[Out] Integral(cot(a + b*x)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\cot(a + b\,x)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)/x^2,x)
```

```
[Out] int(cot(a + b*x)/x^2, x)
```

3.6 $\int x^3 \cot^2(a+bx)\,dx$

Optimal. Leaf size=97

$$-\frac{ix^3}{b}-\frac{x^4}{4}-\frac{x^3\cot(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{3ix\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{3\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^4}$$

```
[Out] -I*x^3/b-1/4*x^4-x^3*cot(b*x+a)/b+3*x^2*ln(1-exp(2*I*(b*x+a)))/b^2-3*I*x*po
lylog(2,exp(2*I*(b*x+a)))/b^3+3/2*polylog(3,exp(2*I*(b*x+a)))/b^4
```

Rubi [A]

time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3801, 3798, 2221, 2611, 2320, 6724, 30}

$$\frac{3\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3ix\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}+\frac{3x^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{x^3\cot(a+bx)}{b}-\frac{ix^3}{b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cot[a + b*x]^2,x]
```

```
[Out] ((-I)*x^3)/b - x^4/4 - (x^3*Cot[a + b*x])/b + (3*x^2*Log[1 - E^((2*I)*(a +
b*x))])/b^2 - ((3*I)*x*PolyLog[2, E^((2*I)*(a + b*x))])/b^3 + (3*PolyLog[3,
 E^((2*I)*(a + b*x))])/(2*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \cot^2(a+bx)\,dx = -\frac{x^3\cot(a+bx)}{b} + \frac{3\int x^2\cot(a+bx)\,dx}{b} - \int x^3\,dx$$

$$= -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3\cot(a+bx)}{b} - \frac{(6i)\int \frac{e^{2i(a+bx)}x^2}{1-e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3\cot(a+bx)}{b} + \frac{3x^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{6\int x\log\left(1-e^{2i(a+bx)}\right)\,dx}{b^2}$$

$$= -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3\cot(a+bx)}{b} + \frac{3x^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{3ix\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3} + \frac{(3i)\int \text{Li}$$

$$= -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3\cot(a+bx)}{b} + \frac{3x^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{3ix\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3} + \frac{3\text{Subst}\big(}{}$$

$$= -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3\cot(a+bx)}{b} + \frac{3x^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{3ix\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3} + \frac{3\text{Li}_3\left(e^{2i}\right.}{2b}$$

Mathematica [**A**]
time = 1.26, size = 120, normalized size = 1.24

$$-\frac{x^4}{4}+\frac{2b^2x^2\left(-\frac{2ibe^{2ia}x}{-1+e^{2ia}}+3\log\left(1-e^{2i(a+bx)}\right)\right)-6ibx\text{PolyLog}\left(2,e^{2i(a+bx)}\right)+3\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^4}+\frac{x^3\csc(a)\csc(a+bx)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cot[a + b*x]^2,x]
```

```
[Out] -1/4*x^4 + (2*b^2*x^2*(((-2*I)*b*E^((2*I)*a)*x)/(-1 + E^((2*I)*a)) + 3*Log[
1 - E^((2*I)*(a + b*x))]) - (6*I)*b*x*PolyLog[2, E^((2*I)*(a + b*x))] + 3*P
olyLog[3, E^((2*I)*(a + b*x))])/(2*b^4) + (x^3*Csc[a]*Csc[a + b*x]*Sin[b*x]
)/b
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(85) = 170.
time = 0.38, size = 231, normalized size = 2.38

method	result
risch	$-\frac{x^4}{4}-\frac{2ix^3}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)}-\frac{3\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a^2}{b^4}+\frac{6ia^2x}{b^3}+\frac{3\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x^2}{b^2}-\frac{6i\,\mathrm{polylog}\left(2,\mathrm{e}^{i(bx+a)}\right)x}{b^3}+\frac{6\,\mathrm{polylog}\left(3,\mathrm{e}^{i(b}\right.}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^4-2*I*x^3/b/(exp(2*I*(b*x+a))-1)-3/b^4*ln(1-exp(I*(b*x+a)))*a^2+6*I/
b^3*a^2*x+3/b^2*ln(1-exp(I*(b*x+a)))*x^2-6*I/b^3*polylog(2,exp(I*(b*x+a)))*
x+6/b^4*polylog(3,exp(I*(b*x+a)))+3/b^2*ln(exp(I*(b*x+a))+1)*x^2-6*I/b^3*po
lylog(2,-exp(I*(b*x+a)))*x+6/b^4*polylog(3,-exp(I*(b*x+a)))+3/b^4*a^2*ln(ex
p(I*(b*x+a))-1)-6/b^4*a^2*ln(exp(I*(b*x+a)))-2*I/b*x^3+4*I/b^4*a^3
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 952 vs. 2(82) = 164.
time = 0.59, size = 952, normalized size = 9.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a + 1/tan(b*x + a))*a^3 - 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 +
 (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a
)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(
b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^2/(cos(2
```

```
*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1) + 2*(-I*(b*x +
 a)^4 + 4*I*(b*x + a)^3*a - 12*((b*x + a)^2 - 2*(b*x + a)*a - ((b*x + a)^2
- 2*(b*x + a)*a)*cos(2*b*x + 2*a) + (-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*sin(
2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 12*((b*x + a)^2 - 2
*(b*x + a)*a - ((b*x + a)^2 - 2*(b*x + a)*a)*cos(2*b*x + 2*a) - (I*(b*x + a
)^2 - 2*I*(b*x + a)*a)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a
) + 1) + (I*(b*x + a)^4 - 4*(b*x + a)^3*(I*a + 2) + 24*(b*x + a)^2*a)*cos(2
*b*x + 2*a) - 24*(b*x*cos(2*b*x + 2*a) + I*b*x*sin(2*b*x + 2*a) - b*x)*dilo
g(-e^(I*b*x + I*a)) - 24*(b*x*cos(2*b*x + 2*a) + I*b*x*sin(2*b*x + 2*a) - b
*x)*dilog(e^(I*b*x + I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a + (I*(b*x
+ a)^2 - 2*I*(b*x + a)*a)*cos(2*b*x + 2*a) - ((b*x + a)^2 - 2*(b*x + a)*a)*
sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
 - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a + (I*(b*x + a)^2 - 2*I*(b*x + a)*a)*
cos(2*b*x + 2*a) - ((b*x + a)^2 - 2*(b*x + a)*a)*sin(2*b*x + 2*a))*log(cos(
b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 24*(I*cos(2*b*x + 2*a)
- sin(2*b*x + 2*a) - I)*polylog(3, -e^(I*b*x + I*a)) - 24*(I*cos(2*b*x + 2*
a) - sin(2*b*x + 2*a) - I)*polylog(3, e^(I*b*x + I*a)) - ((b*x + a)^4 - 4*(
b*x + a)^3*(a - 2*I) - 24*I*(b*x + a)^2*a)*sin(2*b*x + 2*a))/(-4*I*cos(2*b*
x + 2*a) + 4*sin(2*b*x + 2*a) + 4*I))/b^4
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(82) = 164$.
time = 3.26, size = 372, normalized size = 3.84

$$-\frac{b^4x^4\sin(2bx+2a)+4b^3x^3\cos(2bx+2a)+4b^3x^3+6ibx\mathrm{Li}_2(\cos(2bx+2a)+i\sin(2bx+2a))\sin(2bx+2a)-6ibx\mathrm{Li}_2(\cos(2bx+2a)-i\sin(2bx+2a))\sin(2bx+2a)-6a^2\log\left(-\frac{1}{2}\cos(2bx+2a)+\frac{1}{2}i\sin(2bx+2a)+\frac{1}{2}\right)\sin(2bx+2a)-6a^2\log\left(-\frac{1}{2}\cos(2bx+2a)-\frac{1}{2}i\sin(2bx+2a)+\frac{1}{2}\right)\sin(2bx+2a)-6(b^2x^2-a^2)\log(-\cos(2bx+2a)+i\sin(2bx+2a)+1)\sin(2bx+2a)-6(b^2x^2-a^2)\log(-\cos(2bx+2a)-i\sin(2bx+2a)+1)\sin(2bx+2a)-3\,\mathrm{polylog}(3,\cos(2bx+2a)+i\sin(2bx+2a))\sin(2bx+2a)-3\,\mathrm{polylog}(3,\cos(2bx+2a)-i\sin(2bx+2a))\sin(2bx+2a)}{4b^4\sin(2bx+2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4*sin(2*b*x + 2*a) + 4*b^3*x^3*cos(2*b*x + 2*a) + 4*b^3*x^3 + 6
*I*b*x*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 6*I*
b*x*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 6*a^2*l
og(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) -
 6*a^2*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x
+ 2*a) - 6*(b^2*x^2 - a^2)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)*
sin(2*b*x + 2*a) - 6*(b^2*x^2 - a^2)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x +
2*a) + 1)*sin(2*b*x + 2*a) - 3*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x +
2*a))*sin(2*b*x + 2*a) - 3*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)
)*sin(2*b*x + 2*a))/(b^4*sin(2*b*x + 2*a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cot(b*x+a)**2,x)
```

```
[Out] Integral(x**3*cot(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cot(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \cot(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(a + b*x)^2,x)
```

```
[Out] int(x^3*cot(a + b*x)^2, x)
```

3.7 $\int x^2 \cot^2(a+bx)\,dx$

Optimal. Leaf size=74

$$-\frac{ix^2}{b}-\frac{x^3}{3}-\frac{x^2\cot(a+bx)}{b}+\frac{2x\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{i\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}$$

```
[Out] -I*x^2/b-1/3*x^3-x^2*cot(b*x+a)/b+2*x*ln(1-exp(2*I*(b*x+a)))/b^2-I*polylog(
2,exp(2*I*(b*x+a)))/b^3
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3801, 3798, 2221, 2317, 2438, 30}

$$-\frac{i\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}+\frac{2x\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{x^2\cot(a+bx)}{b}-\frac{ix^2}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cot[a + b*x]^2,x]
```

```
[Out] ((-I)*x^2)/b - x^3/3 - (x^2*Cot[a + b*x])/b + (2*x*Log[1 - E^((2*I)*(a + b*
x))])/b^2 - (I*PolyLog[2, E^((2*I)*(a + b*x))])/b^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \cot^2(a+bx)\,dx &= -\frac{x^2\cot(a+bx)}{b}+\frac{2\int x\cot(a+bx)\,dx}{b}-\int x^2\,dx\\
&= -\frac{ix^2}{b}-\frac{x^3}{3}-\frac{x^2\cot(a+bx)}{b}-\frac{(4i)\int\frac{e^{2i(a+bx)}x}{1-e^{2i(a+bx)}}\,dx}{b}\\
&= -\frac{ix^2}{b}-\frac{x^3}{3}-\frac{x^2\cot(a+bx)}{b}+\frac{2x\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{2\int\log\left(1-e^{2i(a+bx)}\right)\,dx}{b^2}\\
&= -\frac{ix^2}{b}-\frac{x^3}{3}-\frac{x^2\cot(a+bx)}{b}+\frac{2x\log\left(1-e^{2i(a+bx)}\right)}{b^2}+\frac{i\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2i(a}\right.}{b^3}\\
&= -\frac{ix^2}{b}-\frac{x^3}{3}-\frac{x^2\cot(a+bx)}{b}+\frac{2x\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{i\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}
\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 181 vs. $2(74)=148$.
time = 6.16, size = 181, normalized size = 2.45

$$-\frac{x^3}{3}+\frac{x^2\csc(a)\csc(a+bx)\sin(bx)}{b}-\frac{\csc(a)\sec(a)\left(b^2e^{i\text{ArcTan}(\tan(a))}x^2+\frac{\left(ibx(-\pi+2\text{ArcTan}(\tan(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx+\text{ArcTan}(\tan(a)))\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)+\pi\log(\cos(bx))+2\text{ArcTan}(\tan(a))\log(\sin(bx+\text{ArcTan}(\tan(a))))+i\text{PolyLog}\left(2,e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)\right)\tan(a)}{\sqrt{1+\tan^2(a)}}\right)}{b^3\sqrt{\sec^2(a)\left(\cos^2(a)+\sin^2(a)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cot[a + b*x]^2,x]
```

```
[Out] -1/3*x^3 + (x^2*Csc[a]*Csc[a + b*x]*Sin[b*x])/b - (Csc[a]*Sec[a]*(b^2*E^(I*
ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*
I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]])
)] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I
```

```
*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))
/(b^3*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(66) = 132$.
time = 0.32, size = 183, normalized size = 2.47

method	result
risch	$-\frac{x^3}{3} - \frac{2ix^2}{b(e^{2i(bx+a)}-1)} - \frac{2ix^2}{b} - \frac{4ixa}{b^2} - \frac{2ia^2}{b^3} + \frac{2\ln(1-e^{i(bx+a)})x}{b^2} + \frac{2\ln(1-e^{i(bx+a)})a}{b^3} - \frac{2i\,\mathrm{polylog}(2,e^{i(bx+a)})}{b^3} + 2\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3-2*I*x^2/b/(exp(2*I*(b*x+a))-1)-2*I/b*x^2-4*I/b^2*x*a-2*I/b^3*a^2+2
/b^2*ln(1-exp(I*(b*x+a)))*x+2/b^3*ln(1-exp(I*(b*x+a)))*a-2*I/b^3*polylog(2,
exp(I*(b*x+a)))+2/b^2*ln(exp(I*(b*x+a))+1)*x-2*I/b^3*polylog(2,-exp(I*(b*x+
a)))-2/b^3*a*ln(exp(I*(b*x+a))-1)+4/b^3*a*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(63) = 126$.
time = 0.58, size = 386, normalized size = 5.22

$$\frac{-i b^3x^3 + 6(bx\cos(2bx+2a) + ibx\sin(2bx+2a) - bx)\arctan(\sin(bx+a), \cos(bx+a)+1) - 6(bx\cos(2bx+2a) + ibx\sin(2bx+2a) - bx)\arctan(\sin(bx+a), -\cos(bx+a)+1) + (ib^3x^3 - 6b^2x^2)\cos(2bx+2a) - 6(\cos(2bx+2a) + i\sin(2bx+2a) - 1)\mathrm{Li}_2(-e^{(ibx+ia)}) - 6(\cos(2bx+2a) + i\sin(2bx+2a) - 1)\mathrm{Li}_2(e^{(ibx+ia)}) - 3(ibx\cos(2bx+2a) - bx\sin(2bx+2a) - ibx)\log(\cos(bx+a)^2 + \sin(bx+a)^2 + 2\cos(bx+a) + 1) - 3(ibx\cos(2bx+2a) - bx\sin(2bx+2a) - ibx)\log(\cos(bx+a)^2 + \sin(bx+a)^2 - 2\cos(bx+a) + 1) - (b^3x^3 + 6ib^2x^2)\sin(2bx+2a)}{-3ib^3\cos(2bx+2a) + 3b^3\sin(2bx+2a) + 3ib^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (-I*b^3*x^3 + 6*(b*x*cos(2*b*x + 2*a) + I*b*x*sin(2*b*x + 2*a) - b*x)*arcta
n2(sin(b*x + a), cos(b*x + a) + 1) - 6*(b*x*cos(2*b*x + 2*a) + I*b*x*sin(2*
b*x + 2*a) - b*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (I*b^3*x^3 - 6
*b^2*x^2)*cos(2*b*x + 2*a) - 6*(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) - 1)*
dilog(-e^(I*b*x + I*a)) - 6*(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) - 1)*dil
og(e^(I*b*x + I*a)) - 3*(I*b*x*cos(2*b*x + 2*a) - b*x*sin(2*b*x + 2*a) - I*
b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 3*(I*b*x*c
os(2*b*x + 2*a) - b*x*sin(2*b*x + 2*a) - I*b*x)*log(cos(b*x + a)^2 + sin(b*
x + a)^2 - 2*cos(b*x + a) + 1) - (b^3*x^3 + 6*I*b^2*x^2)*sin(2*b*x + 2*a))/
(-3*I*b^3*cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) + 3*I*b^3)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(63) = 126$.
time = 3.45, size = 281, normalized size = 3.80

$$-\frac{2b^3x^3\sin(2bx+2a) + 6b^2x^2\cos(2bx+2a) + 6b^2x^2 + 6a\log\left(-\frac{1}{2}\cos(2bx+2a) + \frac{1}{2}i\sin(2bx+2a) + \frac{1}{2}\right)\sin(2bx+2a) + 6a\log\left(-\frac{1}{2}\cos(2bx+2a) - \frac{1}{2}i\sin(2bx+2a) + \frac{1}{2}\right)\sin(2bx+2a) - 6(bx+a)\log(-\cos(2bx+2a) + i\sin(2bx+2a) + 1)\sin(2bx+2a) - 6(bx+a)\log(-\cos(2bx+2a) - i\sin(2bx+2a) + 1)\sin(2bx+2a) + 3i\,\mathrm{Li}_2(\cos(2bx+2a) + i\sin(2bx+2a))\sin(2bx+2a) - 3i\,\mathrm{Li}_2(\cos(2bx+2a) - i\sin(2bx+2a))\sin(2bx+2a)}{6b^3\sin(2bx+2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*x^3*sin(2*b*x + 2*a) + 6*b^2*x^2*cos(2*b*x + 2*a) + 6*b^2*x^2 +
 6*a*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x +
2*a) + 6*a*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*
b*x + 2*a) - 6*(b*x + a)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)*si
n(2*b*x + 2*a) - 6*(b*x + a)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1
)*sin(2*b*x + 2*a) + 3*I*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2
*b*x + 2*a) - 3*I*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*x +
2*a))/(b^3*sin(2*b*x + 2*a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cot(b*x+a)**2,x)
```

```
[Out] Integral(x**2*cot(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cot(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\cot(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(a + b*x)^2,x)
```

```
[Out] int(x^2*cot(a + b*x)^2, x)
```

3.8 $\int x \cot^2(a + bx)\, dx$

Optimal. Leaf size=31

$$-\frac{x^2}{2} - \frac{x \cot(a + bx)}{b} + \frac{\log(\sin(a + bx))}{b^2}$$

```
[Out] -1/2*x^2-x*cot(b*x+a)/b+ln(sin(b*x+a))/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3801, 3556, 30}

$$\frac{\log(\sin(a + bx))}{b^2} - \frac{x \cot(a + bx)}{b} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cot[a + b*x]^2,x]
```

```
[Out] -1/2*x^2 - (x*Cot[a + b*x])/b + Log[Sin[a + b*x]]/b^2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \int x \cot^2(a + bx)\, dx &= -\frac{x \cot(a + bx)}{b} + \frac{\int \cot(a + bx)\, dx}{b} - \int x\, dx \\ &= -\frac{x^2}{2} - \frac{x \cot(a + bx)}{b} + \frac{\log(\sin(a + bx))}{b^2} \end{aligned}$$

Mathematica [A]
time = 0.24, size = 44, normalized size = 1.42

$$-\frac{x^2}{2}-\frac{x\cot(a)}{b}+\frac{\log(\sin(a+bx))}{b^2}+\frac{x\csc(a)\csc(a+bx)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cot[a + b*x]^2,x]
```

```
[Out] -1/2*x^2 - (x*Cot[a])/b + Log[Sin[a + b*x]]/b^2 + (x*Csc[a]*Csc[a + b*x]*Si
n[b*x])/b
```

Maple [A]
time = 0.31, size = 40, normalized size = 1.29

method	result	size
default	$-\frac{x^2}{2}+\frac{-(bx+a)\cot(bx+a)+\ln(\sin(bx+a))+a\cot(bx+a)}{b^2}$	40
norman	$\frac{-\frac{x}{b}-\frac{x^2\tan(bx+a)}{2}}{\tan(bx+a)}+\frac{\ln(\tan(bx+a))}{b^2}-\frac{\ln(1+\tan^2(bx+a))}{2b^2}$	56
risch	$-\frac{x^2}{2}-\frac{2ix}{b}-\frac{2ia}{b^2}-\frac{2ix}{b(e^{2i(bx+a)}-1)}+\frac{\ln(e^{2i(bx+a)}-1)}{b^2}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2+1/b^2*(-(b*x+a)*cot(b*x+a)+ln(sin(b*x+a))+a*cot(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(29) = 58$.
time = 0.50, size = 269, normalized size = 8.68

$$\frac{2\left(bx+a+\frac{1}{\tan(bx+a)}\right)a-\frac{(bx+a)^2\cos(2bx+2a)^2+(bx+a)^2\sin(2bx+2a)^2-2(bx+a)^2\cos(2bx+2a)+(bx+a)^2-\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2+2\cos(bx+a)+1\right)-\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2-2\cos(bx+a)+1\right)+4(bx+a)\sin(2bx+2a)}{\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a + 1/tan(b*x + a))*a - ((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*
x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2
- (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(co
s(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 +
 sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))/(cos(2*b*x + 2
*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1))/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(29) = 58$.
time = 3.68, size = 75, normalized size = 2.42

$$-\frac{b^2x^2\sin(2bx+2a)+2bx\cos(2bx+2a)+2bx-\log\left(-\frac{1}{2}\cos(2bx+2a)+\frac{1}{2}\right)\sin(2bx+2a)}{2b^2\sin(2bx+2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2*sin(2*b*x + 2*a) + 2*b*x*cos(2*b*x + 2*a) + 2*b*x - log(-1/2*
cos(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a))/(b^2*sin(2*b*x + 2*a))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.
time = 0.23, size = 65, normalized size = 2.10

$$\begin{cases} \tilde{\infty}x^2 & \text{for } a=0 \wedge b=0 \\ \frac{x^2\cot^2(a)}{2} & \text{for } b=0 \\ \tilde{\infty}x^2 & \text{for } a=-bx \\ -\frac{x^2}{2}-\frac{x}{b\tan(a+bx)}-\frac{\log(\tan^2(a+bx)+1)}{2b^2}+\frac{\log(\tan(a+bx))}{b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**2, Eq(a, 0) & Eq(b, 0)), (x**2*cot(a)**2/2, Eq(b, 0)), (z
oo*x**2, Eq(a, -b*x)), (-x**2/2 - x/(b*tan(a + b*x)) - log(tan(a + b*x)**2
+ 1)/(2*b**2) + log(tan(a + b*x))/b**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1250 vs. $2(29) = 58$.
time = 0.84, size = 1250, normalized size = 40.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*x^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*x^2*tan(1/2*b*x)*tan(1/2*a)^2
 - b*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*x^2*tan(1/2*b*x) - b^2*x^2*tan(1/2
*a) + b*x*tan(1/2*b*x)^2 + 4*b*x*tan(1/2*b*x)*tan(1/2*a) - log(16*(tan(1/2*
b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/
2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6
```

```
- 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b
*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan
(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan
(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan
(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a) + b*x*tan(1/2*a)
^2 - log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 +
tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^
6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a
)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1
/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*t
an(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2
 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a)
 + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*
a)^2 - b*x + log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2
*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1
/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*t
an(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)
^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)
^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(
1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*ta
n(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)
+ log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan
(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*t
an(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4
 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*
a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(
1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 -
2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) +
tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*a))/(b^2*tan(1/2
*b*x)^2*tan(1/2*a) + b^2*tan(1/2*b*x)*tan(1/2*a)^2 - b^2*tan(1/2*b*x) - b^2
*tan(1/2*a))
```

Mupad [B]
time = 0.45, size = 54, normalized size = 1.74

$$\frac{\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-1\right)}{b^2}-\frac{x\,2\mathrm{i}}{b}-\frac{x^2}{2}-\frac{x\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(a + b*x)^2,x)
```

```
[Out] log(exp(a*2i)*exp(b*x*2i) - 1)/b^2 - (x*2i)/b - x^2/2 - (x*2i)/(b*(exp(a*2i
 + b*x*2i) - 1))
```

3.9 $\int \frac{\cot^2(a+bx)}{x} dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\cot^2(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^2/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^2(a+bx)}{x} dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[a + b*x]^2/x,x]
```

```
[Out] Defer[Int][Cot[a + b*x]^2/x, x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)}{x} dx = \int \frac{\cot^2(a+bx)}{x} dx$$

Mathematica [A]
time = 3.29, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{x} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^2/x,x]
```

```
[Out] Integrate[Cot[a + b*x]^2/x, x]
```

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(bx+a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2/x,x)
```

```
[Out] int(cot(b*x+a)^2/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -(b*x*cos(2*b*x + 2*a)^2*log(x) + b*x*log(x)*sin(2*b*x + 2*a)^2 - 2*b*x*cos
(2*b*x + 2*a)*log(x) + b*x*log(x) - (b^2*x*cos(2*b*x + 2*a)^2 + b^2*x*sin(2
*b*x + 2*a)^2 - 2*b^2*x*cos(2*b*x + 2*a) + b^2*x)*integrate(sin(b*x + a)/(b
^2*x^2*cos(b*x + a)^2 + b^2*x^2*sin(b*x + a)^2 + 2*b^2*x^2*cos(b*x + a) + b
^2*x^2), x) + (b^2*x*cos(2*b*x + 2*a)^2 + b^2*x*sin(2*b*x + 2*a)^2 - 2*b^2*
x*cos(2*b*x + 2*a) + b^2*x)*integrate(sin(b*x + a)/(b^2*x^2*cos(b*x + a)^2
+ b^2*x^2*sin(b*x + a)^2 - 2*b^2*x^2*cos(b*x + a) + b^2*x^2), x) + 2*sin(2*
b*x + 2*a))/(b*x*cos(2*b*x + 2*a)^2 + b*x*sin(2*b*x + 2*a)^2 - 2*b*x*cos(2*
b*x + 2*a) + b*x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^2/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a + bx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2/x,x)
```

```
[Out] Integral(cot(a + b*x)**2/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^2/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cot(a+b\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/x,x)
```

```
[Out] int(cot(a + b*x)^2/x, x)
```

3.10 $\int \frac{\cot^2(a+bx)}{x^2} dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\cot^2(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^2/x^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^2(a+bx)}{x^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[a + b*x]^2/x^2,x]
```

```
[Out] Defer[Int][Cot[a + b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)}{x^2} dx = \int \frac{\cot^2(a+bx)}{x^2} dx$$

Mathematica [A]
time = 3.41, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{x^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^2/x^2,x]
```

```
[Out] Integrate[Cot[a + b*x]^2/x^2, x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(bx+a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2/x^2,x)
```

```
[Out] int(cot(b*x+a)^2/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] (b*x*cos(2*b*x + 2*a)^2 + b*x*sin(2*b*x + 2*a)^2 - 2*b*x*cos(2*b*x + 2*a) +
 b*x + 2*(b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(2*b*x + 2*a)^2 - 2*b^2*x
^2*cos(2*b*x + 2*a) + b^2*x^2)*integrate(sin(b*x + a)/(b^2*x^3*cos(b*x + a)
^2 + b^2*x^3*sin(b*x + a)^2 + 2*b^2*x^3*cos(b*x + a) + b^2*x^3), x) - 2*(b^
2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(2*b*x + 2*a)^2 - 2*b^2*x^2*cos(2*b*x
 + 2*a) + b^2*x^2)*integrate(sin(b*x + a)/(b^2*x^3*cos(b*x + a)^2 + b^2*x^3
*sin(b*x + a)^2 - 2*b^2*x^3*cos(b*x + a) + b^2*x^3), x) - 2*sin(2*b*x + 2*a
))/(b*x^2*cos(2*b*x + 2*a)^2 + b*x^2*sin(2*b*x + 2*a)^2 - 2*b*x^2*cos(2*b*x
 + 2*a) + b*x^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^2/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2/x**2,x)
```

```
[Out] Integral(cot(a + b*x)**2/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^2/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cot(a+b\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/x^2,x)
```

```
[Out] int(cot(a + b*x)^2/x^2, x)
```

3.11 $\int x^3 \cot^3(a+bx)\,dx$

Optimal. Leaf size=202

$$-\frac{3ix^2}{2b^2}-\frac{x^3}{2b}+\frac{ix^4}{4}-\frac{3x^2\cot(a+bx)}{2b^2}-\frac{x^3\cot^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{3i\text{PolyLo}}{}$$

```
[Out] -3/2*I*x^2/b^2-1/2*x^3/b+1/4*I*x^4-3/2*x^2*cot(b*x+a)/b^2-1/2*x^3*cot(b*x+a
)^2/b+3*x*ln(1-exp(2*I*(b*x+a)))/b^3-x^3*ln(1-exp(2*I*(b*x+a)))/b-3/2*I*pol
ylog(2,exp(2*I*(b*x+a)))/b^4+3/2*I*x^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*
x*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*polylog(4,exp(2*I*(b*x+a)))/b^4
```

Rubi [A]
time = 0.21, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3801, 3798, 2221, 2317, 2438, 30, 2611, 6744, 2320, 6724}

$$-\frac{3i\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3i\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}-\frac{3x\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{3x\log\left(1-e^{2i(a+bx)}\right)}{b^3}+\frac{3ix^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{3x^2\cot(a+bx)}{2b^2}-\frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{x^3\cot^2(a+bx)}{2b}-\frac{3ix^2}{2b^2}-\frac{x^3}{2b}+\frac{ix^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cot[a + b*x]^3,x]
```

```
[Out] (((-3*I)/2)*x^2)/b^2 - x^3/(2*b) + (I/4)*x^4 - (3*x^2*Cot[a + b*x])/(2*b^2)
 - (x^3*Cot[a + b*x]^2)/(2*b) + (3*x*Log[1 - E^((2*I)*(a + b*x))])/b^3 - (x
^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*PolyLog[2, E^((2*I)*(a + b*
x))])/b^4 + (((3*I)/2)*x^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*x*Poly
Log[3, E^((2*I)*(a + b*x))])/(2*b^3) - (((3*I)/4)*PolyLog[4, E^((2*I)*(a +
b*x))])/b^4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3 \cot^3(a+bx)\,dx &= -\frac{x^3\cot^2(a+bx)}{2b}+\frac{3\int x^2\cot^2(a+bx)\,dx}{2b}-\int x^3\cot(a+bx)\,dx\\
&= \frac{ix^4}{4}-\frac{3x^2\cot(a+bx)}{2b^2}-\frac{x^3\cot^2(a+bx)}{2b}+2i\int\frac{e^{2i(a+bx)}x^3}{1-e^{2i(a+bx)}}\,dx+\frac{3\int x\cot(a+bx)}{b^2}\\
&= -\frac{3ix^2}{2b^2}-\frac{x^3}{2b}+\frac{ix^4}{4}-\frac{3x^2\cot(a+bx)}{2b^2}-\frac{x^3\cot^2(a+bx)}{2b}-\frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\\
&= -\frac{3ix^2}{2b^2}-\frac{x^3}{2b}+\frac{ix^4}{4}-\frac{3x^2\cot(a+bx)}{2b^2}-\frac{x^3\cot^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\\
&= -\frac{3ix^2}{2b^2}-\frac{x^3}{2b}+\frac{ix^4}{4}-\frac{3x^2\cot(a+bx)}{2b^2}-\frac{x^3\cot^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\\
&= -\frac{3ix^2}{2b^2}-\frac{x^3}{2b}+\frac{ix^4}{4}-\frac{3x^2\cot(a+bx)}{2b^2}-\frac{x^3\cot^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\\
&= -\frac{3ix^2}{2b^2}-\frac{x^3}{2b}+\frac{ix^4}{4}-\frac{3x^2\cot(a+bx)}{2b^2}-\frac{x^3\cot^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2i(a+bx)}\right)}{b^3}-
\end{aligned}$$

Mathematica [A]
time = 6.75, size = 349, normalized size = 1.73

$$-\frac{1}{4}x^4\cot(a)-\frac{x^3\csc^2(a+bx)}{2b}+\frac{1}{4}e^{ia}\csc(a)\left(x^4+\left(-1+e^{-2ia}\right)x^4+\frac{e^{-2ia}\left(-1+e^{2ia}\right)\left(2b^4x^4+4ib^3x^3\log\left(1-e^{2i(a+bx)}\right)+6b^2x^2\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)+6ibx\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)-3\mathrm{PolyLog}\left(4,e^{2i(a+bx)}\right)\right)}{2b^4}\right)+\frac{3x^2\csc(a)\csc(a+bx)\sin(bx)}{2b^2}-\frac{3\csc(a)\sec(a)\left(b^2e^{i\mathrm{ArcTan}(\tan(a))}x^2+\frac{\left(ibx(-\pi+2\mathrm{ArcTan}(\tan(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx+\mathrm{ArcTan}(\tan(a)))\log\left(1-e^{2i(bx+\mathrm{ArcTan}(\tan(a)))}\right)+\pi\log(\cos(bx))+2\mathrm{ArcTan}(\tan(a))\log(\sin(bx+\mathrm{ArcTan}(\tan(a))))+i\mathrm{PolyLog}\left(2,e^{2i(bx+\mathrm{ArcTan}(\tan(a)))}\right)\right)\tan(a)}{\sqrt{1+\tan^2(a)}}\right)}{2b^4\sqrt{\sec^2(a)\left(\cos^2(a)+\sin^2(a)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cot[a + b*x]^3,x]
```

```
[Out] -1/4*(x^4*Cot[a]) - (x^3*Csc[a + b*x]^2)/(2*b) + (E^(I*a)*Csc[a]*(x^4 + (-1
 + E^((-2*I)*a))*x^4 + ((-1 + E^((2*I)*a))*(2*b^4*x^4 + (4*I)*b^3*x^3*Log[1
 - E^((2*I)*(a + b*x))] + 6*b^2*x^2*PolyLog[2, E^((2*I)*(a + b*x))] + (6*I)
*b*x*PolyLog[3, E^((2*I)*(a + b*x))] - 3*PolyLog[4, E^((2*I)*(a + b*x))]))/
(2*b^4*E^((2*I)*a))))/4 + (3*x^2*Csc[a]*Csc[a + b*x]*Sin[b*x])/(2*b^2) - (3
*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[
a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*
I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b
```

```
*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan
[a])/Sqrt[1 + Tan[a]^2]))/(2*b^4*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(167) = 334$.
time = 0.39, size = 444, normalized size = 2.20

method	result
risch	$\frac{2ia^3x}{b^3} - \frac{\ln(1-e^{i(bx+a)})a^3}{b^4} + \frac{3\ln(1-e^{i(bx+a)})x}{b^3} - \frac{\ln(e^{i(bx+a)}+1)x^3}{b} - \frac{\ln(1-e^{i(bx+a)})x^3}{b} + \frac{ix^4}{4} + \frac{3i\,\mathrm{polylog}(2,-e^{i(bx+a)}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*I/b^2*polylog(2,-exp(I*(b*x+a)))*x^2+3*I/b^2*polylog(2,exp(I*(b*x+a)))*x^
2-6*I/b^3*x*a+2*I/b^3*a^3*x-1/b^4*ln(1-exp(I*(b*x+a)))*a^3+3/b^3*ln(1-exp(I
*(b*x+a)))*x+x^2*(2*b*x*exp(2*I*(b*x+a))-3*I*exp(2*I*(b*x+a))+3*I)/b^2/(exp
(2*I*(b*x+a))-1)^2-1/b*ln(exp(I*(b*x+a))+1)*x^3-2/b^4*a^3*ln(exp(I*(b*x+a))
)+3/2*I/b^4*a^4-3*I/b^4*a^2-6*I/b^4*polylog(4,exp(I*(b*x+a)))-6*I/b^4*polyl
og(4,-exp(I*(b*x+a)))-3*I/b^4*polylog(2,exp(I*(b*x+a)))-3*I/b^4*polylog(2,-
exp(I*(b*x+a)))-3*I/b^2*x^2+3/b^3*ln(exp(I*(b*x+a))+1)*x-6/b^3*polylog(3,ex
p(I*(b*x+a)))*x+3/b^4*a*ln(1-exp(I*(b*x+a)))-6/b^3*polylog(3,-exp(I*(b*x+a)
))*x-3/b^4*a*ln(exp(I*(b*x+a))-1)+6/b^4*a*ln(exp(I*(b*x+a)))+1/b^4*a^3*ln(e
xp(I*(b*x+a))-1)-1/b*ln(1-exp(I*(b*x+a)))*x^3+1/4*I*x^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1960 vs. $2(160) = 320$.
time = 0.48, size = 1960, normalized size = 9.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) + 2*((b*x + a)^4 - 4*(b*x
 + a)^3*a + 6*(b*x + a)^2*a^2 + 12*a^2 - 4*((b*x + a)^3 - 3*(b*x + a)^2*a +
 3*(a^2 - 1)*(b*x + a) + ((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(a^2 - 1)*(b*x
+ a) + 3*a)*cos(4*b*x + 4*a) - 2*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(a^2 -
1)*(b*x + a) + 3*a)*cos(2*b*x + 2*a) - (-I*(b*x + a)^3 + 3*I*(b*x + a)^2*a
+ 3*(-I*a^2 + I)*(b*x + a) - 3*I*a)*sin(4*b*x + 4*a) - 2*(I*(b*x + a)^3 - 3
*I*(b*x + a)^2*a + 3*(I*a^2 - I)*(b*x + a) + 3*I*a)*sin(2*b*x + 2*a) + 3*a)
*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 12*(a*cos(4*b*x + 4*a) - 2*a*cos
(2*b*x + 2*a) + I*a*sin(4*b*x + 4*a) - 2*I*a*sin(2*b*x + 2*a) + a)*arctan2(
sin(b*x + a), cos(b*x + a) - 1) + 4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(a^2
 - 1)*(b*x + a) + ((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(a^2 - 1)*(b*x + a))*c
```

```
os(4*b*x + 4*a) - 2*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(a^2 - 1)*(b*x + a))
*cos(2*b*x + 2*a) + (I*(b*x + a)^3 - 3*I*(b*x + a)^2*a + 3*(I*a^2 - I)*(b*x
 + a))*sin(4*b*x + 4*a) + 2*(-I*(b*x + a)^3 + 3*I*(b*x + a)^2*a + 3*(-I*a^2
 + I)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1)
 + ((b*x + a)^4 - 4*(b*x + a)^3*a + 6*(a^2 - 2)*(b*x + a)^2 + 24*(b*x + a)*
a)*cos(4*b*x + 4*a) - 2*((b*x + a)^4 - 4*(b*x + a)^3*(a - I) + 6*(a^2 - 2*I
*a - 1)*(b*x + a)^2 - 12*(-I*a^2 - a)*(b*x + a) + 6*a^2)*cos(2*b*x + 2*a) +
 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2 + ((b*x + a)^2 - 2*(b*x + a)*a + a^2
 - 1)*cos(4*b*x + 4*a) - 2*((b*x + a)^2 - 2*(b*x + a)*a + a^2 - 1)*cos(2*b*
x + 2*a) + (I*(b*x + a)^2 - 2*I*(b*x + a)*a + I*a^2 - I)*sin(4*b*x + 4*a) +
 2*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a - I*a^2 + I)*sin(2*b*x + 2*a) - 1)*dil
og(-e^(I*b*x + I*a)) + 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2 + ((b*x + a)^2
 - 2*(b*x + a)*a + a^2 - 1)*cos(4*b*x + 4*a) - 2*((b*x + a)^2 - 2*(b*x + a)
*a + a^2 - 1)*cos(2*b*x + 2*a) + (I*(b*x + a)^2 - 2*I*(b*x + a)*a + I*a^2 -
 I)*sin(4*b*x + 4*a) + 2*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a - I*a^2 + I)*sin
(2*b*x + 2*a) - 1)*dilog(e^(I*b*x + I*a)) + 2*(I*(b*x + a)^3 - 3*I*(b*x + a
)^2*a + 3*(I*a^2 - I)*(b*x + a) + (I*(b*x + a)^3 - 3*I*(b*x + a)^2*a + 3*(I
*a^2 - I)*(b*x + a) + 3*I*a)*cos(4*b*x + 4*a) + 2*(-I*(b*x + a)^3 + 3*I*(b*
x + a)^2*a + 3*(-I*a^2 + I)*(b*x + a) - 3*I*a)*cos(2*b*x + 2*a) - ((b*x + a
)^3 - 3*(b*x + a)^2*a + 3*(a^2 - 1)*(b*x + a) + 3*a)*sin(4*b*x + 4*a) + 2*(
(b*x + a)^3 - 3*(b*x + a)^2*a + 3*(a^2 - 1)*(b*x + a) + 3*a)*sin(2*b*x + 2*
a) + 3*I*a)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 2*(
I*(b*x + a)^3 - 3*I*(b*x + a)^2*a + 3*(I*a^2 - I)*(b*x + a) + (I*(b*x + a)^
3 - 3*I*(b*x + a)^2*a + 3*(I*a^2 - I)*(b*x + a) + 3*I*a)*cos(4*b*x + 4*a) +
 2*(-I*(b*x + a)^3 + 3*I*(b*x + a)^2*a + 3*(-I*a^2 + I)*(b*x + a) - 3*I*a)*
cos(2*b*x + 2*a) - ((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(a^2 - 1)*(b*x + a) +
 3*a)*sin(4*b*x + 4*a) + 2*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(a^2 - 1)*(b*
x + a) + 3*a)*sin(2*b*x + 2*a) + 3*I*a)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 - 2*cos(b*x + a) + 1) - 24*(cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + I*sin(
4*b*x + 4*a) - 2*I*sin(2*b*x + 2*a) + 1)*polylog(4, -e^(I*b*x + I*a)) - 24*
(cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + I*sin(4*b*x + 4*a) - 2*I*sin(2*b*x
 + 2*a) + 1)*polylog(4, e^(I*b*x + I*a)) + 24*(I*b*x*cos(4*b*x + 4*a) - 2*I
*b*x*cos(2*b*x + 2*a) - b*x*sin(4*b*x + 4*a) + 2*b*x*sin(2*b*x + 2*a) + I*b
*x)*polylog(3, -e^(I*b*x + I*a)) + 24*(I*b*x*cos(4*b*x + 4*a) - 2*I*b*x*cos
(2*b*x + 2*a) - b*x*sin(4*b*x + 4*a) + 2*b*x*sin(2*b*x + 2*a) + I*b*x)*poly
log(3, e^(I*b*x + I*a)) - (-I*(b*x + a)^4 + 4*I*(b*x + a)^3*a - 6*(I*a^2 -
2*I)*(b*x + a)^2 - 24*I*(b*x + a)*a)*sin(4*b*x + 4*a) + 2*(-I*(b*x + a)^4 +
 4*(b*x + a)^3*(I*a + 1) + 6*(-I*a^2 - 2*a + I)*(b*x + a)^2 + 12*(a^2 - I*a
)*(b*x + a) - 6*I*a^2)*sin(2*b*x + 2*a))/(-4*I*cos(4*b*x + 4*a) + 8*I*cos(2
*b*x + 2*a) + 4*sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a) - 4*I))/b^4
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(160) = 320$.
time = 4.15, size = 569, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^3*x^3 + 12*b^2*x^2*sin(2*b*x + 2*a) - 6*(I*b^2*x^2 + (-I*b^2*x^2 +
 I)*cos(2*b*x + 2*a) - I)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - 6*
(-I*b^2*x^2 + (I*b^2*x^2 - I)*cos(2*b*x + 2*a) + I)*dilog(cos(2*b*x + 2*a)
- I*sin(2*b*x + 2*a)) - 4*(a^3 - (a^3 - 3*a)*cos(2*b*x + 2*a) - 3*a)*log(-1
/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) - 4*(a^3 - (a^3 - 3*a)*
cos(2*b*x + 2*a) - 3*a)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a)
+ 1/2) + 4*(b^3*x^3 + a^3 - 3*b*x - (b^3*x^3 + a^3 - 3*b*x - 3*a)*cos(2*b*x
 + 2*a) - 3*a)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 4*(b^3*x^3
 + a^3 - 3*b*x - (b^3*x^3 + a^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*a)*log(
-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) - 3*(I*cos(2*b*x + 2*a) - I)*po
lylog(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - 3*(-I*cos(2*b*x + 2*a) +
I)*polylog(4, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) - 6*(b*x*cos(2*b*x + 2
*a) - b*x)*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - 6*(b*x*cos(2
*b*x + 2*a) - b*x)*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)))/(b^4*
cos(2*b*x + 2*a) - b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cot^3 (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cot(b*x+a)**3,x)
```

```
[Out] Integral(x**3*cot(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cot(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \cot(a + b\, x)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(a + b*x)^3,x)
```

```
[Out] int(x^3*cot(a + b*x)^3, x)
```

3.12 $\int x^2 \cot^3(a+bx)\,dx$

Optimal. Leaf size=126

$$-\frac{x^2}{2b}+\frac{ix^3}{3}-\frac{x\cot(a+bx)}{b^2}-\frac{x^2\cot^2(a+bx)}{2b}-\frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b}+\frac{\log(\sin(a+bx))}{b^3}+\frac{ix\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}$$

```
[Out] -1/2*x^2/b+1/3*I*x^3-x*cot(b*x+a)/b^2-1/2*x^2*cot(b*x+a)^2/b-x^2*ln(1-exp(2
*I*(b*x+a)))/b+ln(sin(b*x+a))/b^3+I*x*polylog(2,exp(2*I*(b*x+a)))/b^2-1/2*p
olylog(3,exp(2*I*(b*x+a)))/b^3
```

Rubi [A]
time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3801, 3556, 30, 3798, 2221, 2611, 2320, 6724}

$$-\frac{\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{\log(\sin(a+bx))}{b^3}+\frac{ix\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{x\cot(a+bx)}{b^2}-\frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{x^2\cot^2(a+bx)}{2b}-\frac{x^2}{2b}+\frac{ix^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cot[a + b*x]^3,x]
```

```
[Out] -1/2*x^2/b + (I/3)*x^3 - (x*Cot[a + b*x])/b^2 - (x^2*Cot[a + b*x]^2)/(2*b)
- (x^2*Log[1 - E^((2*I)*(a + b*x))])/b + Log[Sin[a + b*x]]/b^3 + (I*x*PolyL
og[2, E^((2*I)*(a + b*x))])/b^2 - PolyLog[3, E^((2*I)*(a + b*x))]/(2*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x^2\cot^3(a+bx)\,dx &= -\frac{x^2\cot^2(a+bx)}{2b}+\frac{\int x\cot^2(a+bx)\,dx}{b}-\int x^2\cot(a+bx)\,dx\\ &= \frac{ix^3}{3}-\frac{x\cot(a+bx)}{b^2}-\frac{x^2\cot^2(a+bx)}{2b}+2i\int\frac{e^{2i(a+bx)}x^2}{1-e^{2i(a+bx)}}\,dx+\frac{\int\cot(a+bx)\,dx}{b^2}-\\ &= -\frac{x^2}{2b}+\frac{ix^3}{3}-\frac{x\cot(a+bx)}{b^2}-\frac{x^2\cot^2(a+bx)}{2b}-\frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b}+\frac{\log(\sin(a+}{b^3}\\ &= -\frac{x^2}{2b}+\frac{ix^3}{3}-\frac{x\cot(a+bx)}{b^2}-\frac{x^2\cot^2(a+bx)}{2b}-\frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b}+\frac{\log(\sin(a+}{b^3}\\ &= -\frac{x^2}{2b}+\frac{ix^3}{3}-\frac{x\cot(a+bx)}{b^2}-\frac{x^2\cot^2(a+bx)}{2b}-\frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b}+\frac{\log(\sin(a+}{b^3}\\ &= -\frac{x^2}{2b}+\frac{ix^3}{3}-\frac{x\cot(a+bx)}{b^2}-\frac{x^2\cot^2(a+bx)}{2b}-\frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b}+\frac{\log(\sin(a+}{b^3}\end{aligned}$$

Mathematica [A]
time = 2.68, size = 154, normalized size = 1.22

$$\frac{-6bx\cot(a)-2b^3x^3\cot(a)-3b^2x^2\csc^2(a+bx)+6\log(\sin(a+bx))+e^{-ia}(i+\cot(a))\left(2b^2x^2\left(ibx+bx\cot(a)-3\log\left(1-e^{2i(a+bx)}\right)\right)+6ibx\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)-3\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)\right)\sin(a)+6bx\csc(a)\csc(a+bx)\sin(bx)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cot[a + b*x]^3,x]
```

```
[Out] (-6*b*x*Cot[a] - 2*b^3*x^3*Cot[a] - 3*b^2*x^2*Csc[a + b*x]^2 + 6*Log[Sin[a
+ b*x]] + ((I + Cot[a])*(2*b^2*x^2*(I*b*x + b*x*Cot[a] - 3*Log[1 - E^((2*I)
*(a + b*x))]) + (6*I)*b*x*PolyLog[2, E^((2*I)*(a + b*x))] - 3*PolyLog[3, E^
((2*I)*(a + b*x))])*Sin[a])/E^(I*a) + 6*b*x*Csc[a]*Csc[a + b*x]*Sin[b*x])/(
6*b^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(110) = 220.
time = 0.34, size = 293, normalized size = 2.33

method	result
risch	$\frac{ix^3}{3}+\frac{2x\left(bx\,\mathrm{e}^{2i(bx+a)}-i\mathrm{e}^{2i(bx+a)}+i\right)}{b^2\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}-\frac{4ia^3}{3b^3}+\frac{\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a^2}{b^3}-\frac{2ia^2x}{b^2}+\frac{2a^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^3}-\frac{a^2\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^3}+ 2i$ p

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*x^3+2*x*(b*x*exp(2*I*(b*x+a))-I*exp(2*I*(b*x+a))+I)/b^2/(exp(2*I*(b*x
+a))-1)^2-4/3*I/b^3*a^3+1/b^3*ln(1-exp(I*(b*x+a)))*a^2+2*I/b^2*polylog(2,ex
```

```
p(I*(b*x+a)))*x+2/b^3*a^2*ln(exp(I*(b*x+a)))-1/b^3*a^2*ln(exp(I*(b*x+a))-1)
-2*I/b^2*a^2*x-1/b*ln(1-exp(I*(b*x+a)))*x^2+2*I/b^2*polylog(2,-exp(I*(b*x+a
)))*x-1/b*ln(exp(I*(b*x+a))+1)*x^2-2/b^3*polylog(3,exp(I*(b*x+a)))-2/b^3*po
lylog(3,-exp(I*(b*x+a)))+1/b^3*ln(exp(I*(b*x+a))+1)-2/b^3*ln(exp(I*(b*x+a))
)+1/b^3*ln(exp(I*(b*x+a))-1)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1208 vs. $2(107) = 214$.
time = 0.44, size = 1208, normalized size = 9.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 2*(2*(b*x + a)^3 - 6*(
b*x + a)^2*a - 6*((b*x + a)^2 - 2*(b*x + a)*a + ((b*x + a)^2 - 2*(b*x + a)*
a - 1)*cos(4*b*x + 4*a) - 2*((b*x + a)^2 - 2*(b*x + a)*a - 1)*cos(2*b*x + 2
*a) - (-I*(b*x + a)^2 + 2*I*(b*x + a)*a + I)*sin(4*b*x + 4*a) - 2*(I*(b*x +
 a)^2 - 2*I*(b*x + a)*a - I)*sin(2*b*x + 2*a) - 1)*arctan2(sin(b*x + a), co
s(b*x + a) + 1) + 6*(cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + I*sin(4*b*x +
4*a) - 2*I*sin(2*b*x + 2*a) + 1)*arctan2(sin(b*x + a), cos(b*x + a) - 1) +
6*((b*x + a)^2 - 2*(b*x + a)*a + ((b*x + a)^2 - 2*(b*x + a)*a)*cos(4*b*x +
4*a) - 2*((b*x + a)^2 - 2*(b*x + a)*a)*cos(2*b*x + 2*a) + (I*(b*x + a)^2 -
2*I*(b*x + a)*a)*sin(4*b*x + 4*a) + 2*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*si
n(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 2*((b*x + a)^3 -
 3*(b*x + a)^2*a - 6*b*x - 6*a)*cos(4*b*x + 4*a) - 4*((b*x + a)^3 - 3*(b*x
+ a)^2*(a - I) - 3*(b*x + a)*(2*I*a + 1) - 3*a)*cos(2*b*x + 2*a) + 12*(b*x*
cos(4*b*x + 4*a) - 2*b*x*cos(2*b*x + 2*a) + I*b*x*sin(4*b*x + 4*a) - 2*I*b*
x*sin(2*b*x + 2*a) + b*x)*dilog(-e^(I*b*x + I*a)) + 12*(b*x*cos(4*b*x + 4*a
) - 2*b*x*cos(2*b*x + 2*a) + I*b*x*sin(4*b*x + 4*a) - 2*I*b*x*sin(2*b*x + 2
*a) + b*x)*dilog(e^(I*b*x + I*a)) + 3*(I*(b*x + a)^2 - 2*I*(b*x + a)*a + (I
*(b*x + a)^2 - 2*I*(b*x + a)*a - I)*cos(4*b*x + 4*a) + 2*(-I*(b*x + a)^2 +
2*I*(b*x + a)*a + I)*cos(2*b*x + 2*a) - ((b*x + a)^2 - 2*(b*x + a)*a - 1)*s
in(4*b*x + 4*a) + 2*((b*x + a)^2 - 2*(b*x + a)*a - 1)*sin(2*b*x + 2*a) - I)
*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 3*(I*(b*x + a)
^2 - 2*I*(b*x + a)*a + (I*(b*x + a)^2 - 2*I*(b*x + a)*a - I)*cos(4*b*x + 4*
a) + 2*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a + I)*cos(2*b*x + 2*a) - ((b*x + a)
^2 - 2*(b*x + a)*a - 1)*sin(4*b*x + 4*a) + 2*((b*x + a)^2 - 2*(b*x + a)*a -
 1)*sin(2*b*x + 2*a) - I)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x +
 a) + 1) + 12*(I*cos(4*b*x + 4*a) - 2*I*cos(2*b*x + 2*a) - sin(4*b*x + 4*a)
 + 2*sin(2*b*x + 2*a) + I)*polylog(3, -e^(I*b*x + I*a)) + 12*(I*cos(4*b*x +
 4*a) - 2*I*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a) + I)*p
olylog(3, e^(I*b*x + I*a)) + 2*(I*(b*x + a)^3 - 3*I*(b*x + a)^2*a - 6*I*b*x
 - 6*I*a)*sin(4*b*x + 4*a) + 4*(-I*(b*x + a)^3 + 3*(b*x + a)^2*(I*a + 1) -
```

```
3*(b*x + a)*(2*a - I) + 3*I*a)*sin(2*b*x + 2*a) - 12*a)/(-6*I*cos(4*b*x + 4
*a) + 12*I*cos(2*b*x + 2*a) + 6*sin(4*b*x + 4*a) - 12*sin(2*b*x + 2*a) - 6*
I))/b^3
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(107) = 214$.
time = 4.06, size = 425, normalized size = 3.37

$$\frac{4 b^2x^2 + 4 bx\sin(2bx+2a) - 2(-ibx\cos(2bx+2a) + ibx)\mathrm{Li}_2(\cos(2bx+2a) + i\sin(2bx+2a)) - 2(ibx\cos(2bx+2a) - ibx)\mathrm{Li}_2(\cos(2bx+2a) - i\sin(2bx+2a)) + 2(a^2 - (a^2-1)\cos(2bx+2a) - 1)\log\left(-\frac{1}{2}\cos(2bx+2a) + \frac{1}{2}i\sin(2bx+2a) + \frac{1}{2}\right) + 2(a^2 - (a^2-1)\cos(2bx+2a) - 1)\log\left(-\frac{1}{2}\cos(2bx+2a) - \frac{1}{2}i\sin(2bx+2a) + \frac{1}{2}\right) + 2(b^2x^2 - a^2 - (b^2x^2 - a^2)\cos(2bx+2a))\log(-\cos(2bx+2a) + i\sin(2bx+2a) + 1) + 2(b^2x^2 - a^2 - (b^2x^2 - a^2)\cos(2bx+2a))\log(-\cos(2bx+2a) - i\sin(2bx+2a) + 1) - (\cos(2bx+2a) - 1)\mathrm{polylog}(3, \cos(2bx+2a) + i\sin(2bx+2a)) - (\cos(2bx+2a) - 1)\mathrm{polylog}(3, \cos(2bx+2a) - i\sin(2bx+2a))}{4(b^3\cos(2bx+2a) - b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*x^2 + 4*b*x*sin(2*b*x + 2*a) - 2*(-I*b*x*cos(2*b*x + 2*a) + I*b*
x)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - 2*(I*b*x*cos(2*b*x + 2*a)
 - I*b*x)*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 2*(a^2 - (a^2 - 1)
*cos(2*b*x + 2*a) - 1)*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) +
 1/2) + 2*(a^2 - (a^2 - 1)*cos(2*b*x + 2*a) - 1)*log(-1/2*cos(2*b*x + 2*a)
- 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^2*x^2 - a^2 - (b^2*x^2 - a^2)*cos(2*
b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 2*(b^2*x^2 -
a^2 - (b^2*x^2 - a^2)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x
 + 2*a) + 1) - (cos(2*b*x + 2*a) - 1)*polylog(3, cos(2*b*x + 2*a) + I*sin(2
*b*x + 2*a)) - (cos(2*b*x + 2*a) - 1)*polylog(3, cos(2*b*x + 2*a) - I*sin(2
*b*x + 2*a)))/(b^3*cos(2*b*x + 2*a) - b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cot^3(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cot(b*x+a)**3,x)
```

```
[Out] Integral(x**2*cot(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cot(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \cot(a + b\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(a + b*x)^3,x)
```

```
[Out] int(x^2*cot(a + b*x)^3, x)
```

3.13 $\int x \cot^3(a+bx)\,dx$

Optimal. Leaf size=91

$$-\frac{x}{2b}+\frac{ix^2}{2}-\frac{\cot(a+bx)}{2b^2}-\frac{x\cot^2(a+bx)}{2b}-\frac{x\log\left(1-e^{2i(a+bx)}\right)}{b}+\frac{i\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}$$

```
[Out] -1/2*x/b+1/2*I*x^2-1/2*cot(b*x+a)/b^2-1/2*x*cot(b*x+a)^2/b-x*ln(1-exp(2*I*(
b*x+a)))/b+1/2*I*polylog(2,exp(2*I*(b*x+a)))/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.700$, Rules used = {3801, 3554, 8, 3798, 2221, 2317, 2438}

$$\frac{i\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{\cot(a+bx)}{2b^2}-\frac{x\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{x\cot^2(a+bx)}{2b}-\frac{x}{2b}+\frac{ix^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cot[a + b*x]^3,x]
```

```
[Out] -1/2*x/b + (I/2)*x^2 - Cot[a + b*x]/(2*b^2) - (x*Cot[a + b*x]^2)/(2*b) - (x
*Log[1 - E^((2*I)*(a + b*x))])/b + ((I/2)*PolyLog[2, E^((2*I)*(a + b*x))])/
b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x\cot^3(a+bx)\,dx &= -\frac{x\cot^2(a+bx)}{2b}+\frac{\int \cot^2(a+bx)\,dx}{2b}-\int x\cot(a+bx)\,dx\\
&= \frac{ix^2}{2}-\frac{\cot(a+bx)}{2b^2}-\frac{x\cot^2(a+bx)}{2b}+2i\int \frac{e^{2i(a+bx)}x}{1-e^{2i(a+bx)}}\,dx-\frac{\int 1\,dx}{2b}\\
&= -\frac{x}{2b}+\frac{ix^2}{2}-\frac{\cot(a+bx)}{2b^2}-\frac{x\cot^2(a+bx)}{2b}-\frac{x\log\left(1-e^{2i(a+bx)}\right)}{b}+\frac{\int \log\left(1-e^{2i(a\right.}}{b}\\
&= -\frac{x}{2b}+\frac{ix^2}{2}-\frac{\cot(a+bx)}{2b^2}-\frac{x\cot^2(a+bx)}{2b}-\frac{x\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i\text{Subst}\left(\int \frac{\log(1-}{x}\right.}{}\\
&= -\frac{x}{2b}+\frac{ix^2}{2}-\frac{\cot(a+bx)}{2b^2}-\frac{x\cot^2(a+bx)}{2b}-\frac{x\log\left(1-e^{2i(a+bx)}\right)}{b}+\frac{i\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}
\end{aligned}$$

Mathematica [A]
time = 4.42, size = 179, normalized size = 1.97

$$\frac{-ib\pi x-b^2x^2\cot(a)-bx\csc^2(a+bx)-\pi\log\left(1+e^{-2ibx}\right)-2bx\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)+\pi\log(\cos(bx))+2\text{ArcTan}(\tan(a))\left(ibx-\log\left(1-e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)+\log(\sin(bx+\text{ArcTan}(\tan(a))))\right)+i\text{PolyLog}\left(2,e^{2i(bx+\text{ArcTan}(\tan(a)))}\right)+b^2e^{i\text{ArcTan}(\tan(a))}x^2\cot(a)\sqrt{\sec^2(a)}+\csc(a)\csc(a+bx)\sin(bx)}{2b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Cot[a + b*x]^3,x]
```

```
[Out] ((-I)*b*Pi*x - b^2*x^2*Cot[a] - b*x*Csc[a + b*x]^2 - Pi*Log[1 + E^((-2*I)*b
*x)] - 2*b*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] +
 2*ArcTan[Tan[a]]*(I*b*x - Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Log[
Sin[b*x + ArcTan[Tan[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))
] + b^2*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2] + Csc[a]*Csc[a + b*x
]*Sin[b*x])/(2*b^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(75) = 150.
time = 0.29, size = 197, normalized size = 2.16

method	result
risch	$\frac{i x^{2}}{2}+\frac{2 b x\,{\mathrm e}^{2 i \left(b x +a \right)}-i {\mathrm e}^{2 i \left(b x +a \right)}+i}{b^{2} \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)^{2}}+\frac{2 i x a}{b}+\frac{i a^{2}}{b^{2}}-\frac{\ln \left(1-{\mathrm e}^{i \left(b x +a \right)}\right) x}{b}-\frac{\ln \left(1-{\mathrm e}^{i \left(b x +a \right)}\right) a}{b^{2}}+\frac{i \operatorname{polylog}\left(2, {\mathrm e}^{i \left(b x +a \right)}\right)}{b^{2}}-\frac{\ln \left({\mathrm e}^{i (b x}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*x^2+(2*b*x*exp(2*I*(b*x+a))-I*exp(2*I*(b*x+a))+I)/b^2/(exp(2*I*(b*x+a
))-1)^2+2*I/b*x*a+I/b^2*a^2-1/b*ln(1-exp(I*(b*x+a)))*x-1/b^2*ln(1-exp(I*(b*
x+a)))*a+I/b^2*polylog(2,exp(I*(b*x+a)))-1/b*ln(exp(I*(b*x+a))+1)*x+I/b^2*p
olylog(2,-exp(I*(b*x+a)))+1/b^2*a*ln(exp(I*(b*x+a))-1)-2/b^2*a*ln(exp(I*(b*
x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 586 vs. 2(72) = 144.
time = 0.39, size = 586, normalized size = 6.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] (b^2*x^2*cos(4*b*x + 4*a) + I*b^2*x^2*sin(4*b*x + 4*a) + b^2*x^2 - 2*(b*x*c
os(4*b*x + 4*a) - 2*b*x*cos(2*b*x + 2*a) + I*b*x*sin(4*b*x + 4*a) - 2*I*b*x
*sin(2*b*x + 2*a) + b*x)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*(b*x*c
os(4*b*x + 4*a) - 2*b*x*cos(2*b*x + 2*a) + I*b*x*sin(4*b*x + 4*a) - 2*I*b*x
*sin(2*b*x + 2*a) + b*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(b^2*
x^2 + 2*I*b*x + 1)*cos(2*b*x + 2*a) + 2*(cos(4*b*x + 4*a) - 2*cos(2*b*x + 2
*a) + I*sin(4*b*x + 4*a) - 2*I*sin(2*b*x + 2*a) + 1)*dilog(-e^(I*b*x + I*a)
) + 2*(cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + I*sin(4*b*x + 4*a) - 2*I*sin
(2*b*x + 2*a) + 1)*dilog(e^(I*b*x + I*a)) - (-I*b*x*cos(4*b*x + 4*a) + 2*I*
b*x*cos(2*b*x + 2*a) + b*x*sin(4*b*x + 4*a) - 2*b*x*sin(2*b*x + 2*a) - I*b*
x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-I*b*x*cos(
4*b*x + 4*a) + 2*I*b*x*cos(2*b*x + 2*a) + b*x*sin(4*b*x + 4*a) - 2*b*x*sin(
```

```
2*b*x + 2*a) - I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a)
+ 1) + 2*(-I*b^2*x^2 + 2*b*x - I)*sin(2*b*x + 2*a) + 2)/(-2*I*b^2*cos(4*b*x
 + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b
*x + 2*a) - 2*I*b^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(72) = 144$.
time = 3.82, size = 291, normalized size = 3.20

$$\frac{4\,bx + (i\,\cos\left(2\,bx + 2\,a\right) - i){\rm Li}_2(\cos\left(2\,bx + 2\,a\right) + i\,\sin\left(2\,bx + 2\,a\right)) + (-i\,\cos\left(2\,bx + 2\,a\right) + i){\rm Li}_2(\cos\left(2\,bx + 2\,a\right) - i\,\sin\left(2\,bx + 2\,a\right)) + 2\,(a\cos\left(2\,bx + 2\,a\right) - a)\log\left(-\frac{1}{2}\,\cos\left(2\,bx + 2\,a\right) + \frac{1}{2}i\,\sin\left(2\,bx + 2\,a\right) + \frac{1}{2}\right) + 2\,(a\cos\left(2\,bx + 2\,a\right) - a)\log\left(-\frac{1}{2}\,\cos\left(2\,bx + 2\,a\right) - \frac{1}{2}i\,\sin\left(2\,bx + 2\,a\right) + \frac{1}{2}\right) + 2\,(bx - (bx + a)\cos\left(2\,bx + 2\,a\right) + a)\log\left(-\cos\left(2\,bx + 2\,a\right) + i\,\sin\left(2\,bx + 2\,a\right) + 1\right) + 2\,(bx - (bx + a)\cos\left(2\,bx + 2\,a\right) + a)\log\left(-\cos\left(2\,bx + 2\,a\right) - i\,\sin\left(2\,bx + 2\,a\right) + 1\right) + 2\,\sin\left(2\,bx + 2\,a\right)}{4\,(b^2\cos\left(2\,bx + 2\,a\right) - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*x + (I*cos(2*b*x + 2*a) - I)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x
+ 2*a)) + (-I*cos(2*b*x + 2*a) + I)*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x +
2*a)) + 2*(a*cos(2*b*x + 2*a) - a)*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*
b*x + 2*a) + 1/2) + 2*(a*cos(2*b*x + 2*a) - a)*log(-1/2*cos(2*b*x + 2*a) -
1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b*x - (b*x + a)*cos(2*b*x + 2*a) + a)*lo
g(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 2*(b*x - (b*x + a)*cos(2*b*
x + 2*a) + a)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 2*sin(2*b*x
 + 2*a))/(b^2*cos(2*b*x + 2*a) - b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\cot^3\left(a + bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)**3,x)
```

```
[Out] Integral(x*cot(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cot(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\mathrm{cot}\left(a+b\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(a + b*x)^3,x)
```

```
[Out] int(x*cot(a + b*x)^3, x)
```

3.14 $\int \frac{\cot^3(a+bx)}{x} \, dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\cot^3(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^3/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^3(a+bx)}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[a + b*x]^3/x,x]
[Out] Defer[Int][Cot[a + b*x]^3/x, x]
```

Rubi steps

$$\int \frac{\cot^3(a+bx)}{x} \, dx = \int \frac{\cot^3(a+bx)}{x} \, dx$$

Mathematica [A]
time = 4.83, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^3/x,x]
[Out] Integrate[Cot[a + b*x]^3/x, x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(bx+a)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^3/x,x)
```

```
[Out] int(cot(b*x+a)^3/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -(4*b*x*cos(2*b*x + 2*a)^2 + 4*b*x*sin(2*b*x + 2*a)^2 - 2*b*x*cos(2*b*x + 2
*a) - (2*b*x*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (b^2*x
^2*cos(4*b*x + 4*a)^2 + 4*b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(4*b*x +
4*a)^2 - 4*b^2*x^2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^2*sin(2*b*x
+ 2*a)^2 - 4*b^2*x^2*cos(2*b*x + 2*a) + b^2*x^2 - 2*(2*b^2*x^2*cos(2*b*x +
2*a) - b^2*x^2)*cos(4*b*x + 4*a))*integrate((b^2*x^2 - 1)*sin(b*x + a)/(b^2
*x^3*cos(b*x + a)^2 + b^2*x^3*sin(b*x + a)^2 + 2*b^2*x^3*cos(b*x + a) + b^2
*x^3), x) + (b^2*x^2*cos(4*b*x + 4*a)^2 + 4*b^2*x^2*cos(2*b*x + 2*a)^2 + b^
2*x^2*sin(4*b*x + 4*a)^2 - 4*b^2*x^2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*
b^2*x^2*sin(2*b*x + 2*a)^2 - 4*b^2*x^2*cos(2*b*x + 2*a) + b^2*x^2 - 2*(2*b^
2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*cos(4*b*x + 4*a))*integrate((b^2*x^2 - 1)
*sin(b*x + a)/(b^2*x^3*cos(b*x + a)^2 + b^2*x^3*sin(b*x + a)^2 - 2*b^2*x^3*
cos(b*x + a) + b^2*x^3), x) - (2*b*x*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) -
1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))/(b^2*x^2*cos(4*b*x + 4*a)^2 + 4*b^2
*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(4*b*x + 4*a)^2 - 4*b^2*x^2*sin(4*b*x
+ 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^2*sin(2*b*x + 2*a)^2 - 4*b^2*x^2*cos(2*b*
x + 2*a) + b^2*x^2 - 2*(2*b^2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*cos(4*b*x + 4
*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^3/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{3}\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**3/x,x)
```

```
[Out] Integral(cot(a + b*x)**3/x, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^3/x, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cot(a+b\,x)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3/x,x)
```

```
[Out] int(cot(a + b*x)^3/x, x)
```

3.15 $\int \frac{\cot^3(a+bx)}{x^2}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\cot^3(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^3/x^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^3(a+bx)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[a + b*x]^3/x^2,x]
```

```
[Out] Defer[Int][Cot[a + b*x]^3/x^2, x]
```

Rubi steps

$$\int \frac{\cot^3(a+bx)}{x^2}\,dx = \int \frac{\cot^3(a+bx)}{x^2}\,dx$$

Mathematica [A]
time = 5.29, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^3/x^2,x]
```

```
[Out] Integrate[Cot[a + b*x]^3/x^2, x]
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\cot^3\left(bx+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^3/x^2,x)
```

```
[Out] int(cot(b*x+a)^3/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -(4*b*x*cos(2*b*x + 2*a)^2 + 4*b*x*sin(2*b*x + 2*a)^2 - 2*b*x*cos(2*b*x + 2
*a) - 2*(b*x*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (b^2*x
^3*cos(4*b*x + 4*a)^2 + 4*b^2*x^3*cos(2*b*x + 2*a)^2 + b^2*x^3*sin(4*b*x +
4*a)^2 - 4*b^2*x^3*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^3*sin(2*b*x
+ 2*a)^2 - 4*b^2*x^3*cos(2*b*x + 2*a) + b^2*x^3 - 2*(2*b^2*x^3*cos(2*b*x +
2*a) - b^2*x^3)*cos(4*b*x + 4*a))*integrate((b^2*x^2 - 3)*sin(b*x + a)/(b^2
*x^4*cos(b*x + a)^2 + b^2*x^4*sin(b*x + a)^2 + 2*b^2*x^4*cos(b*x + a) + b^2
*x^4), x) + (b^2*x^3*cos(4*b*x + 4*a)^2 + 4*b^2*x^3*cos(2*b*x + 2*a)^2 + b^
2*x^3*sin(4*b*x + 4*a)^2 - 4*b^2*x^3*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*
b^2*x^3*sin(2*b*x + 2*a)^2 - 4*b^2*x^3*cos(2*b*x + 2*a) + b^2*x^3 - 2*(2*b^
2*x^3*cos(2*b*x + 2*a) - b^2*x^3)*cos(4*b*x + 4*a))*integrate((b^2*x^2 - 3)
*sin(b*x + a)/(b^2*x^4*cos(b*x + a)^2 + b^2*x^4*sin(b*x + a)^2 - 2*b^2*x^4*
cos(b*x + a) + b^2*x^4), x) - 2*(b*x*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) -
1)*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))/(b^2*x^3*cos(4*b*x + 4*a)^2 + 4*b
^2*x^3*cos(2*b*x + 2*a)^2 + b^2*x^3*sin(4*b*x + 4*a)^2 - 4*b^2*x^3*sin(4*b*
x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^3*sin(2*b*x + 2*a)^2 - 4*b^2*x^3*cos(2*
b*x + 2*a) + b^2*x^3 - 2*(2*b^2*x^3*cos(2*b*x + 2*a) - b^2*x^3)*cos(4*b*x +
 4*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^3/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a + bx)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**3/x**2,x)
```

```
[Out] Integral(cot(a + b*x)**3/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^3/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cot(a+b\,x)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3/x^2,x)
```

```
[Out] int(cot(a + b*x)^3/x^2, x)
```

3.16 $\int \frac{(c+dx)^3}{a+ia\cot(e+fx)}\,dx$

Optimal. Leaf size=189

$$-\frac{3id^3x}{8af^3}-\frac{3d(c+dx)^2}{8af^2}+\frac{i(c+dx)^3}{4af}+\frac{(c+dx)^4}{8ad}-\frac{3d^3}{8f^4(a+ia\cot(e+fx))}+\frac{3id^2(c+dx)}{4f^3(a+ia\cot(e+fx))}+\frac{3\ldots}{4f^2(a+\ldots}$$

```
[Out] -3/8*I*d^3*x/a/f^3-3/8*d*(d*x+c)^2/a/f^2+1/4*I*(d*x+c)^3/a/f+1/8*(d*x+c)^4/
a/d-3/8*d^3/f^4/(a+I*a*cot(f*x+e))+3/4*I*d^2*(d*x+c)/f^3/(a+I*a*cot(f*x+e))
+3/4*d*(d*x+c)^2/f^2/(a+I*a*cot(f*x+e))-1/2*I*(d*x+c)^3/f/(a+I*a*cot(f*x+e)
)
```

Rubi [A]
time = 0.15, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3804, 3560, 8}

$$\frac{3id^2(c+dx)}{4f^3(a+ia\cot(e+fx))}+\frac{3d(c+dx)^2}{4f^2(a+ia\cot(e+fx))}-\frac{i(c+dx)^3}{2f(a+ia\cot(e+fx))}-\frac{3d(c+dx)^2}{8af^2}+\frac{i(c+dx)^3}{4af}+\frac{(c+dx)^4}{8ad}-\frac{3d^3}{8f^4(a+ia\cot(e+fx))}-\frac{3id^3x}{8af^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] (((-3*I)/8)*d^3*x)/(a*f^3) - (3*d*(c + d*x)^2)/(8*a*f^2) + ((I/4)*(c + d*x)
^3)/(a*f) + (c + d*x)^4/(8*a*d) - (3*d^3)/(8*f^4*(a + I*a*Cot[e + f*x])) +
(((3*I)/4)*d^2*(c + d*x))/(f^3*(a + I*a*Cot[e + f*x])) + (3*d*(c + d*x)^2)/
(4*f^2*(a + I*a*Cot[e + f*x])) - ((I/2)*(c + d*x)^3)/(f*(a + I*a*Cot[e + f*
x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3804

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[a*d*(m/(2*b*f))
, Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m/
(2*b*f*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 + b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+ia\cot(e+fx)}\,dx = \frac{(c+dx)^4}{8ad} - \frac{i(c+dx)^3}{2f(a+ia\cot(e+fx))} + \frac{(3id)\int \frac{(c+dx)^2}{a+ia\cot(e+fx)}\,dx}{2f}$$

$$= \frac{i(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} + \frac{3d(c+dx)^2}{4f^2(a+ia\cot(e+fx))} - \frac{i(c+dx)^3}{2f(a+ia\cot(e+fx))} - \frac{(3$$

$$= -\frac{3d(c+dx)^2}{8af^2} + \frac{i(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} + \frac{3id^2(c+dx)}{4f^3(a+ia\cot(e+fx))} + \frac{3d(c}{4f^2(a+ia}$$

$$= -\frac{3d(c+dx)^2}{8af^2} + \frac{i(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d^3}{8f^4(a+ia\cot(e+fx))} + \frac{3id^2(}{4f^3(a+ia}$$

$$= -\frac{3id^3x}{8af^3} - \frac{3d(c+dx)^2}{8af^2} + \frac{i(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d^3}{8f^4(a+ia\cot(e+fx))} + \frac{}{4j}$$

Mathematica [A]
time = 0.44, size = 246, normalized size = 1.30

$$\frac{2f^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+i(4c^3f^3+6c^2df^2(i+2fx)+6cd^2f(-1+2ifx+2f^2x^2)+d^3(-3i-6fx+6if^2x^2+4f^3x^3))\cos(2fx)(\cos(2e)+i\sin(2e))-(4c^3f^3+6c^2df^2(i+2fx)+6cd^2f(-1+2ifx+2f^2x^2)+d^3(-3i-6fx+6if^2x^2+4f^3x^3))(\cos(2e)+i\sin(2e))\sin(2fx)}{16af^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] (2*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + I*(4*c^3*f^3 + 6*c^2
*d*f^2*(I + 2*f*x) + 6*c*d^2*f*(-1 + (2*I)*f*x + 2*f^2*x^2) + d^3*(-3*I - 6
*f*x + (6*I)*f^2*x^2 + 4*f^3*x^3))*Cos[2*f*x]*(Cos[2*e] + I*Sin[2*e]) - (4*
c^3*f^3 + 6*c^2*d*f^2*(I + 2*f*x) + 6*c*d^2*f*(-1 + (2*I)*f*x + 2*f^2*x^2)
+ d^3*(-3*I - 6*f*x + (6*I)*f^2*x^2 + 4*f^3*x^3))*(Cos[2*e] + I*Sin[2*e])*S
in[2*f*x])/(16*a*f^4)
```

Maple [A]
time = 0.80, size = 170, normalized size = 0.90

method	result
risch	$\frac{d^3x^4}{8a}+\frac{d^2c\,x^3}{2a}+\frac{3d\,c^2x^2}{4a}+\frac{c^3x}{2a}+\frac{c^4}{8ad}+\frac{i(4d^3x^3f^3+12c\,d^2f^3x^2+6id^3f^2x^2+12c^2d\,f^3x+12ic\,d^2f^2x+4c^3f^3+6ic^2d\,f^2-6d^3fx}{16f^4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+I*a*cot(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a*d^3*x^4+1/2/a*d^2*c*x^3+3/4/a*d*c^2*x^2+1/2/a*c^3*x+1/8/a/d*c^4+1/16*
I*(4*d^3*x^3*f^3+6*I*d^3*f^2*x^2+12*c*d^2*f^3*x^2+12*I*c*d^2*f^2*x+12*c^2*d
```

```
*f^3*x+6*I*c^2*d*f^2+4*c^3*f^3-6*d^3*f*x-3*I*d^3-6*c*d^2*f)/f^4/a*exp(2*I*(
f*x+e))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 3.95, size = 155, normalized size = 0.82

$$\frac{2\,d^3f^4x^4+8\,cd^2f^4x^3+12\,c^2df^4x^2+8\,c^3f^4x+(4i\,d^3f^3x^3+4i\,c^3f^3-6\,c^2df^2-6i\,cd^2f+3\,d^3-6\,(-2i\,cd^2f^3+d^3f^2)x^2-6\,(-2i\,c^2df^3+2\,cd^2f^2+i\,d^3f)x)e^{(2i\,fx+2i\,e)}}{16\,af^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*d^3*f^4*x^4 + 8*c*d^2*f^4*x^3 + 12*c^2*d*f^4*x^2 + 8*c^3*f^4*x + (4
*I*d^3*f^3*x^3 + 4*I*c^3*f^3 - 6*c^2*d*f^2 - 6*I*c*d^2*f + 3*d^3 - 6*(-2*I*
c*d^2*f^3 + d^3*f^2)*x^2 - 6*(-2*I*c^2*d*f^3 + 2*c*d^2*f^2 + I*d^3*f)*x)*e^
(2*I*f*x + 2*I*e))/(a*f^4)
```

Sympy [A]
time = 0.19, size = 314, normalized size = 1.66

$$\begin{cases} \frac{\left(4ic^3f^3e^{2ie}+12ic^2df^3xe^{2ie}-6c^2df^2e^{2ie}+12icd^2f^3x^2e^{2ie}-12cd^2f^2xe^{2ie}-6icd^2fe^{2ie}+4id^3f^3x^3e^{2ie}-6d^3f^2x^2e^{2ie}-6id^3fxe^{2ie}+3d^3e^{2ie}\right)e^{2ifx}}{16af^4} & \text{for}\: af^4 \neq 0 \\ -\frac{c^3xe^{2ie}}{2a}-\frac{3c^2dx^2e^{2ie}}{4a}-\frac{cd^2x^3e^{2ie}}{2a}-\frac{d^3x^4e^{2ie}}{8a} & \text{otherwise} \end{cases} + \frac{c^3x}{2a} + \frac{3c^2dx^2}{4a} + \frac{cd^2x^3}{2a} + \frac{d^3x^4}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+I*a*cot(f*x+e)),x)
```

```
[Out] Piecewise((((4*I*c**3*f**3*exp(2*I*e) + 12*I*c**2*d*f**3*x*exp(2*I*e) - 6*c*
*2*d*f**2*exp(2*I*e) + 12*I*c*d**2*f**3*x**2*exp(2*I*e) - 12*c*d**2*f**2*x*
exp(2*I*e) - 6*I*c*d**2*f*exp(2*I*e) + 4*I*d**3*f**3*x**3*exp(2*I*e) - 6*d*
*3*f**2*x**2*exp(2*I*e) - 6*I*d**3*f*x*exp(2*I*e) + 3*d**3*exp(2*I*e))*exp(
2*I*f*x)/(16*a*f**4), Ne(a*f**4, 0)), (-c**3*x*exp(2*I*e)/(2*a) - 3*c**2*d*
x**2*exp(2*I*e)/(4*a) - c*d**2*x**3*exp(2*I*e)/(2*a) - d**3*x**4*exp(2*I*e)
/(8*a), True)) + c**3*x/(2*a) + 3*c**2*d*x**2/(4*a) + c*d**2*x**3/(2*a) + d
**3*x**4/(8*a)
```

Giac [A]
time = 0.41, size = 233, normalized size = 1.23

$$\frac{2\,d^3f^4x^4+8\,cd^2f^4x^3+4i\,d^3f^3x^3e^{(2i\,fx+2i\,e)}+12\,c^2df^4x^2+12i\,cd^2f^3x^2e^{(2i\,fx+2i\,e)}+8\,c^3f^4x+12i\,c^2df^3xe^{(2i\,fx+2i\,e)}-6\,d^3f^2x^2e^{(2i\,fx+2i\,e)}+4i\,c^3f^3e^{(2i\,fx+2i\,e)}-12\,cd^2f^2xe^{(2i\,fx+2i\,e)}-6\,c^2df^2e^{(2i\,fx+2i\,e)}-6i\,d^3fxe^{(2i\,fx+2i\,e)}-6i\,cd^2fe^{(2i\,fx+2i\,e)}+3\,d^3e^{(2i\,fx+2i\,e)}}{16\,af^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/16*(2*d^3*f^4*x^4 + 8*c*d^2*f^4*x^3 + 4*I*d^3*f^3*x^3*e^(2*I*f*x + 2*I*e)
 + 12*c^2*d*f^4*x^2 + 12*I*c*d^2*f^3*x^2*e^(2*I*f*x + 2*I*e) + 8*c^3*f^4*x
+ 12*I*c^2*d*f^3*x*e^(2*I*f*x + 2*I*e) - 6*d^3*f^2*x^2*e^(2*I*f*x + 2*I*e)
+ 4*I*c^3*f^3*e^(2*I*f*x + 2*I*e) - 12*c*d^2*f^2*x*e^(2*I*f*x + 2*I*e) - 6*
c^2*d*f^2*e^(2*I*f*x + 2*I*e) - 6*I*d^3*f*x*e^(2*I*f*x + 2*I*e) - 6*I*c*d^2
*f*e^(2*I*f*x + 2*I*e) + 3*d^3*e^(2*I*f*x + 2*I*e))/(a*f^4)
```

Mupad [B]
time = 1.24, size = 423, normalized size = 2.24

$$\frac{3 d^3 \cos(2 e+2 f x)+8 c^3 f^4 x-4 c^3 f^3 \sin(2 e+2 f x)+2 d^3 f^4 x^4-6 c^2 d f^2 \cos(2 e+2 f x)+12 c^2 d f^4 x^2+8 c d^2 f^4 x^3-6 d^3 f^2 x^2 \cos(2 e+2 f x)-4 d^3 f^3 x^3 \sin(2 e+2 f x)+6 c d^2 f \sin(2 e+2 f x)+6 d^3 f x \sin(2 e+2 f x)-12 c d^2 f^2 x \cos(2 e+2 f x)-12 c^2 d f^3 x \sin(2 e+2 f x)-12 c d^2 f^3 x^2 \sin(2 e+2 f x)+d^3 \sin(2 e+2 f x)\, 3i+c^3 f^3 \cos(2 e+2 f x)\, 4i-c^2 d f^2 \sin(2 e+2 f x)\, 6i+d^3 f^3 x^3 \cos(2 e+2 f x)\, 4i-d^3 f^2 x^2 \sin(2 e+2 f x)\, 6i-c d^2 f \cos(2 e+2 f x)\, 6i-d^3 f x \cos(2 e+2 f x)\, 6i+c^2 d f^3 x \cos(2 e+2 f x)\, 12i-c d^2 f^2 x \sin(2 e+2 f x)\, 12i+c d^2 f^3 x^2 \cos(2 e+2 f x)\, 12i}{16 a f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*cot(e + f*x)*1i),x)
```

```
[Out] (3*d^3*cos(2*e + 2*f*x) + d^3*sin(2*e + 2*f*x)*3i + 8*c^3*f^4*x + c^3*f^3*c
os(2*e + 2*f*x)*4i - 4*c^3*f^3*sin(2*e + 2*f*x) + 2*d^3*f^4*x^4 - 6*c^2*d*f
^2*cos(2*e + 2*f*x) - c^2*d*f^2*sin(2*e + 2*f*x)*6i + 12*c^2*d*f^4*x^2 + 8*
c*d^2*f^4*x^3 - 6*d^3*f^2*x^2*cos(2*e + 2*f*x) + d^3*f^3*x^3*cos(2*e + 2*f*
x)*4i - d^3*f^2*x^2*sin(2*e + 2*f*x)*6i - 4*d^3*f^3*x^3*sin(2*e + 2*f*x) -
c*d^2*f*cos(2*e + 2*f*x)*6i + 6*c*d^2*f*sin(2*e + 2*f*x) - d^3*f*x*cos(2*e
+ 2*f*x)*6i + 6*d^3*f*x*sin(2*e + 2*f*x) - 12*c*d^2*f^2*x*cos(2*e + 2*f*x)
+ c^2*d*f^3*x*cos(2*e + 2*f*x)*12i - c*d^2*f^2*x*sin(2*e + 2*f*x)*12i - 12*
c^2*d*f^3*x*sin(2*e + 2*f*x) + c*d^2*f^3*x^2*cos(2*e + 2*f*x)*12i - 12*c*d^
2*f^3*x^2*sin(2*e + 2*f*x))/(16*a*f^4)
```

3.17 $\int \frac{(c+dx)^2}{a+ia\cot(e+fx)}\,dx$

Optimal. Leaf size=137

$$-\frac{d^2x}{4af^2}+\frac{i(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}+\frac{id^2}{4f^3(a+ia\cot(e+fx))}+\frac{d(c+dx)}{2f^2(a+ia\cot(e+fx))}-\frac{i(c+dx)^2}{2f(a+ia\cot(e+fx))}$$

```
[Out] -1/4*d^2*x/a/f^2+1/4*I*(d*x+c)^2/a/f+1/6*(d*x+c)^3/a/d+1/4*I*d^2/f^3/(a+I*a
*cot(f*x+e))+1/2*d*(d*x+c)/f^2/(a+I*a*cot(f*x+e))-1/2*I*(d*x+c)^2/f/(a+I*a*
cot(f*x+e))
```

Rubi [A]
time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3804, 3560, 8}

$$\frac{d(c+dx)}{2f^2(a+ia\cot(e+fx))}-\frac{i(c+dx)^2}{2f(a+ia\cot(e+fx))}+\frac{i(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}+\frac{id^2}{4f^3(a+ia\cot(e+fx))}-\frac{d^2x}{4af^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] -1/4*(d^2*x)/(a*f^2) + ((I/4)*(c + d*x)^2)/(a*f) + (c + d*x)^3/(6*a*d) + ((
I/4)*d^2)/(f^3*(a + I*a*Cot[e + f*x])) + (d*(c + d*x))/(2*f^2*(a + I*a*Cot[
e + f*x])) - ((I/2)*(c + d*x)^2)/(f*(a + I*a*Cot[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3804

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[a*d*(m/(2*b*f))
, Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m/
(2*b*f*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 + b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+ia\cot(e+fx)}\,dx = \frac{(c+dx)^3}{6ad} - \frac{i(c+dx)^2}{2f(a+ia\cot(e+fx))} + \frac{(id)\int \frac{c+dx}{a+ia\cot(e+fx)}\,dx}{f}$$

$$= \frac{i(c+dx)^2}{4af} + \frac{(c+dx)^3}{6ad} + \frac{d(c+dx)}{2f^2(a+ia\cot(e+fx))} - \frac{i(c+dx)^2}{2f(a+ia\cot(e+fx))} - \frac{d}{}$$

$$= \frac{i(c+dx)^2}{4af} + \frac{(c+dx)^3}{6ad} + \frac{id^2}{4f^3(a+ia\cot(e+fx))} + \frac{d(c+dx)}{2f^2(a+ia\cot(e+fx))} - $$

$$= -\frac{d^2x}{4af^2} + \frac{i(c+dx)^2}{4af} + \frac{(c+dx)^3}{6ad} + \frac{id^2}{4f^3(a+ia\cot(e+fx))} + \frac{d(c+dx)}{2f^2(a+ia\cot(e-}$$

Mathematica [A]
time = 0.28, size = 149, normalized size = 1.09

$$\frac{4f^3x(3c^2+3cdx+d^2x^2)+3((1+i)cf+d(-1+(1+i)fx))((1+i)cf+d(i+(1+i)fx))\cos(2fx)(\cos(2e)+i\sin(2e))+3i((1+i)cf+d(-1+(1+i)fx))((1+i)cf+d(i+(1+i)fx))(\cos(2e)+i\sin(2e))\sin(2fx)}{24af^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] (4*f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 3*((1 + I)*c*f + d*(-1 + (1 + I)*f*x
))*((1 + I)*c*f + d*(I + (1 + I)*f*x))*Cos[2*f*x]*(Cos[2*e] + I*Sin[2*e]) +
 (3*I)*((1 + I)*c*f + d*(-1 + (1 + I)*f*x))*((1 + I)*c*f + d*(I + (1 + I)*f
*x))*(Cos[2*e] + I*Sin[2*e])*Sin[2*f*x])/(24*a*f^3)
```

Maple [A]
time = 0.76, size = 108, normalized size = 0.79

method	result	size
risch	$\frac{d^2x^3}{6a}+\frac{dc\,x^2}{2a}+\frac{c^2x}{2a}+\frac{c^3}{6ad}+\frac{i(2d^2x^2f^2+4cd\,f^2x+2id^2fx+2c^2f^2+2icdf-d^2)\,e^{2i(fx+e)}}{8f^3a}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+I*a*cot(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a*d^2*x^3+1/2/a*d*c*x^2+1/2/a*c^2*x+1/6/a/d*c^3+1/8*I*(2*d^2*x^2*f^2+2*
I*d^2*f*x+4*c*d*f^2*x+2*I*c*d*f+2*c^2*f^2-d^2)/f^3/a*exp(2*I*(f*x+e))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.79, size = 97, normalized size = 0.71

$$\frac{4\,d^2f^3x^3+12\,cdf^3x^2+12\,c^2f^3x-3\,(-2i\,d^2f^2x^2-2i\,c^2f^2+2\,cdf+i\,d^2+2\,(-2i\,cdf^2+d^2f)x)e^{(2i\,fx+2i\,e)}}{24\,af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*d^2*f^3*x^3 + 12*c*d*f^3*x^2 + 12*c^2*f^3*x - 3*(-2*I*d^2*f^2*x^2 -
 2*I*c^2*f^2 + 2*c*d*f + I*d^2 + 2*(-2*I*c*d*f^2 + d^2*f)*x)*e^(2*I*f*x + 2
*I*e))/(a*f^3)
```

Sympy [A]
time = 0.15, size = 194, normalized size = 1.42

$$\begin{cases} \frac{(2ic^2f^2e^{2ie}+4icdf^2xe^{2ie}-2cdfe^{2ie}+2id^2f^2x^2e^{2ie}-2d^2fxe^{2ie}-id^2e^{2ie})e^{2ifx}}{8af^3} & \text{for } af^3\neq 0 \\ -\frac{c^2xe^{2ie}}{2a}-\frac{cdx^2e^{2ie}}{2a}-\frac{d^2x^3e^{2ie}}{6a} & \text{otherwise} \end{cases}+\frac{c^2x}{2a}+\frac{cdx^2}{2a}+\frac{d^2x^3}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+I*a*cot(f*x+e)),x)
```

```
[Out] Piecewise(((2*I*c**2*f**2*exp(2*I*e) + 4*I*c*d*f**2*x*exp(2*I*e) - 2*c*d*f*
exp(2*I*e) + 2*I*d**2*f**2*x**2*exp(2*I*e) - 2*d**2*f*x*exp(2*I*e) - I*d**2
*exp(2*I*e))*exp(2*I*f*x)/(8*a*f**3), Ne(a*f**3, 0)), (-c**2*x*exp(2*I*e)/(
2*a) - c*d*x**2*exp(2*I*e)/(2*a) - d**2*x**3*exp(2*I*e)/(6*a), True)) + c**
2*x/(2*a) + c*d*x**2/(2*a) + d**2*x**3/(6*a)
```

Giac [A]
time = 0.42, size = 137, normalized size = 1.00

$$\frac{4\,d^2f^3x^3+12\,cdf^3x^2+6i\,d^2f^2x^2e^{(2i\,fx+2i\,e)}+12\,c^2f^3x+12i\,cdf^2xe^{(2i\,fx+2i\,e)}+6i\,c^2f^2e^{(2i\,fx+2i\,e)}-6\,d^2fxe^{(2i\,fx+2i\,e)}-6\,cdfe^{(2i\,fx+2i\,e)}-3i\,d^2e^{(2i\,fx+2i\,e)}}{24\,af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/24*(4*d^2*f^3*x^3 + 12*c*d*f^3*x^2 + 6*I*d^2*f^2*x^2*e^(2*I*f*x + 2*I*e)
+ 12*c^2*f^3*x + 12*I*c*d*f^2*x*e^(2*I*f*x + 2*I*e) + 6*I*c^2*f^2*e^(2*I*f*
x + 2*I*e) - 6*d^2*f*x*e^(2*I*f*x + 2*I*e) - 6*c*d*f*e^(2*I*f*x + 2*I*e) -
3*I*d^2*e^(2*I*f*x + 2*I*e))/(a*f^3)
```

Mupad [B]
time = 0.77, size = 241, normalized size = 1.76

$$-\frac{6\,c^2\,f^2\,\sin\left(2\,e+2\,f\,x\right)-12\,c^2\,f^3\,x-3\,d^2\,\sin\left(2\,e+2\,f\,x\right)-4\,d^2\,f^3\,x^3+6\,c\,d\,f\,\cos\left(2\,e+2\,f\,x\right)+6\,d^2\,f^2\,x^2\,\sin\left(2\,e+2\,f\,x\right)-12\,c\,d\,f^3\,x^2+6\,d^2\,f\,x\,\cos\left(2\,e+2\,f\,x\right)+12\,c\,d\,f^2\,x\,\sin\left(2\,e+2\,f\,x\right)+d^2\,\cos\left(2\,e+2\,f\,x\right)\,3\mathrm{i}-c^2\,f^2\,\cos\left(2\,e+2\,f\,x\right)\,6\mathrm{i}+c\,d\,f\,\sin\left(2\,e+2\,f\,x\right)\,6\mathrm{i}-d^2\,f^2\,x^2\,\cos\left(2\,e+2\,f\,x\right)\,6\mathrm{i}+d^2\,f\,x\,\sin\left(2\,e+2\,f\,x\right)\,6\mathrm{i}-c\,d\,f^2\,x\,\cos\left(2\,e+2\,f\,x\right)\,12\mathrm{i}}{24\,a\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*cot(e + f*x)*1i),x)
```

```
[Out] -(d^2*cos(2*e + 2*f*x)*3i - 3*d^2*sin(2*e + 2*f*x) - 12*c^2*f^3*x - c^2*f^2
*cos(2*e + 2*f*x)*6i + 6*c^2*f^2*sin(2*e + 2*f*x) - 4*d^2*f^3*x^3 + 6*c*d*f
*cos(2*e + 2*f*x) + c*d*f*sin(2*e + 2*f*x)*6i - d^2*f^2*x^2*cos(2*e + 2*f*x
)*6i + 6*d^2*f^2*x^2*sin(2*e + 2*f*x) - 12*c*d*f^3*x^2 + 6*d^2*f*x*cos(2*e
+ 2*f*x) + d^2*f*x*sin(2*e + 2*f*x)*6i - c*d*f^2*x*cos(2*e + 2*f*x)*12i + 1
2*c*d*f^2*x*sin(2*e + 2*f*x))/(24*a*f^3)
```

3.18 $\int \frac{c+dx}{a+ia\cot(e+fx)}\,dx$

Optimal. Leaf size=84

$$\frac{idx}{4af}+\frac{(c+dx)^2}{4ad}+\frac{d}{4f^2(a+ia\cot(e+fx))}-\frac{i(c+dx)}{2f(a+ia\cot(e+fx))}$$

```
[Out] 1/4*I*d*x/a/f+1/4*(d*x+c)^2/a/d+1/4*d/f^2/(a+I*a*cot(f*x+e))-1/2*I*(d*x+c)/
f/(a+I*a*cot(f*x+e))
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3804, 3560, 8}

$$-\frac{i(c+dx)}{2f(a+ia\cot(e+fx))}+\frac{(c+dx)^2}{4ad}+\frac{d}{4f^2(a+ia\cot(e+fx))}+\frac{idx}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] ((I/4)*d*x)/(a*f) + (c + d*x)^2/(4*a*d) + d/(4*f^2*(a + I*a*Cot[e + f*x]))
- ((I/2)*(c + d*x))/(f*(a + I*a*Cot[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3804

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[a*d*(m/(2*b*f))
, Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m/
(2*b*f*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 + b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{a+ia\cot(e+fx)}\,dx = \frac{(c+dx)^2}{4ad} - \frac{i(c+dx)}{2f(a+ia\cot(e+fx))} + \frac{(id)\int \frac{1}{a+ia\cot(e+fx)}\,dx}{2f}$$
$$= \frac{(c+dx)^2}{4ad} + \frac{d}{4f^2(a+ia\cot(e+fx))} - \frac{i(c+dx)}{2f(a+ia\cot(e+fx))} + \frac{(id)\int 1\,dx}{4af}$$
$$= \frac{idx}{4af} + \frac{(c+dx)^2}{4ad} + \frac{d}{4f^2(a+ia\cot(e+fx))} - \frac{i(c+dx)}{2f(a+ia\cot(e+fx))}$$

Mathematica [A]
time = 0.20, size = 107, normalized size = 1.27

$$\frac{(\cos(e+fx)+i\sin(e+fx))\left((2cf(i+2fx)+d(-1+2ifx+2f^2x^2))\cos(e+fx)-i(2cf(-i+2fx)+d(1-2ifx+2f^2x^2))\sin(e+fx)\right)}{8af^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] ((Cos[e + f*x] + I*Sin[e + f*x])*((2*c*f*(I + 2*f*x) + d*(-1 + (2*I)*f*x +
2*f^2*x^2))*Cos[e + f*x] - I*(2*c*f*(-I + 2*f*x) + d*(1 - (2*I)*f*x + 2*f^2
*x^2))*Sin[e + f*x]))/(8*a*f^2)
```

Maple [A]
time = 0.69, size = 50, normalized size = 0.60

method	result	size
risch	$\frac{dx^2}{4a}+\frac{cx}{2a}+\frac{i(2dfx+2cf+id)e^{2i(fx+e)}}{8af^2}$	50
norman	$\frac{\frac{dx^2}{4a}-\frac{-2icf+d}{4af^2}+\frac{dx^2\left(\tan^2(fx+e)\right)}{4a}-\frac{(2cf+id)\tan(fx+e)}{4f^2a}+\frac{(2cf+id)x}{4fa}-\frac{dx\tan(fx+e)}{2af}+\frac{(2cf-id)x\left(\tan^2(fx+e)\right)}{4fa}}{1+\tan^2(fx+e)}$	139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+I*a*cot(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*d*x^2+1/2/a*c*x+1/8*I*(2*d*f*x+I*d+2*c*f)/a/f^2*exp(2*I*(f*x+e))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 5.00, size = 49, normalized size = 0.58

$$\frac{2\,df^2x^2 + 4\,cf^2x + (2i\,dfx + 2i\,cf - d)e^{(2i\,fx+2i\,e)}}{8\,af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*d*f^2*x^2 + 4*c*f^2*x + (2*I*d*f*x + 2*I*c*f - d)*e^(2*I*f*x + 2*I*e
))/(a*f^2)
```

Sympy [A]
time = 0.11, size = 100, normalized size = 1.19

$$\begin{cases} \frac{\left(2icfe^{2ie}+2idfxe^{2ie}-de^{2ie}\right)e^{2ifx}}{8af^2} & \text{for}\ af^2 \neq 0 \\ -\frac{cxe^{2ie}}{2a} - \frac{dx^2e^{2ie}}{4a} & \text{otherwise} \end{cases} + \frac{cx}{2a} + \frac{dx^2}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e)),x)
```

```
[Out] Piecewise((((2*I*c*f*exp(2*I*e) + 2*I*d*f*x*exp(2*I*e) - d*exp(2*I*e))*exp(2
*I*f*x)/(8*a*f**2), Ne(a*f**2, 0)), (-c*x*exp(2*I*e)/(2*a) - d*x**2*exp(2*I
*e)/(4*a), True)) + c*x/(2*a) + d*x**2/(4*a)
```

Giac [A]
time = 0.44, size = 64, normalized size = 0.76

$$\frac{2\,df^2x^2 + 4\,cf^2x + 2i\,dfxe^{(2i\,fx+2i\,e)} + 2i\,cfe^{(2i\,fx+2i\,e)} - de^{(2i\,fx+2i\,e)}}{8\,af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/8*(2*d*f^2*x^2 + 4*c*f^2*x + 2*I*d*f*x*e^(2*I*f*x + 2*I*e) + 2*I*c*f*e^(2
*I*f*x + 2*I*e) - d*e^(2*I*f*x + 2*I*e))/(a*f^2)
```

Mupad [B]
time = 0.38, size = 105, normalized size = 1.25

$$-\frac{d\,\cos\left(2\,e+2\,f\,x\right)-2\,d\,f^2\,x^2+2\,c\,f\,\sin\left(2\,e+2\,f\,x\right)-4\,c\,f^2\,x+2\,d\,f\,x\,\sin\left(2\,e+2\,f\,x\right)+d\,\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}-c\,f\,\cos\left(2\,e+2\,f\,x\right)\,2{}\mathrm{i}-d\,f\,x\,\cos\left(2\,e+2\,f\,x\right)\,2{}\mathrm{i}}{8\,a\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*cot(e + f*x)*1i),x)
```

```
[Out] -(d*cos(2*e + 2*f*x) + d*sin(2*e + 2*f*x)*1i - 2*d*f^2*x^2 - c*f*cos(2*e +
2*f*x)*2i + 2*c*f*sin(2*e + 2*f*x) - 4*c*f^2*x - d*f*x*cos(2*e + 2*f*x)*2i
+ 2*d*f*x*sin(2*e + 2*f*x))/(8*a*f^2)
```

3.19 $\int \frac{1}{(c+dx)(a+ia\cot(e+fx))}\,dx$

Optimal. Leaf size=161

$$-\frac{\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{2ad}+\frac{\log(c+dx)}{2ad}-\frac{i\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{2ad}-\frac{i\cos\left(2e-\right.}{}$$

```
[Out] -1/2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a/d+1/2*ln(d*x+c)/a/d-1/2*I*cos(-2
*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a/d+1/2*I*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)
/a/d-1/2*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a/d
```

Rubi [A]
time = 0.22, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3807, 3384, 3380, 3383}

$$-\frac{i\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{2ad}-\frac{\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{2ad}+\frac{\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{2ad}-\frac{i\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{2ad}+\frac{\log(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + I*a*Cot[e + f*x])),x]
```

```
[Out] -1/2*(Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(a*d) + Log[c +
d*x]/(2*a*d) - ((I/2)*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/
(a*d) - ((I/2)*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a*d) +
 (Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*a*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3807

```
Int[1/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Symb
ol] :> Simp[Log[c + d*x]/(2*a*d), x] + (Dist[1/(2*a), Int[Cos[2*e + 2*f*x]/
(c + d*x), x], x] + Dist[1/(2*b), Int[Sin[2*e + 2*f*x]/(c + d*x), x], x]) /
; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+dx)(a+ia\cot(e+fx))}\,dx &= \frac{\log(c+dx)}{2ad} + \frac{i\int \frac{\sin\left(2\left(e+\frac{\pi}{2}\right)+2fx\right)}{c+dx}\,dx}{2a} + \frac{\int \frac{\cos\left(2\left(e+\frac{\pi}{2}\right)+2fx\right)}{c+dx}\,dx}{2a}\\ &= \frac{\log(c+dx)}{2ad} - \frac{\left(i\cos\left(2e-\frac{2cf}{d}\right)\right)\int \frac{\sin\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx}{2a} - \frac{\cos\left(2e-\frac{2cf}{d}\right)\int}{2a}\\ &= -\frac{\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)}{2ad} + \frac{\log(c+dx)}{2ad} - \frac{i\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)\sin\left(\right.}{2ad}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 77, normalized size = 0.48

$$\frac{\log(c+dx)-\left(\cos\left(2e-\frac{2cf}{d}\right)+i\sin\left(2e-\frac{2cf}{d}\right)\right)\left(\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)+i\text{Si}\left(\frac{2f(c+dx)}{d}\right)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + I*a*Cot[e + f*x])),x]
[Out] (Log[c + d*x] - (Cos[2*e - (2*c*f)/d] + I*Sin[2*e - (2*c*f)/d])*(CosIntegra
l[(2*f*(c + d*x))/d] + I*SinIntegral[(2*f*(c + d*x))/d]))/(2*a*d)
```

Maple [A]
time = 0.66, size = 67, normalized size = 0.42

method	result	size
risch	$\frac{\ln(dx+c)}{2ad}+\frac{e^{-\frac{2i(cf-de)}{d}}\text{expIntegral}\left(1,-2ifx-2ie-\frac{2(icf-ide)}{d}\right)}{2ad}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+I*a*cot(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(d*x+c)/a/d+1/2/a/d*exp(-2*I*(c*f-d*e)/d)*Ei(1,-2*I*f*x-2*I*e-2*(I*c*
f-I*d*e)/d)
```

Maxima [A]
time = 0.32, size = 119, normalized size = 0.74

$$\frac{f\cos\left(\frac{2(cf-de)}{d}\right)E_1\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)-ifE_1\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\sin\left(\frac{2(cf-de)}{d}\right)+f\log\left((fx+e)d+cf-de\right)}{2\,adf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(f*cos(2*(c*f - d*e)/d)*exp_integral_e(1, 2*(-I*(f*x + e)*d - I*c*f + I
*d*e)/d) - I*f*exp_integral_e(1, 2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d)*sin(
2*(c*f - d*e)/d) + f*log((f*x + e)*d + c*f - d*e))/(a*d*f)
```

Fricas [A]
time = 2.72, size = 54, normalized size = 0.34

$$-\frac{\operatorname{Ei}\left(-\frac{2(-idfx-icf)}{d}\right)e^{\left(-\frac{2(icf-ide)}{d}\right)}-\log\left(\frac{dx+c}{d}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(Ei(-2*(-I*d*f*x - I*c*f)/d)*e^(-2*(I*c*f - I*d*e)/d) - log((d*x + c)/
d))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{c\cot(e+fx)-ic+dx\cot(e+fx)-idx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e)),x)
```

```
[Out] -I*Integral(1/(c*cot(e + f*x) - I*c + d*x*cot(e + f*x) - I*d*x), x)/a
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(147) = 294.
time = 0.43, size = 351, normalized size = 2.18

$$-\frac{\cos(e)^2\cos\left(\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2(dfx+cf)}{d}\right)+2i\cos(e)\cos\left(\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2(dfx+cf)}{d}\right)\sin(e)-\cos\left(\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2(dfx+cf)}{d}\right)\sin(e)^2-i\cos(e)^2\operatorname{Ci}\left(\frac{2(dfx+cf)}{d}\right)\sin\left(\frac{2cf}{d}\right)+2\cos(e)\operatorname{Ci}\left(\frac{2(dfx+cf)}{d}\right)\sin(e)\sin\left(\frac{2cf}{d}\right)+i\operatorname{Ci}\left(\frac{2(dfx+cf)}{d}\right)\sin(e)^2\sin\left(\frac{2cf}{d}\right)+i\cos(e)^2\cos\left(\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2(dfx+cf)}{d}\right)-2\cos(e)\cos\left(\frac{2cf}{d}\right)\sin(e)\operatorname{Si}\left(\frac{2(dfx+cf)}{d}\right)-i\cos\left(\frac{2cf}{d}\right)\sin(e)^2\operatorname{Si}\left(\frac{2(dfx+cf)}{d}\right)+\cos(e)^2\sin\left(\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2(dfx+cf)}{d}\right)+2i\cos(e)\sin(e)\sin\left(\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2(dfx+cf)}{d}\right)-\sin(e)^2\sin\left(\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2(dfx+cf)}{d}\right)-\log(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*(cos(e)^2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d) + 2*I*cos(e)*co
s(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) - cos(2*c*f/d)*cos_integr
al(2*(d*f*x + c*f)/d)*sin(e)^2 - I*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d)
*sin(2*c*f/d) + 2*cos(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)*sin(2*c*f/d
) + I*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2*sin(2*c*f/d) + I*cos(e)^2*co
s(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - 2*cos(e)*cos(2*c*f/d)*sin(e)*s
in_integral(2*(d*f*x + c*f)/d) - I*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*
f*x + c*f)/d) + cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 2*I
*cos(e)*sin(e)*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - sin(e)^2*sin(
2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - log(d*x + c))/(a*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+a\cot(e+f\,x)\ 1\mathrm{i})\ (c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)),x)
```

```
[Out] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)), x)
```

3.20 $\int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))}\,dx$

Optimal. Leaf size=166

$$-\frac{if\cos\left(2e-\frac{2cf}{d}\right)\operatorname{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{ad^2}-\frac{1}{d(c+dx)(a+ia\cot(e+fx))}+\frac{f\operatorname{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2\right.}{ad^2}$$

```
[Out] -I*f*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a/d^2-1/d/(d*x+c)/(a+I*a*cot(f*x+e
))+f*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a/d^2-f*Ci(2*c*f/d+2*f*x)*sin(-2*e
+2*c*f/d)/a/d^2-I*f*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a/d^2
```

Rubi [A]
time = 0.17, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3805, 3384, 3380, 3383}

$$\frac{f\operatorname{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{ad^2}-\frac{if\operatorname{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{ad^2}+\frac{if\sin\left(2e-\frac{2cf}{d}\right)\operatorname{Si}\left(2xf+\frac{2cf}{d}\right)}{ad^2}+\frac{f\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Si}\left(2xf+\frac{2cf}{d}\right)}{ad^2}-\frac{1}{d(c+dx)(a+ia\cot(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])),x]
```

```
[Out] ((-I)*f*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(a*d^2) - 1/(d
*(c + d*x)*(a + I*a*Cot[e + f*x])) + (f*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[
2*e - (2*c*f)/d])/(a*d^2) + (f*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d +
 2*f*x])/(a*d^2) + (I*f*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x]
)/(a*d^2)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3805

```
Int[1/(((c_.) + (d_.)*(x_))^2*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Sy
mbol] :> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Dist[f/(a*d)
, Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Dist[f/(b*d), Int[Cos[2*e + 2*f*
x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))}\,dx &= -\frac{1}{d(c+dx)(a+ia\cot(e+fx))}+\frac{(if)\int\frac{\cos\left(2\left(e+\frac{\pi}{2}\right)+2fx\right)}{c+dx}\,dx}{ad}-\frac{f\int\frac{\sin}{}}{}\\ &= -\frac{1}{d(c+dx)(a+ia\cot(e+fx))}-\frac{\left(if\cos\left(2e-\frac{2cf}{d}\right)\right)\int\frac{\cos\left(\frac{2cf}{d}+2fx\right)}{c+dx}}{ad}\\ &= -\frac{if\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)}{ad^2}-\frac{1}{d(c+dx)(a+ia\cot(e+fx))}+\end{aligned}$$

Mathematica [A]

time = 1.27, size = 215, normalized size = 1.30

$$\frac{\left(\cos\left(e+f\left(-\frac{c}{d}+x\right)\right)+i\sin\left(e+f\left(-\frac{c}{d}+x\right)\right)\right)\left(d\left(-\cos\left(e+f\left(-\frac{c}{d}+x\right)\right)+\cos\left(e+f\left(\frac{c}{d}+x\right)\right)+i\left(\sin\left(e+f\left(-\frac{c}{d}+x\right)\right)+\sin\left(e+f\left(\frac{c}{d}+x\right)\right)\right)\right)+2f(c+dx)\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)\left(-i\cos\left(e-\frac{f(c+dx)}{d}\right)+\sin\left(e-\frac{f(c+dx)}{d}\right)\right)+2f(c+dx)\left(\cos\left(e-\frac{f(c+dx)}{d}\right)+i\sin\left(e-\frac{f(c+dx)}{d}\right)\right)\operatorname{Si}\left(\frac{2f(c+dx)}{d}\right)\right)}{2ad^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])),x]
```

```
[Out] ((Cos[e + f*(-(c/d) + x)] + I*Sin[e + f*(-(c/d) + x)])*(d*(-Cos[e + f*(-(c/
d) + x)] + Cos[e + f*(c/d + x)] + I*(Sin[e + f*(-(c/d) + x)] + Sin[e + f*(c
/d + x)])) + 2*f*(c + d*x)*CosIntegral[(2*f*(c + d*x))/d]*((-I)*Cos[e - (f*
(c + d*x))/d] + Sin[e - (f*(c + d*x))/d]) + 2*f*(c + d*x)*(Cos[e - (f*(c +
d*x))/d] + I*Sin[e - (f*(c + d*x))/d])*SinIntegral[(2*f*(c + d*x))/d]))/(2*
a*d^2*(c + d*x))
```

Maple [A]

time = 0.64, size = 105, normalized size = 0.63

method	result	size
risch	$-\frac{1}{2d(dx+c)a}+\frac{if\,\mathrm{e}^{2i(fx+e)}}{2a\,d^2\left(ifx+\frac{icf}{d}\right)}+\frac{if\,\mathrm{e}^{-\frac{2i(cf-de)}{d}}\,\text{expIntegral}\left(1,-2ifx-2ie-\frac{2(icf-ide)}{d}\right)}{a\,d^2}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+I*a*cot(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d/(d*x+c)/a+1/2*I/a*f/d^2*exp(2*I*(f*x+e))/(I*f*x+I/d*c*f)+I/a*f/d^2*e
xp(-2*I*(c*f-d*e)/d)*Ei(1,-2*I*f*x-2*I*e-2*(I*c*f-I*d*e)/d)
```

Maxima [A]
time = 0.35, size = 129, normalized size = 0.78

$$\frac{f^2\cos\left(\frac{2\,(cf-de)}{d}\right)E_2\left(\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)-i\,f^2E_2\left(\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)\sin\left(\frac{2\,(cf-de)}{d}\right)-f^2}{2\,((fx+e)ad^2+acdf-ad^2e)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(f^2*cos(2*(c*f - d*e)/d)*exp_integral_e(2, 2*(-I*(f*x + e)*d - I*c*f +
 I*d*e)/d) - I*f^2*exp_integral_e(2, 2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d)*
sin(2*(c*f - d*e)/d) - f^2)/(((f*x + e)*a*d^2 + a*c*d*f - a*d^2*e)*f)
```

Fricas [A]
time = 2.10, size = 76, normalized size = 0.46

$$-\frac{2\,(i\,dfx+i\,cf)\mathrm{Ei}\left(-\frac{2\,(-i\,dfx-i\,cf)}{d}\right)e^{\left(-\frac{2\,(i\,cf-i\,de)}{d}\right)}-de^{(2i\,fx+2i\,e)}+d}{2\,(ad^3x+acd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(I*d*f*x + I*c*f)*Ei(-2*(-I*d*f*x - I*c*f)/d)*e^(-2*(I*c*f - I*d*e)
/d) - d*e^(2*I*f*x + 2*I*e) + d)/(a*d^3*x + a*c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{c^2\cot{(e+fx)}-ic^2+2cdx\cot{(e+fx)}-2icdx+d^2x^2\cot{(e+fx)}-id^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+I*a*cot(f*x+e)),x)
```

```
[Out] -I*Integral(1/(c**2*cot(e + f*x) - I*c**2 + 2*c*d*x*cot(e + f*x) - 2*I*c*d*
x + d**2*x**2*cot(e + f*x) - I*d**2*x**2), x)/a
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(161) = 322.
time = 1.41, size = 357, normalized size = 2.15

$$-\frac{i\left(-2i\,(dx+c)\left(\frac{i\,de}{dx+c}-\frac{i\,cf}{dx+c}+i\,f\right)f^2\mathrm{Ei}\left(\frac{2\left((dx+c)\left(\frac{i\,de}{dx+c}-\frac{i\,cf}{dx+c}+i\,f\right)-i\,de+i\,cf\right)}{d}\right)e^{\left(-\frac{2\,(-i\,de+i\,cf)}{d}\right)}-2\,def^2\mathrm{Ei}\left(\frac{2\left((dx+c)\left(\frac{i\,de}{dx+c}-\frac{i\,cf}{dx+c}+i\,f\right)-i\,de+i\,cf\right)}{d}\right)e^{\left(-\frac{2\,(-i\,de+i\,cf)}{d}\right)}+2\,cf^3\mathrm{Ei}\left(\frac{2\left((dx+c)\left(\frac{i\,de}{dx+c}-\frac{i\,cf}{dx+c}+i\,f\right)-i\,de+i\,cf\right)}{d}\right)e^{\left(-\frac{2\,(-i\,de+i\,cf)}{d}\right)}+i\,df^2e^{\left(-\frac{2\,(dx+c)\left(-\frac{i\,de}{dx+c}+\frac{i\,cf}{dx+c}-i\,f\right)}{d}\right)}\right)d^2}{2\,\left(-i\,(dx+c)d^4\left(\frac{i\,de}{dx+c}-\frac{i\,cf}{dx+c}+i\,f\right)-d^5e+cd^4f\right)af}-\frac{1}{2\,(dx+c)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*I*(-2*I*(d*x + c)*(I*d*e/(d*x + c) - I*c*f/(d*x + c) + I*f)*f^2*Ei(2*(
(d*x + c)*(I*d*e/(d*x + c) - I*c*f/(d*x + c) + I*f) - I*d*e + I*c*f)/d)*e^(
-2*(-I*d*e + I*c*f)/d) - 2*d*e*f^2*Ei(2*((d*x + c)*(I*d*e/(d*x + c) - I*c*f
/(d*x + c) + I*f) - I*d*e + I*c*f)/d)*e^(-2*(-I*d*e + I*c*f)/d) + 2*c*f^3*E
i(2*((d*x + c)*(I*d*e/(d*x + c) - I*c*f/(d*x + c) + I*f) - I*d*e + I*c*f)/d
)*e^(-2*(-I*d*e + I*c*f)/d) + I*d*f^2*e^(-2*(d*x + c)*(-I*d*e/(d*x + c) + I
*c*f/(d*x + c) - I*f)/d))*d^2/((-I*(d*x + c)*d^4*(I*d*e/(d*x + c) - I*c*f/(
d*x + c) + I*f) - d^5*e + c*d^4*f)*a*f) - 1/2/((d*x + c)*a*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\cot\left(e+f\,x\right)\,1\mathrm{i}\right)\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)^2), x)
```

3.21 $\int \frac{1}{(c+dx)^3(a+ia\cot(e+fx))}\,dx$

Optimal. Leaf size=227

$$\frac{if}{2ad^2(c+dx)}+\frac{f^2\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{ad^3}-\frac{1}{2d(c+dx)^2(a+ia\cot(e+fx))}-\frac{i}{d^2(c+dx)(a+}$$

```
[Out] 1/2*I*f/a/d^2/(d*x+c)+f^2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a/d^3-1/2/d/(
d*x+c)^2/(a+I*a*cot(f*x+e))-I*f/d^2/(d*x+c)/(a+I*a*cot(f*x+e))+I*f^2*cos(-2
*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a/d^3-I*f^2*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/
d)/a/d^3+f^2*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a/d^3
```

Rubi [A]
time = 0.23, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3806, 3805, 3384, 3380, 3383}

$$\frac{if^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{ad^3}+\frac{f^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{ad^3}-\frac{f^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{ad^3}+\frac{if^2\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{ad^3}-\frac{if}{d^2(c+dx)(a+ia\cot(e+fx))}+\frac{if}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a+ia\cot(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^3*(a + I*a*Cot[e + f*x])),x]
```

```
[Out] ((I/2)*f)/(a*d^2*(c + d*x)) + (f^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)
/d + 2*f*x])/(a*d^3) - 1/(2*d*(c + d*x)^2*(a + I*a*Cot[e + f*x])) - (I*f)/(
d^2*(c + d*x)*(a + I*a*Cot[e + f*x])) + (I*f^2*CosIntegral[(2*c*f)/d + 2*f*
x]*Sin[2*e - (2*c*f)/d])/(a*d^3) + (I*f^2*Cos[2*e - (2*c*f)/d]*SinIntegral[
(2*c*f)/d + 2*f*x])/(a*d^3) - (f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)
/d + 2*f*x])/(a*d^3)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3805

```
Int[1/(((c_.) + (d_.)*(x_))^2*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Sy
mbol] :> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Dist[f/(a*d)
, Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Dist[f/(b*d), Int[Cos[2*e + 2*f*
x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3806

```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sym
bol] :> Simp[f*((c + d*x)^(m + 2)/(b*d^2*(m + 1)*(m + 2))), x] + (Dist[2*b*
(f/(a*d*(m + 1))), Int[(c + d*x)^(m + 1)/(a + b*Tan[e + f*x]), x], x] + Sim
p[(c + d*x)^(m + 1)/(d*(m + 1)*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3(a+ia\cot(e+fx))}\,dx = \frac{if}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+ia\cot(e+fx))} + \frac{(if)\int \frac{1}{(c+dx)^2(a+ia}}{d}$$

$$= \frac{if}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+ia\cot(e+fx))} - \frac{if}{d^2(c+dx)(a+i}$$

$$= \frac{if}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+ia\cot(e+fx))} - \frac{if}{d^2(c+dx)(a+i}$$

$$= \frac{if}{2ad^2(c+dx)} + \frac{f^2\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)}{ad^3} - \frac{1}{2d(c+dx)^2(a+}$$

Mathematica [A]
time = 1.61, size = 283, normalized size = 1.25

$$\frac{\left(\cos\left(e+f\left(-\frac{c}{d}+x\right)\right)+i\sin\left(e+f\left(-\frac{c}{d}+x\right)\right)\right)\left(4f^2(c+dx)^2\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)\left(\cos\left(e-\frac{f(c+dx)}{d}\right)+i\sin\left(e-\frac{f(c+dx)}{d}\right)\right)+i\left(d\left(id\cos\left(e+f\left(-\frac{c}{d}+x\right)\right)+(-id+2cf+2dfx)\cos\left(e+f\left(\frac{c}{d}+x\right)\right)+d\sin\left(e+f\left(-\frac{c}{d}+x\right)\right)+d\sin\left(e+f\left(\frac{c}{d}+x\right)\right)+2icf\sin\left(e+f\left(\frac{c}{d}+x\right)\right)+2idfx\sin\left(e+f\left(\frac{c}{d}+x\right)\right)\right)+4f^2(c+dx)^2\left(\cos\left(e-\frac{f(c+dx)}{d}\right)+i\sin\left(e-\frac{f(c+dx)}{d}\right)\right)\text{Si}\left(\frac{2f(c+dx)}{d}\right)\right)\right)}{4ad^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^3*(a + I*a*Cot[e + f*x])),x]
```

```
[Out] ((Cos[e + f*(-(c/d) + x)] + I*Sin[e + f*(-(c/d) + x)])*(4*f^2*(c + d*x)^2*C
osIntegral[(2*f*(c + d*x))/d]*(Cos[e - (f*(c + d*x))/d] + I*Sin[e - (f*(c +
 d*x))/d]) + I*(d*(I*d*Cos[e + f*(-(c/d) + x)] + ((-I)*d + 2*c*f + 2*d*f*x)
*Cos[e + f*(c/d + x)] + d*Sin[e + f*(-(c/d) + x)] + d*Sin[e + f*(c/d + x)]
+ (2*I)*c*f*Sin[e + f*(c/d + x)] + (2*I)*d*f*x*Sin[e + f*(c/d + x)]) + 4*f^
2*(c + d*x)^2*(Cos[e - (f*(c + d*x))/d] + I*Sin[e - (f*(c + d*x))/d])*SinIn
tegral[(2*f*(c + d*x))/d])))/(4*a*d^3*(c + d*x)^2)
```

Maple [A]
time = 0.69, size = 143, normalized size = 0.63

method	result	size
risch	$-\frac{1}{4d(dx+c)^2 a}-\frac{f^2 e^{2i(fx+e)}}{4a\,d^3\left(ifx+\frac{icf}{d}\right)^2}-\frac{f^2 e^{2i(fx+e)}}{2a\,d^3\left(ifx+\frac{icf}{d}\right)}-\frac{f^2 e^{-\frac{2i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,-2ifx-2ie-\frac{2(icf-ide)}{d}\right)}{a\,d^3}$	143

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3/(a+I*a*cot(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d/(d*x+c)^2/a-1/4*f^2/a/d^3*exp(2*I*(f*x+e))/(I*f*x+I/d*c*f)^2-1/2*f^2
/a/d^3*exp(2*I*(f*x+e))/(I*f*x+I/d*c*f)-f^2/a/d^3*exp(-2*I*(c*f-d*e)/d)*Ei(
1,-2*I*f*x-2*I*e-2*(I*c*f-I*d*e)/d)
```

Maxima [A]
time = 0.36, size = 169, normalized size = 0.74

$$\frac{2\,f^3\cos\left(\frac{2\,(cf-de)}{d}\right)E_3\left(\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)-2i\,f^3E_3\left(\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)\sin\left(\frac{2\,(cf-de)}{d}\right)-f^3}{4\left((fx+e)^2ad^3+ac^2df^2-2\,acd^2fe+ad^3e^2+2\,(acd^2f-ad^3e)(fx+e)\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*f^3*cos(2*(c*f - d*e)/d)*exp_integral_e(3, 2*(-I*(f*x + e)*d - I*c*f
 + I*d*e)/d) - 2*I*f^3*exp_integral_e(3, 2*(-I*(f*x + e)*d - I*c*f + I*d*e)
/d)*sin(2*(c*f - d*e)/d) - f^3)/(((f*x + e)^2*a*d^3 + a*c^2*d*f^2 - 2*a*c*d
^2*f*e + a*d^3*e^2 + 2*(a*c*d^2*f - a*d^3*e)*(f*x + e))*f)
```

Fricas [A]
time = 2.98, size = 122, normalized size = 0.54

$$\frac{4\,(d^2f^2x^2+2\,cdf^2x+c^2f^2)\mathrm{Ei}\left(-\frac{2\,(-i\,dfx-i\,cf)}{d}\right)e^{\left(-\frac{2\,(i\,cf-i\,de)}{d}\right)}-d^2+(2i\,d^2fx+2i\,cdf+d^2)e^{(2i\,fx+2i\,e)}}{4\,(ad^5x^2+2\,acd^4x+ac^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*Ei(-2*(-I*d*f*x - I*c*f)/d)*e^
(-2*(I*c*f - I*d*e)/d) - d^2 + (2*I*d^2*f*x + 2*I*c*d*f + d^2)*e^(2*I*f*x +
 2*I*e))/(a*d^5*x^2 + 2*a*c*d^4*x + a*c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{c^3\cot(e+fx)-ic^3+3c^2dx\cot(e+fx)-3ic^2dx+3cd^2x^2\cot(e+fx)-3icd^2x^2+d^3x^3\cot(e+fx)-id^3x^3}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3/(a+I*a*cot(f*x+e)),x)
```

```
[Out] -I*Integral(1/(c**3*cot(e + f*x) - I*c**3 + 3*c**2*d*x*cot(e + f*x) - 3*I*c
**2*d*x + 3*c*d**2*x**2*cot(e + f*x) - 3*I*c*d**2*x**2 + d**3*x**3*cot(e +
f*x) - I*d**3*x**3), x)/a
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1558 vs. $2(210) = 420$.
time = 0.43, size = 1558, normalized size = 6.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*(4*d^2*f^2*x^2*cos(e)^2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d) +
8*I*d^2*f^2*x^2*cos(e)*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)
- 4*d^2*f^2*x^2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2 - 4*I
*d^2*f^2*x^2*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) + 8*d^2*
f^2*x^2*cos(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)*sin(2*c*f/d) + 4*I*d^
2*f^2*x^2*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2sin(2*c*f/d) + 4*I*d^2*f
^2*x^2*cos(e)^2*cos(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - 8*d^2*f^2*x^
2*cos(e)*cos(2*c*f/d)*sin(e)*sin_integral(2*(d*f*x + c*f)/d) - 4*I*d^2*f^2*
x^2*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) + 4*d^2*f^2*x^2*c
os(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 8*I*d^2*f^2*x^2*cos(
e)*sin(e)*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - 4*d^2*f^2*x^2*sin(
e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 8*c*d*f^2*x*cos(e)^2*co
s(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d) + 16*I*c*d*f^2*x*cos(e)*cos(2*c*
f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) - 8*c*d*f^2*x*cos(2*c*f/d)*cos_
integral(2*(d*f*x + c*f)/d)*sin(e)^2 - 8*I*c*d*f^2*x*cos(e)^2*cos_integral(
2*(d*f*x + c*f)/d)*sin(2*c*f/d) + 16*c*d*f^2*x*cos(e)*cos_integral(2*(d*f*x
 + c*f)/d)*sin(e)*sin(2*c*f/d) + 8*I*c*d*f^2*x*cos_integral(2*(d*f*x + c*f)
/d)*sin(e)^2sin(2*c*f/d) + 8*I*c*d*f^2*x*cos(e)^2*cos(2*c*f/d)*sin_integra
l(2*(d*f*x + c*f)/d) - 16*c*d*f^2*x*cos(e)*cos(2*c*f/d)*sin(e)*sin_integral
(2*(d*f*x + c*f)/d) - 8*I*c*d*f^2*x*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d
*f*x + c*f)/d) + 8*c*d*f^2*x*cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x +
c*f)/d) + 16*I*c*d*f^2*x*cos(e)*sin(e)*sin(2*c*f/d)*sin_integral(2*(d*f*x +
 c*f)/d) - 8*c*d*f^2*x*sin(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d
) + 4*c^2*f^2*cos(e)^2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d) + 8*I*c
^2*f^2*cos(e)*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) - 4*c^2*f
^2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2 - 4*I*c^2*f^2*cos(
e)^2*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) + 8*c^2*f^2*cos(e)*cos_in
tegral(2*(d*f*x + c*f)/d)*sin(e)*sin(2*c*f/d) + 4*I*c^2*f^2*cos_integral(2*
(d*f*x + c*f)/d)*sin(e)^2*sin(2*c*f/d) + 4*I*c^2*f^2*cos(e)^2*cos(2*c*f/d)*
```

```
sin_integral(2*(d*f*x + c*f)/d) - 8*c^2*f^2*cos(e)*cos(2*c*f/d)*sin(e)*sin_
integral(2*(d*f*x + c*f)/d) - 4*I*c^2*f^2*cos(2*c*f/d)*sin(e)^2*sin_integra
l(2*(d*f*x + c*f)/d) + 4*c^2*f^2*cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*
x + c*f)/d) + 8*I*c^2*f^2*cos(e)*sin(e)*sin(2*c*f/d)*sin_integral(2*(d*f*x
+ c*f)/d) - 4*c^2*f^2*sin(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d)
 + 2*I*d^2*f*x*cos(2*f*x)*cos(e)^2 - 2*d^2*f*x*cos(e)^2*sin(2*f*x) - 4*d^2*
f*x*cos(2*f*x)*cos(e)*sin(e) - 4*I*d^2*f*x*cos(e)*sin(2*f*x)*sin(e) - 2*I*d
^2*f*x*cos(2*f*x)*sin(e)^2 + 2*d^2*f*x*sin(2*f*x)*sin(e)^2 + 2*I*c*d*f*cos(
2*f*x)*cos(e)^2 - 2*c*d*f*cos(e)^2*sin(2*f*x) - 4*c*d*f*cos(2*f*x)*cos(e)*s
in(e) - 4*I*c*d*f*cos(e)*sin(2*f*x)*sin(e) - 2*I*c*d*f*cos(2*f*x)*sin(e)^2
+ 2*c*d*f*sin(2*f*x)*sin(e)^2 + d^2*cos(2*f*x)*cos(e)^2 + I*d^2*cos(e)^2*si
n(2*f*x) + 2*I*d^2*cos(2*f*x)*cos(e)*sin(e) - 2*d^2*cos(e)*sin(2*f*x)*sin(e
) - d^2*cos(2*f*x)*sin(e)^2 - I*d^2*sin(2*f*x)*sin(e)^2 - d^2)/(a*d^5*x^2 +
 2*a*c*d^4*x + a*c^2*d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + a\cot(e + f\,x)\ 1\mathrm{i})\ (c + d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)^3),x)
[Out] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)^3), x)
```

3.22 $\int \frac{(c+dx)^3}{(a+ia\cot(e+fx))^2}\,dx$

Optimal. Leaf size=270

$$\frac{3d^3e^{2ie+2ifx}}{16a^2f^4}-\frac{3d^3e^{4ie+4ifx}}{512a^2f^4}-\frac{3id^2e^{2ie+2ifx}(c+dx)}{8a^2f^3}+\frac{3id^2e^{4ie+4ifx}(c+dx)}{128a^2f^3}-\frac{3de^{2ie+2ifx}(c+dx)^2}{8a^2f^2}+\frac{3de^{4ie+4ifx}(}{64a^2}$$

```
[Out] 3/16*d^3*exp(2*I*e+2*I*f*x)/a^2/f^4-3/512*d^3*exp(4*I*e+4*I*f*x)/a^2/f^4-3/
8*I*d^2*exp(2*I*e+2*I*f*x)*(d*x+c)/a^2/f^3+3/128*I*d^2*exp(4*I*e+4*I*f*x)*(
d*x+c)/a^2/f^3-3/8*d*exp(2*I*e+2*I*f*x)*(d*x+c)^2/a^2/f^2+3/64*d*exp(4*I*e+
4*I*f*x)*(d*x+c)^2/a^2/f^2+1/4*I*exp(2*I*e+2*I*f*x)*(d*x+c)^3/a^2/f-1/16*I*
exp(4*I*e+4*I*f*x)*(d*x+c)^3/a^2/f+1/16*(d*x+c)^4/a^2/d
```

Rubi [A]
time = 0.19, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3810, 2207, 2225}

$$-\frac{3id^2(c+dx)e^{2ie+2ifx}}{8a^2f^3}+\frac{3id^2(c+dx)e^{4ie+4ifx}}{128a^2f^3}-\frac{3d(c+dx)^2e^{2ie+2ifx}}{8a^2f^2}+\frac{3d(c+dx)^2e^{4ie+4ifx}}{64a^2f^2}+\frac{i(c+dx)^3e^{2ie+2ifx}}{4a^2f}-\frac{i(c+dx)^3e^{4ie+4ifx}}{16a^2f}+\frac{(c+dx)^4}{16a^2d}+\frac{3d^3e^{2ie+2ifx}}{16a^2f^4}-\frac{3d^3e^{4ie+4ifx}}{512a^2f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] (3*d^3*E^((2*I)*e + (2*I)*f*x))/(16*a^2*f^4) - (3*d^3*E^((4*I)*e + (4*I)*f*
x))/(512*a^2*f^4) - (((3*I)/8)*d^2*E^((2*I)*e + (2*I)*f*x)*(c + d*x))/(a^2*
f^3) + (((3*I)/128)*d^2*E^((4*I)*e + (4*I)*f*x)*(c + d*x))/(a^2*f^3) - (3*d
*E^((2*I)*e + (2*I)*f*x)*(c + d*x)^2)/(8*a^2*f^2) + (3*d*E^((4*I)*e + (4*I)
*f*x)*(c + d*x)^2)/(64*a^2*f^2) + ((I/4)*E^((2*I)*e + (2*I)*f*x)*(c + d*x)^
3)/(a^2*f) - ((I/16)*E^((4*I)*e + (4*I)*f*x)*(c + d*x)^3)/(a^2*f) + (c + d*
x)^4/(16*a^2*d)
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3810

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^3}{(a+ia\cot(e+fx))^2}\,dx &= \int \left(\frac{(c+dx)^3}{4a^2}-\frac{e^{2ie+2ifx}(c+dx)^3}{2a^2}+\frac{e^{4ie+4ifx}(c+dx)^3}{4a^2}\right)dx \\
&= \frac{(c+dx)^4}{16a^2d}+\frac{\int e^{4ie+4ifx}(c+dx)^3\,dx}{4a^2}-\frac{\int e^{2ie+2ifx}(c+dx)^3\,dx}{2a^2} \\
&= \frac{ie^{2ie+2ifx}(c+dx)^3}{4a^2f}-\frac{ie^{4ie+4ifx}(c+dx)^3}{16a^2f}+\frac{(c+dx)^4}{16a^2d}+\frac{(3id)\int e^{4ie+4ifx}(c+d\ldots}{16a^2f} \\
&= -\frac{3de^{2ie+2ifx}(c+dx)^2}{8a^2f^2}+\frac{3de^{4ie+4ifx}(c+dx)^2}{64a^2f^2}+\frac{ie^{2ie+2ifx}(c+dx)^3}{4a^2f}-\frac{ie^{4ie+4if\ldots}}{16\ldots} \\
&= -\frac{3id^2e^{2ie+2ifx}(c+dx)}{8a^2f^3}+\frac{3id^2e^{4ie+4ifx}(c+dx)}{128a^2f^3}-\frac{3de^{2ie+2ifx}(c+dx)^2}{8a^2f^2}+\frac{3de^{4i\ldots}}{\ldots} \\
&= \frac{3d^3e^{2ie+2ifx}}{16a^2f^4}-\frac{3d^3e^{4ie+4ifx}}{512a^2f^4}-\frac{3id^2e^{2ie+2ifx}(c+dx)}{8a^2f^3}+\frac{3id^2e^{4ie+4ifx}(c+dx)}{128a^2f^3}-\ldots
\end{aligned}
$$

Mathematica [A]
time = 1.42, size = 362, normalized size = 1.34

$$
\frac{(\cos(2(e+fx))+i\sin(2(e+fx)))\left((32c^3f^3(-i+4fx)+24c^2df^2(1-4ifx+8f^2x^2)+4cd^2f(3i+12fx-24if^2x^2+32f^3x^3)+d^3(-3+12ifx+24f^2x^2-32if^3x^3+32f^4x^4))\cos(2(e+fx))-i(-32(4c^3f^3+6c^2df^2(i+2fx)+6cd^2f(-1+2ifx+2f^2x^2)+d^3(-3i-6fx+6if^2x^2+4f^3x^3))+(32c^3f^3(i+4fx)+24c^2df^2(-1+4ifx+8f^2x^2)+4cd^2f(-3i-12fx+24if^2x^2+32f^3x^3)+d^3(3-12ifx-24f^2x^2+32if^3x^3+32f^4x^4))\sin(2(e+fx)))\right)}{512a^2f^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] ((Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*((32*c^3*f^3*(-I + 4*f*x) + 24*c^2
*d*f^2*(1 - (4*I)*f*x + 8*f^2*x^2) + 4*c*d^2*f*(3*I + 12*f*x - (24*I)*f^2*x
^2 + 32*f^3*x^3) + d^3*(-3 + (12*I)*f*x + 24*f^2*x^2 - (32*I)*f^3*x^3 + 32*
f^4*x^4))*Cos[2*(e + f*x)] - I*(-32*(4*c^3*f^3 + 6*c^2*d*f^2*(I + 2*f*x) +
6*c*d^2*f*(-1 + (2*I)*f*x + 2*f^2*x^2) + d^3*(-3*I - 6*f*x + (6*I)*f^2*x^2
+ 4*f^3*x^3)) + (32*c^3*f^3*(I + 4*f*x) + 24*c^2*d*f^2*(-1 + (4*I)*f*x + 8*
f^2*x^2) + 4*c*d^2*f*(-3*I - 12*f*x + (24*I)*f^2*x^2 + 32*f^3*x^3) + d^3*(3
 - (12*I)*f*x - 24*f^2*x^2 + (32*I)*f^3*x^3 + 32*f^4*x^4))*Sin[2*(e + f*x)]
)))/(512*a^2*f^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2276 vs. 2(224) = 448.
time = 1.14, size = 2277, normalized size = 8.43

method	result
risch	$\frac{d^3x^4}{16a^2}+\frac{d^2c\,x^3}{4a^2}+\frac{3d\,c^2x^2}{8a^2}+\frac{c^3x}{4a^2}+\frac{c^4}{16a^2d}-\frac{i(32d^3x^3f^3+96c\,d^2f^3x^2+24id^3f^2x^2+96c^2d\,f^3x+48ic\,d^2f^2x+32c^3f^3+24ic^2d\,f}{512a^2f^4}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+I*a*cot(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/f*(-12*I/f^2*c*d^2*e*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)-3/32*f*x-3/32*e)-3/2*I/f*c^2*d*e*sin(f*x+e)^4+3/2*
I/f^2*c*d^2*e^2*sin(f*x+e)^4-3/f*c^2*d*((f*x+e)*(-1/2*cos(f*x+e)*sin(f*x+e)
+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)-3/f^2*c*d^2*((f*x+e)^2*(-1/
2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/2*(f*x+e)*cos(f*x+e)^2+1/4*cos(f*x
+e)*sin(f*x+e)+1/4*f*x+1/4*e-1/3*(f*x+e)^3)-2/f^3*d^3*e^3*(-1/4*cos(f*x+e)^
3*sin(f*x+e)+1/8*cos(f*x+e)*sin(f*x+e)+1/8*f*x+1/8*e)-6/f^3*d^3*e*((f*x+e)^
2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/8*(f*x+e)*cos(f*x+e)^2+1/16*
cos(f*x+e)*sin(f*x+e)+7/64*f*x+7/64*e-1/12*(f*x+e)^3-(f*x+e)^2*(-1/4*(sin(f
*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/8*(f*x+e)*sin(f*x+e)^4-
1/32*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e))-3/f^3*d^3*e^2*((f*x+e)*(-1/2
*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)+3/f^3
*d^3*e*((f*x+e)^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/2*(f*x+e)*co
s(f*x+e)^2+1/4*cos(f*x+e)*sin(f*x+e)+1/4*f*x+1/4*e-1/3*(f*x+e)^3)+2*I/f^3*d
^3*(1/4*(f*x+e)^3*sin(f*x+e)^4-3/4*(f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*
x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/32*(f*x+e)*sin(f*x+e)^4-3/128*(sin(f*x+e)
^3+3/2*sin(f*x+e))*cos(f*x+e)-27/256*f*x-27/256*e+9/32*(f*x+e)*cos(f*x+e)^2
-9/64*cos(f*x+e)*sin(f*x+e)+3/16*(f*x+e)^3)+1/f^3*d^3*e^3*(-1/2*cos(f*x+e)*
sin(f*x+e)+1/2*f*x+1/2*e)+6/f^2*c*d^2*((f*x+e)^2*(-1/2*cos(f*x+e)*sin(f*x+e
)+1/2*f*x+1/2*e)-1/8*(f*x+e)*cos(f*x+e)^2+1/16*cos(f*x+e)*sin(f*x+e)+7/64*f
*x+7/64*e-1/12*(f*x+e)^3-(f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(
f*x+e)+3/8*f*x+3/8*e)-1/8*(f*x+e)*sin(f*x+e)^4-1/32*(sin(f*x+e)^3+3/2*sin(f
*x+e))*cos(f*x+e))+6/f^3*d^3*e^2*((f*x+e)*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f
*x+1/2*e)-1/16*(f*x+e)^2+1/4*sin(f*x+e)^2-(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*s
in(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/64*(2*sin(f*x+e)^2+3)^2)+6/f*c^2*d*(
(f*x+e)*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/16*(f*x+e)^2+1/4*sin(f
*x+e)^2-(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*
e)-1/64*(2*sin(f*x+e)^2+3)^2)-c^3*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e
)+2*c^3*(-1/4*cos(f*x+e)^3*sin(f*x+e)+1/8*cos(f*x+e)*sin(f*x+e)+1/8*f*x+1/8
*e)+6*I/f*c^2*d*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e)
)*cos(f*x+e)-3/32*f*x-3/32*e)+1/2*I*c^3*sin(f*x+e)^4+2/f^3*d^3*((f*x+e)^3*(
-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-3/16*(f*x+e)^2*cos(f*x+e)^2+3/8*(
f*x+e)*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-21/128*(f*x+e)^2-3/32*sin(
f*x+e)^2-3/32*(f*x+e)^4-(f*x+e)^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f
*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2*sin(f*x+e)^4+3/8*(f*x+e)*(-1/4*(sin(f*x
+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3/512*(2*sin(f*x+e)^2+3)^2)
```

```
-12/f^2*c*d^2*e*((f*x+e)*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/16*(f
*x+e)^2+1/4*sin(f*x+e)^2-(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*
x+e)+3/8*f*x+3/8*e)-1/64*(2*sin(f*x+e)^2+3)^2)-1/f^3*d^3*((f*x+e)^3*(-1/2*c
os(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-3/4*(f*x+e)^2*cos(f*x+e)^2+3/2*(f*x+e)*
(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-3/8*(f*x+e)^2-3/8*sin(f*x+e)^2-3/
8*(f*x+e)^4)+3/f*c^2*d*e*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-6/f*c^2
*d*e*(-1/4*cos(f*x+e)^3*sin(f*x+e)+1/8*cos(f*x+e)*sin(f*x+e)+1/8*f*x+1/8*e)
-1/2*I/f^3*d^3*e^3*sin(f*x+e)^4+6/f^2*c*d^2*e^2*(-1/4*cos(f*x+e)^3*sin(f*x+
e)+1/8*cos(f*x+e)*sin(f*x+e)+1/8*f*x+1/8*e)+6*I/f^3*d^3*e^2*(1/4*(f*x+e)*si
n(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/32*f*x-3/32*e)-6
*I/f^3*d^3*e*(1/4*(f*x+e)^2*sin(f*x+e)^4-1/2*(f*x+e)*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3/32*(f*x+e)^2-1/128*(2*sin(f*x+e)^
2+3)^2)+6*I/f^2*c*d^2*(1/4*(f*x+e)^2*sin(f*x+e)^4-1/2*(f*x+e)*(-1/4*(sin(f*
x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3/32*(f*x+e)^2-1/128*(2*si
n(f*x+e)^2+3)^2)+6/f^2*c*d^2*e*((f*x+e)*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x
+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)-3/f^2*c*d^2*e^2*(-1/2*cos(f*x+e)*si
n(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 4.57, size = 259, normalized size = 0.96

$$\frac{32 d^3 f^4 x^4 + 128 c d^2 f^4 x^3 + 192 c^2 d f^4 x^2 + 128 c^3 f^4 x + \left(-32 i d^3 f^3 x^3 - 32 i c^3 f^3 + 24 c^2 d f^2 + 12 i c d^2 f - 3 d^3 - 24 \left(4 i c d^2 f^3 - d^3 f^2\right) x^2 - 12 \left(8 i c^2 d f^3 - 4 c d^2 f^2 - i d^3 f\right) x\right) e^{(4 i f x + 4 i e)} - 32 \left(-4 i d^3 f^3 x^3 - 4 i c^3 f^3 + 6 c^2 d f^2 + 6 i c d^2 f - 3 d^3 + 6 \left(-2 i c d^2 f^3 + d^3 f^2\right) x^2 + 6 \left(-2 i c^2 d f^3 + 2 c d^2 f^2 + i d^3 f\right) x\right) e^{(2 i f x + 2 i e)}}{512 a^2 f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/512*(32*d^3*f^4*x^4 + 128*c*d^2*f^4*x^3 + 192*c^2*d*f^4*x^2 + 128*c^3*f^4
*x + (-32*I*d^3*f^3*x^3 - 32*I*c^3*f^3 + 24*c^2*d*f^2 + 12*I*c*d^2*f - 3*d^
3 - 24*(4*I*c*d^2*f^3 - d^3*f^2)*x^2 - 12*(8*I*c^2*d*f^3 - 4*c*d^2*f^2 - I*
d^3*f)*x)*e^(4*I*f*x + 4*I*e) - 32*(-4*I*d^3*f^3*x^3 - 4*I*c^3*f^3 + 6*c^2*
d*f^2 + 6*I*c*d^2*f - 3*d^3 + 6*(-2*I*c*d^2*f^3 + d^3*f^2)*x^2 + 6*(-2*I*c^
2*d*f^3 + 2*c*d^2*f^2 + I*d^3*f)*x)*e^(2*I*f*x + 2*I*e))/(a^2*f^4)
```

Sympy [A]
time = 0.32, size = 651, normalized size = 2.41

$$\begin{cases} \frac{\left(2048 i a^2 c^3 f^7 e^{2ie} + 6144 i a^2 c^2 d f^7 x e^{2ie} - 3072 a^2 c^2 d f^6 e^{2ie} + 6144 i a^2 c d^2 f^7 x^2 e^{2ie} - 6144 a^2 c d^2 f^6 x e^{2ie} - 3072 i a^2 c d^2 f^5 e^{2ie} + 2048 i a^2 d^3 f^7 x^3 e^{2ie} - 3072 a^2 d^3 f^6 x^2 e^{2ie} - 3072 i a^2 d^3 f^5 x e^{2ie} + 1536 a^2 d^3 f^4 e^{2ie}\right) e^{2ifx} + \left(-512 i a^2 c^3 f^7 e^{4ie} - 1536 i a^2 c^2 d f^7 x e^{4ie} + 384 a^2 c^2 d f^6 e^{4ie} - 1536 i a^2 c d^2 f^7 x^2 e^{4ie} + 768 a^2 c d^2 f^6 x e^{4ie} + 192 i a^2 c d^2 f^5 e^{4ie} - 512 i a^2 d^3 f^7 x^3 e^{4ie} + 384 a^2 d^3 f^6 x^2 e^{4ie} + 192 i a^2 d^3 f^5 x e^{4ie} - 48 a^2 d^3 f^4 e^{4ie}\right) e^{4ifx}}{8192 a^4 f^8} & \text{for}\ a^4 f^8 \neq 0 \\ \frac{x^4 \left(d^3 e^{4ie} - 2 d^3 e^{2ie}\right)}{16 a^2} + \frac{x^3 \left(c d^2 e^{4ie} - 2 c d^2 e^{2ie}\right)}{4 a^2} + \frac{x^2 \cdot \left(3 c^2 d e^{4ie} - 6 c^2 d e^{2ie}\right)}{8 a^2} + \frac{x \left(c^3 e^{4ie} - 2 c^3 e^{2ie}\right)}{4 a^2} & \text{otherwise} \end{cases} + \frac{c^3 x}{4 a^2} + \frac{3 c^2 d x^2}{8 a^2} + \frac{c d^2 x^3}{4 a^2} + \frac{d^3 x^4}{16 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+I*a*cot(f*x+e))**2,x)
```

```
[Out] Piecewise((((2048*I*a**2*c**3*f**7*exp(2*I*e) + 6144*I*a**2*c**2*d*f**7*x*e
xp(2*I*e) - 3072*a**2*c**2*d*f**6*exp(2*I*e) + 6144*I*a**2*c*d**2*f**7*x**2
*exp(2*I*e) - 6144*a**2*c*d**2*f**6*x*exp(2*I*e) - 3072*I*a**2*c*d**2*f**5*
exp(2*I*e) + 2048*I*a**2*d**3*f**7*x**3*exp(2*I*e) - 3072*a**2*d**3*f**6*x*
*2*exp(2*I*e) - 3072*I*a**2*d**3*f**5*x*exp(2*I*e) + 1536*a**2*d**3*f**4*ex
p(2*I*e))*exp(2*I*f*x) + (-512*I*a**2*c**3*f**7*exp(4*I*e) - 1536*I*a**2*c*
*2*d*f**7*x*exp(4*I*e) + 384*a**2*c**2*d*f**6*exp(4*I*e) - 1536*I*a**2*c*d*
*2*f**7*x**2*exp(4*I*e) + 768*a**2*c*d**2*f**6*x*exp(4*I*e) + 192*I*a**2*c*
d**2*f**5*exp(4*I*e) - 512*I*a**2*d**3*f**7*x**3*exp(4*I*e) + 384*a**2*d**3
*f**6*x**2*exp(4*I*e) + 192*I*a**2*d**3*f**5*x*exp(4*I*e) - 48*a**2*d**3*f*
*4*exp(4*I*e))*exp(4*I*f*x))/(8192*a**4*f**8), Ne(a**4*f**8, 0)), (x**4*(d*
*3*exp(4*I*e) - 2*d**3*exp(2*I*e))/(16*a**2) + x**3*(c*d**2*exp(4*I*e) - 2*
c*d**2*exp(2*I*e))/(4*a**2) + x**2*(3*c**2*d*exp(4*I*e) - 6*c**2*d*exp(2*I*
e))/(8*a**2) + x*(c**3*exp(4*I*e) - 2*c**3*exp(2*I*e))/(4*a**2), True)) + c
**3*x/(4*a**2) + 3*c**2*d*x**2/(8*a**2) + c*d**2*x**3/(4*a**2) + d**3*x**4/
(16*a**2)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(204) = 408$.
time = 0.45, size = 413, normalized size = 1.53

$$\frac{32\,d^3f^4x^4+128\,cd^2f^4x^3-32i\,d^3f^3x^3e^{(4i\,fx+4i\,e)}+128i\,d^3f^3x^3e^{(2i\,fx+2i\,e)}+192\,c^2df^4x^2-96i\,cd^2f^3x^2e^{(4i\,fx+4i\,e)}+384i\,cd^2f^3x^2e^{(2i\,fx+2i\,e)}+128\,c^3f^4x-96i\,c^2df^3xe^{(4i\,fx+4i\,e)}+24\,d^3f^2x^2e^{(4i\,fx+4i\,e)}+384i\,c^2df^3xe^{(2i\,fx+2i\,e)}-192\,d^3f^2x^2e^{(2i\,fx+2i\,e)}-32i\,c^3f^3e^{(4i\,fx+4i\,e)}+48\,cd^2f^2xe^{(4i\,fx+4i\,e)}+128i\,c^3f^3e^{(2i\,fx+2i\,e)}-384\,cd^2f^2xe^{(2i\,fx+2i\,e)}+24\,c^2df^2e^{(4i\,fx+4i\,e)}+12i\,d^3fxe^{(4i\,fx+4i\,e)}-192\,c^2df^2e^{(2i\,fx+2i\,e)}-192i\,d^3fxe^{(2i\,fx+2i\,e)}+12i\,cd^2fe^{(4i\,fx+4i\,e)}-192i\,cd^2fe^{(2i\,fx+2i\,e)}-3\,d^3e^{(4i\,fx+4i\,e)}+96\,d^3e^{(2i\,fx+2i\,e)}}{512\,a^2f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/512*(32*d^3*f^4*x^4 + 128*c*d^2*f^4*x^3 - 32*I*d^3*f^3*x^3*e^(4*I*f*x + 4
*I*e) + 128*I*d^3*f^3*x^3*e^(2*I*f*x + 2*I*e) + 192*c^2*d*f^4*x^2 - 96*I*c*
d^2*f^3*x^2*e^(4*I*f*x + 4*I*e) + 384*I*c*d^2*f^3*x^2*e^(2*I*f*x + 2*I*e) +
 128*c^3*f^4*x - 96*I*c^2*d*f^3*x*e^(4*I*f*x + 4*I*e) + 24*d^3*f^2*x^2*e^(4
*I*f*x + 4*I*e) + 384*I*c^2*d*f^3*x*e^(2*I*f*x + 2*I*e) - 192*d^3*f^2*x^2*e
^(2*I*f*x + 2*I*e) - 32*I*c^3*f^3*e^(4*I*f*x + 4*I*e) + 48*c*d^2*f^2*x*e^(4
*I*f*x + 4*I*e) + 128*I*c^3*f^3*e^(2*I*f*x + 2*I*e) - 384*c*d^2*f^2*x*e^(2*
I*f*x + 2*I*e) + 24*c^2*d*f^2*e^(4*I*f*x + 4*I*e) + 12*I*d^3*f*x*e^(4*I*f*x
 + 4*I*e) - 192*c^2*d*f^2*e^(2*I*f*x + 2*I*e) - 192*I*d^3*f*x*e^(2*I*f*x +
2*I*e) + 12*I*c*d^2*f*e^(4*I*f*x + 4*I*e) - 192*I*c*d^2*f*e^(2*I*f*x + 2*I*
e) - 3*d^3*e^(4*I*f*x + 4*I*e) + 96*d^3*e^(2*I*f*x + 2*I*e))/(a^2*f^4)
```

Mupad [B]
time = 1.12, size = 294, normalized size = 1.09

$$\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\left(-\frac{\left(-4\,c^3\,f^3-c^2\,d\,f^2\,6\mathrm{i}+6\,c\,d^2\,f+d^3\,3\mathrm{i}\right)\,1\mathrm{i}}{16\,a^2\,f^4}+\frac{d^3\,x^3\,1\mathrm{i}}{4\,a^2\,f}+\frac{d\,x\,\left(2\,c^2\,f^2+c\,d\,f\,2\mathrm{i}-d^2\right)\,3\mathrm{i}}{8\,a^2\,f^3}+\frac{d^2\,x^2\,\left(2\,c\,f+d\,1\mathrm{i}\right)\,3\mathrm{i}}{8\,a^2\,f^2}\right)-\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\left(-\frac{\left(-32\,c^3\,f^3-c^2\,d\,f^2\,24\mathrm{i}+12\,c\,d^2\,f+d^3\,3\mathrm{i}\right)\,1\mathrm{i}}{512\,a^2\,f^4}+\frac{d^3\,x^3\,1\mathrm{i}}{16\,a^2\,f}+\frac{d\,x\,\left(8\,c^2\,f^2+c\,d\,f\,4\mathrm{i}-d^2\right)\,3\mathrm{i}}{128\,a^2\,f^3}+\frac{d^2\,x^2\,\left(4\,c\,f+d\,1\mathrm{i}\right)\,3\mathrm{i}}{64\,a^2\,f^2}\right)+\frac{c^3\,x}{4\,a^2}+\frac{d^3\,x^4}{16\,a^2}+\frac{3\,c^2\,d\,x^2}{8\,a^2}+\frac{c\,d^2\,x^3}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*cot(e + f*x)*1i)^2,x)
```

```
[Out] exp(e*2i + f*x*2i)*((d^3*x^3*1i)/(4*a^2*f) - ((d^3*3i - 4*c^3*f^3 - c^2*d*f
^2*6i + 6*c*d^2*f)*1i)/(16*a^2*f^4) + (d*x*(2*c^2*f^2 - d^2 + c*d*f*2i)*3i)
/(8*a^2*f^3) + (d^2*x^2*(d*1i + 2*c*f)*3i)/(8*a^2*f^2)) - exp(e*4i + f*x*4i
)*((d^3*x^3*1i)/(16*a^2*f) - ((d^3*3i - 32*c^3*f^3 - c^2*d*f^2*24i + 12*c*d
^2*f)*1i)/(512*a^2*f^4) + (d*x*(8*c^2*f^2 - d^2 + c*d*f*4i)*3i)/(128*a^2*f^
3) + (d^2*x^2*(d*1i + 4*c*f)*3i)/(64*a^2*f^2)) + (c^3*x)/(4*a^2) + (d^3*x^4
)/(16*a^2) + (3*c^2*d*x^2)/(8*a^2) + (c*d^2*x^3)/(4*a^2)
```

3.23 $\int \frac{(c+dx)^2}{(a+ia\cot(e+fx))^2}\,dx$

Optimal. Leaf size=202

$$-\frac{id^2e^{2ie+2ifx}}{8a^2f^3}+\frac{id^2e^{4ie+4ifx}}{128a^2f^3}-\frac{de^{2ie+2ifx}(c+dx)}{4a^2f^2}+\frac{de^{4ie+4ifx}(c+dx)}{32a^2f^2}+\frac{ie^{2ie+2ifx}(c+dx)^2}{4a^2f}-\frac{ie^{4ie+4ifx}(c+dx)^2}{16a^2f}$$

```
[Out] -1/8*I*d^2*exp(2*I*e+2*I*f*x)/a^2/f^3+1/128*I*d^2*exp(4*I*e+4*I*f*x)/a^2/f^
3-1/4*d*exp(2*I*e+2*I*f*x)*(d*x+c)/a^2/f^2+1/32*d*exp(4*I*e+4*I*f*x)*(d*x+c
)/a^2/f^2+1/4*I*exp(2*I*e+2*I*f*x)*(d*x+c)^2/a^2/f-1/16*I*exp(4*I*e+4*I*f*x
)*(d*x+c)^2/a^2/f+1/12*(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.14, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3810, 2207, 2225}

$$-\frac{d(c+dx)e^{2ie+2ifx}}{4a^2f^2}+\frac{d(c+dx)e^{4ie+4ifx}}{32a^2f^2}+\frac{i(c+dx)^2e^{2ie+2ifx}}{4a^2f}-\frac{i(c+dx)^2e^{4ie+4ifx}}{16a^2f}+\frac{(c+dx)^3}{12a^2d}-\frac{id^2e^{2ie+2ifx}}{8a^2f^3}+\frac{id^2e^{4ie+4ifx}}{128a^2f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] ((-1/8*I)*d^2*E^((2*I)*e + (2*I)*f*x))/(a^2*f^3) + ((I/128)*d^2*E^((4*I)*e
+ (4*I)*f*x))/(a^2*f^3) - (d*E^((2*I)*e + (2*I)*f*x)*(c + d*x))/(4*a^2*f^2)
 + (d*E^((4*I)*e + (4*I)*f*x)*(c + d*x))/(32*a^2*f^2) + ((I/4)*E^((2*I)*e +
 (2*I)*f*x)*(c + d*x)^2)/(a^2*f) - ((I/16)*E^((4*I)*e + (4*I)*f*x)*(c + d*x
)^2)/(a^2*f) + (c + d*x)^3/(12*a^2*d)
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3810

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
```

```
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+ia\cot(e+fx))^2}\,dx = \int \left(\frac{(c+dx)^2}{4a^2} - \frac{e^{2ie+2ifx}(c+dx)^2}{2a^2} + \frac{e^{4ie+4ifx}(c+dx)^2}{4a^2}\right) dx$$

$$= \frac{(c+dx)^3}{12a^2d} + \frac{\int e^{4ie+4ifx}(c+dx)^2\,dx}{4a^2} - \frac{\int e^{2ie+2ifx}(c+dx)^2\,dx}{2a^2}$$

$$= \frac{ie^{2ie+2ifx}(c+dx)^2}{4a^2f} - \frac{ie^{4ie+4ifx}(c+dx)^2}{16a^2f} + \frac{(c+dx)^3}{12a^2d} + \frac{(id)\int e^{4ie+4ifx}(c+dx}{8a^2f}$$

$$= -\frac{de^{2ie+2ifx}(c+dx)}{4a^2f^2} + \frac{de^{4ie+4ifx}(c+dx)}{32a^2f^2} + \frac{ie^{2ie+2ifx}(c+dx)^2}{4a^2f} - \frac{ie^{4ie+4ifx}(c+}{16a^2f}$$

$$= -\frac{id^2e^{2ie+2ifx}}{8a^2f^3} + \frac{id^2e^{4ie+4ifx}}{128a^2f^3} - \frac{de^{2ie+2ifx}(c+dx)}{4a^2f^2} + \frac{de^{4ie+4ifx}(c+dx)}{32a^2f^2} + \frac{ie^{2ie+}}{}$$

Mathematica [**A**]
time = 0.55, size = 255, normalized size = 1.26

$$\frac{32f^3x(3c^2+3cdx+d^2x^2)+48((1+i)cf+d(-1+(1+i)fx))((1+i)cf+d(i+(1+i)fx))\cos(2fx)(\cos(2e)+i\sin(2e))-3((2+2i)cf+d(-1+(2+2i)fx))((2+2i)cf+d(i+(2+2i)fx))\cos(4fx)(\cos(4e)+i\sin(4e))+48i((1+i)cf+d(-1+(1+i)fx))((1+i)cf+d(i+(1+i)fx))(\cos(2e)+i\sin(2e))\sin(2fx)-3(d-(2+2i)cf-(2+2i)dfx)(d+(2-2i)cf+(2-2i)dfx)(\cos(4e)+i\sin(4e))\sin(4fx)}{384a^2f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] (32*f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 48*((1 + I)*c*f + d*(-1 + (1 + I)*f
*x))*((1 + I)*c*f + d*(I + (1 + I)*f*x))*Cos[2*f*x]*(Cos[2*e] + I*Sin[2*e])
 - 3*((2 + 2*I)*c*f + d*(-1 + (2 + 2*I)*f*x))*((2 + 2*I)*c*f + d*(I + (2 +
2*I)*f*x))*Cos[4*f*x]*(Cos[4*e] + I*Sin[4*e]) + (48*I)*((1 + I)*c*f + d*(-1
 + (1 + I)*f*x))*((1 + I)*c*f + d*(I + (1 + I)*f*x))*(Cos[2*e] + I*Sin[2*e]
)*Sin[2*f*x] - 3*(d - (2 + 2*I)*c*f - (2 + 2*I)*d*f*x)*(d + (2 - 2*I)*c*f +
 (2 - 2*I)*d*f*x)*(Cos[4*e] + I*Sin[4*e])*Sin[4*f*x])/(384*a^2*f^3)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1080 vs. 2(166) = 332.
time = 0.94, size = 1081, normalized size = 5.35

method	result
risch	$\frac{d^2x^3}{12a^2} + \frac{dc\,x^2}{4a^2} + \frac{c^2x}{4a^2} + \frac{c^3}{12a^2d} - \frac{i(8d^2x^2f^2+16cd\,f^2x+4id^2fx+8c^2f^2+4icdf-d^2)\,\mathrm{e}^{4i(fx+e)}}{128a^2f^3} + \frac{i(2d^2x^2f^2+4cd\,f^2x+2id^2fx+2}{8a^2f^3}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+I*a*cot(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/f*(4*I/f*c*d*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*
x+e))*cos(f*x+e)-3/32*f*x-3/32*e)-4*I/f^2*d^2*e*(1/4*(f*x+e)*sin(f*x+e)^4+1
/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/32*f*x-3/32*e)-I/f*c*d*e*sin
(f*x+e)^4+1/2*I/f^2*d^2*e^2*sin(f*x+e)^4+1/2*I*c^2*sin(f*x+e)^4+2*I/f^2*d^2
*(1/4*(f*x+e)^2*sin(f*x+e)^4-1/2*(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e)
)*cos(f*x+e)+3/8*f*x+3/8*e)+3/32*(f*x+e)^2-1/128*(2*sin(f*x+e)^2+3)^2)+2*c^
2*(-1/4*cos(f*x+e)^3*sin(f*x+e)+1/8*cos(f*x+e)*sin(f*x+e)+1/8*f*x+1/8*e)-4/
f*c*d*e*(-1/4*cos(f*x+e)^3*sin(f*x+e)+1/8*cos(f*x+e)*sin(f*x+e)+1/8*f*x+1/8
*e)+4/f*c*d*((f*x+e)*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/16*(f*x+e
)^2+1/4*sin(f*x+e)^2-(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)
+3/8*f*x+3/8*e)-1/64*(2*sin(f*x+e)^2+3)^2)+2/f^2*d^2*e^2*(-1/4*cos(f*x+e)^3
*sin(f*x+e)+1/8*cos(f*x+e)*sin(f*x+e)+1/8*f*x+1/8*e)-4/f^2*d^2*e*((f*x+e)*(
-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/16*(f*x+e)^2+1/4*sin(f*x+e)^2-(
f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/64*(
2*sin(f*x+e)^2+3)^2)+2/f^2*d^2*((f*x+e)^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f
*x+1/2*e)-1/8*(f*x+e)*cos(f*x+e)^2+1/16*cos(f*x+e)*sin(f*x+e)+7/64*f*x+7/64
*e-1/12*(f*x+e)^3-(f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+
3/8*f*x+3/8*e)-1/8*(f*x+e)*sin(f*x+e)^4-1/32*(sin(f*x+e)^3+3/2*sin(f*x+e))*
cos(f*x+e))-c^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+2/f*c*d*e*(-1/2*
cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2/f*c*d*((f*x+e)*(-1/2*cos(f*x+e)*sin(
f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)-1/f^2*d^2*e^2*(-1/2*c
os(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+2/f^2*d^2*e*((f*x+e)*(-1/2*cos(f*x+e)*s
in(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)-1/f^2*d^2*((f*x+e)
^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/2*(f*x+e)*cos(f*x+e)^2+1/4*
cos(f*x+e)*sin(f*x+e)+1/4*f*x+1/4*e-1/3*(f*x+e)^3))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 3.87, size = 156, normalized size = 0.77

$$\frac{32\,d^2f^3x^3+96\,cdf^3x^2+96\,c^2f^3x-3\,(8i\,d^2f^2x^2+8i\,c^2f^2-4\,cdf-i\,d^2+4\,(4i\,cdf^2-d^2f)x)e^{(4i\,fx+4i\,e)}-48\,(-2i\,d^2f^2x^2-2i\,c^2f^2+2\,cdf+i\,d^2+2\,(-2i\,cdf^2+d^2f)x)e^{(2i\,fx+2i\,e)}}{384\,a^2f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/384*(32*d^2*f^3*x^3 + 96*c*d*f^3*x^2 + 96*c^2*f^3*x - 3*(8*I*d^2*f^2*x^2
+ 8*I*c^2*f^2 - 4*c*d*f - I*d^2 + 4*(4*I*c*d*f^2 - d^2*f)*x)*e^(4*I*f*x + 4
*I*e) - 48*(-2*I*d^2*f^2*x^2 - 2*I*c^2*f^2 + 2*c*d*f + I*d^2 + 2*(-2*I*c*d*
f^2 + d^2*f)*x)*e^(2*I*f*x + 2*I*e))/(a^2*f^3)
```

Sympy [A]
time = 0.25, size = 405, normalized size = 2.00

$$\begin{cases} \frac{\left(256ia^2c^2f^5e^{2ie}+512ia^2cdf^5xe^{2ie}-256a^2cdf^4e^{2ie}+256ia^2d^2f^5x^2e^{2ie}-256a^2d^2f^4xe^{2ie}-128ia^2d^2f^3e^{2ie}\right)e^{2ifx}+\left(-64ia^2c^2f^5e^{4ie}-128ia^2cdf^5xe^{4ie}+32a^2cdf^4e^{4ie}-64ia^2d^2f^5x^2e^{4ie}+32a^2d^2f^4xe^{4ie}+8ia^2d^2f^3e^{4ie}\right)e^{4ifx}}{1024a^4f^6} & \text{for}\ a^4f^6 \neq 0 \\ \frac{x^3\left(d^2e^{4ie}-2d^2e^{2ie}\right)}{12a^2}+\frac{x^2\left(cde^{4ie}-2cde^{2ie}\right)}{4a^2}+\frac{x\left(c^2e^{4ie}-2c^2e^{2ie}\right)}{4a^2} & \text{otherwise} \end{cases} + \frac{c^2x}{4a^2}+\frac{cdx^2}{4a^2}+\frac{d^2x^3}{12a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+I*a*cot(f*x+e))**2,x)
```

```
[Out] Piecewise((((256*I*a**2*c**2*f**5*exp(2*I*e) + 512*I*a**2*c*d*f**5*x*exp(2*
I*e) - 256*a**2*c*d*f**4*exp(2*I*e) + 256*I*a**2*d**2*f**5*x**2*exp(2*I*e)
- 256*a**2*d**2*f**4*x*exp(2*I*e) - 128*I*a**2*d**2*f**3*exp(2*I*e))*exp(2*
I*f*x) + (-64*I*a**2*c**2*f**5*exp(4*I*e) - 128*I*a**2*c*d*f**5*x*exp(4*I*e
) + 32*a**2*c*d*f**4*exp(4*I*e) - 64*I*a**2*d**2*f**5*x**2*exp(4*I*e) + 32*
a**2*d**2*f**4*x*exp(4*I*e) + 8*I*a**2*d**2*f**3*exp(4*I*e))*exp(4*I*f*x))/
(1024*a**4*f**6), Ne(a**4*f**6, 0)), (x**3*(d**2*exp(4*I*e) - 2*d**2*exp(2*
I*e))/(12*a**2) + x**2*(c*d*exp(4*I*e) - 2*c*d*exp(2*I*e))/(4*a**2) + x*(c*
*2*exp(4*I*e) - 2*c**2*exp(2*I*e))/(4*a**2), True)) + c**2*x/(4*a**2) + c*d
*x**2/(4*a**2) + d**2*x**3/(12*a**2)
```

Giac [A]
time = 0.46, size = 235, normalized size = 1.16

$$\frac{32\,d^2f^3x^3+96\,cdf^3x^2-24i\,d^2f^2x^2e^{(4i\,fx+4i\,e)}+96i\,d^2f^2x^2e^{(2i\,fx+2i\,e)}+96\,c^2f^3x-48i\,cdf^2xe^{(4i\,fx+4i\,e)}+192i\,cdf^2xe^{(2i\,fx+2i\,e)}-24i\,c^2f^2e^{(4i\,fx+4i\,e)}+12\,d^2fxe^{(4i\,fx+4i\,e)}+96i\,c^2f^2e^{(2i\,fx+2i\,e)}-96\,d^2fxe^{(2i\,fx+2i\,e)}+12\,cdfe^{(4i\,fx+4i\,e)}-96\,cdfe^{(2i\,fx+2i\,e)}+3i\,d^2e^{(4i\,fx+4i\,e)}-48i\,d^2e^{(2i\,fx+2i\,e)}}{384\,a^2f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/384*(32*d^2*f^3*x^3 + 96*c*d*f^3*x^2 - 24*I*d^2*f^2*x^2*e^(4*I*f*x + 4*I*
e) + 96*I*d^2*f^2*x^2*e^(2*I*f*x + 2*I*e) + 96*c^2*f^3*x - 48*I*c*d*f^2*x*e
^(4*I*f*x + 4*I*e) + 192*I*c*d*f^2*x*e^(2*I*f*x + 2*I*e) - 24*I*c^2*f^2*e^(
4*I*f*x + 4*I*e) + 12*d^2*f*x*e^(4*I*f*x + 4*I*e) + 96*I*c^2*f^2*e^(2*I*f*x
 + 2*I*e) - 96*d^2*f*x*e^(2*I*f*x + 2*I*e) + 12*c*d*f*e^(4*I*f*x + 4*I*e) -
 96*c*d*f*e^(2*I*f*x + 2*I*e) + 3*I*d^2*e^(4*I*f*x + 4*I*e) - 48*I*d^2*e^(2
*I*f*x + 2*I*e))/(a^2*f^3)
```

Mupad [B]
time = 0.58, size = 186, normalized size = 0.92

$$\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\left(\frac{\left(2\,c^2\,f^2+c\,d\,f\,2\mathrm{i}-d^2\right)\,1\mathrm{i}}{8\,a^2\,f^3}+\frac{d^2\,x^2\,1\mathrm{i}}{4\,a^2\,f}+\frac{d\,x\,\left(2\,c\,f+d\,1\mathrm{i}\right)\,1\mathrm{i}}{4\,a^2\,f^2}\right)-\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\left(\frac{\left(8\,c^2\,f^2+c\,d\,f\,4\mathrm{i}-d^2\right)\,1\mathrm{i}}{128\,a^2\,f^3}+\frac{d^2\,x^2\,1\mathrm{i}}{16\,a^2\,f}+\frac{d\,x\,\left(4\,c\,f+d\,1\mathrm{i}\right)\,1\mathrm{i}}{32\,a^2\,f^2}\right)+\frac{c^2\,x}{4\,a^2}+\frac{d^2\,x^3}{12\,a^2}+\frac{c\,d\,x^2}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*cot(e + f*x)*1i)^2,x)

[Out] exp(e*2i + f*x*2i)*(((2*c^2*f^2 - d^2 + c*d*f*2i)*1i)/(8*a^2*f^3) + (d^2*x^
2*1i)/(4*a^2*f) + (d*x*(d*1i + 2*c*f)*1i)/(4*a^2*f^2)) - exp(e*4i + f*x*4i)
*(((8*c^2*f^2 - d^2 + c*d*f*4i)*1i)/(128*a^2*f^3) + (d^2*x^2*1i)/(16*a^2*f)
 + (d*x*(d*1i + 4*c*f)*1i)/(32*a^2*f^2)) + (c^2*x)/(4*a^2) + (d^2*x^3)/(12*
a^2) + (c*d*x^2)/(4*a^2)
```

3.24 $\int \frac{c+dx}{(a+ia\cot(e+fx))^2}\,dx$

Optimal. Leaf size=151

$$\frac{3idx}{16a^2f}-\frac{dx^2}{8a^2}+\frac{x(c+dx)}{4a^2}+\frac{d}{16f^2(a+ia\cot(e+fx))^2}-\frac{i(c+dx)}{4f(a+ia\cot(e+fx))^2}+\frac{3d}{16f^2\left(a^2+ia^2\cot(e+fx)\right)}-$$

```
[Out] 3/16*I*d*x/a^2/f-1/8*d*x^2/a^2+1/4*x*(d*x+c)/a^2+1/16*d/f^2/(a+I*a*cot(f*x+
e))^2-1/4*I*(d*x+c)/f/(a+I*a*cot(f*x+e))^2+3/16*d/f^2/(a^2+I*a^2*cot(f*x+e)
)-1/4*I*(d*x+c)/f/(a^2+I*a^2*cot(f*x+e))
```

Rubi [A]
time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3560, 8, 3811}

$$-\frac{i(c+dx)}{4f\left(a^2+ia^2\cot(e+fx)\right)}+\frac{x(c+dx)}{4a^2}+\frac{3d}{16f^2\left(a^2+ia^2\cot(e+fx)\right)}+\frac{3idx}{16a^2f}-\frac{dx^2}{8a^2}-\frac{i(c+dx)}{4f(a+ia\cot(e+fx))^2}+\frac{d}{16f^2(a+ia\cot(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] (((3*I)/16)*d*x)/(a^2*f) - (d*x^2)/(8*a^2) + (x*(c + d*x))/(4*a^2) + d/(16*
f^2*(a + I*a*Cot[e + f*x])^2) - ((I/4)*(c + d*x))/(f*(a + I*a*Cot[e + f*x])
^2) + (3*d)/(16*f^2*(a^2 + I*a^2*Cot[e + f*x])) - ((I/4)*(c + d*x))/(f*(a^2
 + I*a^2*Cot[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3811

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Dist[(c + d*x)
^m, u, x] - Dist[d*m, Int[Dist[(c + d*x)^(m - 1), u, x], x], x]] /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{(a+ia\cot(e+fx))^2}\,dx = \frac{x(c+dx)}{4a^2} - \frac{i(c+dx)}{4f(a+ia\cot(e+fx))^2} - \frac{i(c+dx)}{4f\left(a^2+ia^2\cot(e+fx)\right)} - d\int \Big(\frac{x}{4a}$$

$$= -\frac{dx^2}{8a^2} + \frac{x(c+dx)}{4a^2} - \frac{i(c+dx)}{4f(a+ia\cot(e+fx))^2} - \frac{i(c+dx)}{4f\left(a^2+ia^2\cot(e+fx)\right)} +$$

$$= -\frac{dx^2}{8a^2} + \frac{x(c+dx)}{4a^2} + \frac{d}{16f^2(a+ia\cot(e+fx))^2} - \frac{i(c+dx)}{4f(a+ia\cot(e+fx))^2} +$$

$$= \frac{idx}{8a^2f} - \frac{dx^2}{8a^2} + \frac{x(c+dx)}{4a^2} + \frac{d}{16f^2(a+ia\cot(e+fx))^2} - \frac{i(c+dx)}{4f(a+ia\cot(e+f}$$

$$= \frac{3idx}{16a^2f} - \frac{dx^2}{8a^2} + \frac{x(c+dx)}{4a^2} + \frac{d}{16f^2(a+ia\cot(e+fx))^2} - \frac{i(c+dx)}{4f(a+ia\cot(e+}$$

Mathematica [A]
time = 0.50, size = 165, normalized size = 1.09

$$\frac{-8de^2+16cef+16cf^2x+8df^2x^2+8i(2cf+d(i+2fx))\cos(2(e+fx))+(d-4icf-4idfx)\cos(4(e+fx))-8id\sin(2(e+fx))-16cf\sin(2(e+fx))-16dfx\sin(2(e+fx))+id\sin(4(e+fx))+4cf\sin(4(e+fx))+4dfx\sin(4(e+fx))}{64a^2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] (-8*d*e^2 + 16*c*e*f + 16*c*f^2*x + 8*d*f^2*x^2 + (8*I)*(2*c*f + d*(I + 2*f
*x))*Cos[2*(e + f*x)] + (d - (4*I)*c*f - (4*I)*d*f*x)*Cos[4*(e + f*x)] - (8
*I)*d*Sin[2*(e + f*x)] - 16*c*f*Sin[2*(e + f*x)] - 16*d*f*x*Sin[2*(e + f*x)
] + I*d*Sin[4*(e + f*x)] + 4*c*f*Sin[4*(e + f*x)] + 4*d*f*x*Sin[4*(e + f*x)
])/(64*a^2*f^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(130) = 260.
time = 0.79, size = 396, normalized size = 2.62

method	result
risch	$\frac{d\,x^2}{8a^2}+\frac{cx}{4a^2}-\frac{i(4dfx+4cf+id)e^{4i(fx+e)}}{64a^2f^2}+\frac{i(2dfx+2cf+id)e^{2i(fx+e)}}{8a^2f^2}$
default	$-\frac{\frac{ic\left(\sin^4(fx+e)\right)}{2}-\frac{ide\left(\sin^4(fx+e)\right)}{2f}+\frac{2id\left(\frac{(fx+e)\left(\sin^4(fx+e)\right)}{4}+\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{16}-\frac{3fx}{32}-\frac{3e}{32}\right)}{f}+2c\left(-\frac{\left(\cos^3(fx+e)\right)}{4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+I*a*cot(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/f*(1/2*I*c*sin(f*x+e)^4-1/2*I/f*d*e*sin(f*x+e)^4+2*I/f*d*(1/4*(f*x+e
)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/32*f*x-3/32*
e)+2*c*(-1/4*cos(f*x+e)^3*sin(f*x+e)+1/8*cos(f*x+e)*sin(f*x+e)+1/8*f*x+1/8*
e)-2/f*d*e*(-1/4*cos(f*x+e)^3*sin(f*x+e)+1/8*cos(f*x+e)*sin(f*x+e)+1/8*f*x+
1/8*e)+2/f*d*((f*x+e)*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/16*(f*x+
e)^2+1/4*sin(f*x+e)^2-(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e
)+3/8*f*x+3/8*e)-1/64*(2*sin(f*x+e)^2+3)^2)-c*(-1/2*cos(f*x+e)*sin(f*x+e)+1
/2*f*x+1/2*e)+1/f*d*e*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/f*d*((f*
x+e)*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e
)^2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 3.72, size = 70, normalized size = 0.46

$$\frac{8\,df^2x^2 + 16\,cf^2x + (-4i\,dfx - 4i\,cf + d)e^{(4i\,fx+4i\,e)} - 8\,(-2i\,dfx - 2i\,cf + d)e^{(2i\,fx+2i\,e)}}{64\,a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/64*(8*d*f^2*x^2 + 16*c*f^2*x + (-4*I*d*f*x - 4*I*c*f + d)*e^(4*I*f*x + 4*
I*e) - 8*(-2*I*d*f*x - 2*I*c*f + d)*e^(2*I*f*x + 2*I*e))/(a^2*f^2)
```

Sympy [**A**]
time = 0.18, size = 212, normalized size = 1.40

$$\begin{cases} \frac{(128ia^2cf^3e^{2ie}+128ia^2df^3xe^{2ie}-64a^2df^2e^{2ie})e^{2ifx}+(-32ia^2cf^3e^{4ie}-32ia^2df^3xe^{4ie}+8a^2df^2e^{4ie})e^{4ifx}}{512a^4f^4} & \text{for } a^4f^4 \neq 0 \\ \frac{x^2(de^{4ie}-2de^{2ie})}{8a^2} + \frac{x(ce^{4ie}-2ce^{2ie})}{4a^2} & \text{otherwise} \end{cases} + \frac{cx}{4a^2} + \frac{dx^2}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))**2,x)
```

```
[Out] Piecewise((((128*I*a**2*c*f**3*exp(2*I*e) + 128*I*a**2*d*f**3*x*exp(2*I*e)
- 64*a**2*d*f**2*exp(2*I*e))*exp(2*I*f*x) + (-32*I*a**2*c*f**3*exp(4*I*e) -
```

```
 32*I*a**2*d*f**3*x*exp(4*I*e) + 8*a**2*d*f**2*exp(4*I*e))*exp(4*I*f*x))/(5
12*a**4*f**4), Ne(a**4*f**4, 0)), (x**2*(d*exp(4*I*e) - 2*d*exp(2*I*e))/(8*
a**2) + x*(c*exp(4*I*e) - 2*c*exp(2*I*e))/(4*a**2), True)) + c*x/(4*a**2) +
 d*x**2/(8*a**2)
```

Giac [A]
time = 0.47, size = 102, normalized size = 0.68

$$\frac{8\,df^2x^2 + 16\,cf^2x - 4i\,dfxe^{(4i\,fx+4i\,e)} + 16i\,dfxe^{(2i\,fx+2i\,e)} - 4i\,cfe^{(4i\,fx+4i\,e)} + 16i\,cfe^{(2i\,fx+2i\,e)} + de^{(4i\,fx+4i\,e)} - 8\,de^{(2i\,fx+2i\,e)}}{64\,a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/64*(8*d*f^2*x^2 + 16*c*f^2*x - 4*I*d*f*x*e^(4*I*f*x + 4*I*e) + 16*I*d*f*x
*e^(2*I*f*x + 2*I*e) - 4*I*c*f*e^(4*I*f*x + 4*I*e) + 16*I*c*f*e^(2*I*f*x +
2*I*e) + d*e^(4*I*f*x + 4*I*e) - 8*d*e^(2*I*f*x + 2*I*e))/(a^2*f^2)
```

Mupad [B]
time = 0.34, size = 102, normalized size = 0.68

$$\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\left(\frac{\left(2\,c\,f+d\,1\mathrm{i}\right)\,1\mathrm{i}}{8\,a^2\,f^2}+\frac{d\,x\,1\mathrm{i}}{4\,a^2\,f}\right)-\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\left(\frac{\left(4\,c\,f+d\,1\mathrm{i}\right)\,1\mathrm{i}}{64\,a^2\,f^2}+\frac{d\,x\,1\mathrm{i}}{16\,a^2\,f}\right)+\frac{d\,x^2}{8\,a^2}+\frac{c\,x}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*cot(e + f*x)*1i)^2,x)
```

```
[Out] exp(e*2i + f*x*2i)*(((d*1i + 2*c*f)*1i)/(8*a^2*f^2) + (d*x*1i)/(4*a^2*f)) -
 exp(e*4i + f*x*4i)*(((d*1i + 4*c*f)*1i)/(64*a^2*f^2) + (d*x*1i)/(16*a^2*f)
) + (d*x^2)/(8*a^2) + (c*x)/(4*a^2)
```

3.25 $\int \frac{1}{(c+dx)(a+ia\cot(e+fx))^2}\,dx$

Optimal. Leaf size=305

$$-\frac{\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{2a^2d}+\frac{\cos\left(4e-\frac{4cf}{d}\right)\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)}{4a^2d}+\frac{\log(c+dx)}{4a^2d}+\frac{i\text{CosIntegra}}{}$$

[Out] 1/4*Ci(4*c*f/d+4*f*x)*cos(-4*e+4*c*f/d)/a^2/d-1/2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a^2/d+1/4*ln(d*x+c)/a^2/d-1/2*I*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a^2/d+1/4*I*cos(-4*e+4*c*f/d)*Si(4*c*f/d+4*f*x)/a^2/d-1/4*I*Ci(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^2/d+1/4*Si(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^2/d+1/2*I*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a^2/d-1/2*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a^2/d

Rubi [A]
time = 0.55, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3809, 3384, 3380, 3383, 3393}

$$-\frac{i\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{2a^2d}+\frac{i\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\sin\left(4e-\frac{4cf}{d}\right)}{4a^2d}-\frac{\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{2a^2d}+\frac{\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\cos\left(4e-\frac{4cf}{d}\right)}{4a^2d}+\frac{\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{2a^2d}-\frac{\sin\left(4e-\frac{4cf}{d}\right)\text{Si}\left(4xf+\frac{4cf}{d}\right)}{4a^2d}-\frac{i\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{2a^2d}+\frac{i\cos\left(4e-\frac{4cf}{d}\right)\text{Si}\left(4xf+\frac{4cf}{d}\right)}{4a^2d}+\frac{\log(c+dx)}{4a^2d}$$

Antiderivative was successfully verified.

[In] Int[1/((c + d*x)*(a + I*a*Cot[e + f*x])^2),x]

[Out] -1/2*(Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(a^2*d) + (Cos[4*e - (4*c*f)/d]*CosIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d) + Log[c + d*x]/(4*a^2*d) + ((I/4)*CosIntegral[(4*c*f)/d + 4*f*x]*Sin[4*e - (4*c*f)/d])/(a^2*d) - ((I/2)*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a^2*d) - ((I/2)*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^2*d) + (Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) + ((I/4)*Cos[4*e - (4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(a^2*d) - (Sin[4*e - (4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d)

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3809

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(c+dx)(a+ia\cot(e+fx))^2}\,dx &= \int \left(\frac{1}{4a^2(c+dx)} - \frac{\cos(2e+2fx)}{2a^2(c+dx)} + \frac{\cos^2(2e+2fx)}{4a^2(c+dx)} - \frac{i\sin(2e+2f}{2a^2(c+dx} \right.\\
&= \frac{\log(c+dx)}{4a^2d} + \frac{i\int\frac{\sin(4e+4fx)}{c+dx}\,dx}{4a^2} - \frac{i\int\frac{\sin(2e+2fx)}{c+dx}\,dx}{2a^2} + \frac{\int\frac{\cos^2(2e+2fx)}{c+dx}\,dx}{4a^2}\\
&= \frac{\log(c+dx)}{4a^2d} - \frac{\int\left(\frac{1}{2(c+dx)} - \frac{\cos(4e+4fx)}{2(c+dx)}\right)dx}{4a^2} + \frac{\int\left(\frac{1}{2(c+dx)} + \frac{\cos(4e+4fx)}{2(c+dx)}\right.}{4a^2}\\
&= -\frac{\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} + \frac{\log(c+dx)}{4a^2d} + \frac{i\operatorname{Ci}\left(\frac{4cf}{d}+4fx\right)\sin}{4a^2d}\\
&= -\frac{\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} + \frac{\log(c+dx)}{4a^2d} + \frac{i\operatorname{Ci}\left(\frac{4cf}{d}+4fx\right)\sin}{4a^2d}\\
&= -\frac{\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} + \frac{\log(c+dx)}{4a^2d} + \frac{i\operatorname{Ci}\left(\frac{4cf}{d}+4fx\right)\sin}{4a^2d}
\end{aligned}$$

Mathematica [A]
time = 0.55, size = 136, normalized size = 0.45

$$\frac{\log(c+dx) - 2\left(\cos\left(2e-\frac{2cf}{d}\right) + i\sin\left(2e-\frac{2cf}{d}\right)\right)\left(\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right) + i\text{Si}\left(\frac{2f(c+dx)}{d}\right)\right) + \left(\cos\left(4e-\frac{4cf}{d}\right) + i\sin\left(4e-\frac{4cf}{d}\right)\right)\left(\text{CosIntegral}\left(\frac{4f(c+dx)}{d}\right) + i\text{Si}\left(\frac{4f(c+dx)}{d}\right)\right)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + I*a*Cot[e + f*x])^2),x]
```

```
[Out] (Log[c + d*x] - 2*(Cos[2*e - (2*c*f)/d] + I*Sin[2*e - (2*c*f)/d])*(CosInteg
ral[(2*f*(c + d*x))/d] + I*SinIntegral[(2*f*(c + d*x))/d]) + (Cos[4*e - (4*
c*f)/d] + I*Sin[4*e - (4*c*f)/d])*(CosIntegral[(4*f*(c + d*x))/d] + I*SinIn
tegral[(4*f*(c + d*x))/d]))/(4*a^2*d)
```

Maple [A]
time = 0.75, size = 378, normalized size = 1.24

method	result
risch	$\frac{\ln(dx+c)}{4a^2d}+\frac{\mathrm{e}^{-\frac{2i(cf-de)}{d}}\operatorname{expIntegral}\left(1,-2ifx-2ie-\frac{2(icf-ide)}{d}\right)}{2a^2d}-\frac{\mathrm{e}^{-\frac{4i(cf-de)}{d}}\operatorname{expIntegral}\left(1,-4ifx-4ie-\frac{4(icf-ide)}{d}\right)}{4a^2d}$
default	$-\frac{if\left(\frac{2\operatorname{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\operatorname{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}\right)}{4}-\frac{if\left(\frac{4\operatorname{sinIntegral}\left(4fx+4e+\frac{4cf-4}{d}\right.}{d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+I*a*cot(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/f*(1/4*I*f*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2*Ci(
2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)-1/16*I*f*(4*Si(4*f*x+4*e+4*(
c*f-d*e)/d)*cos(4*(c*f-d*e)/d)/d-4*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d
*e)/d)/d)-1/4*f*ln(c*f-d*e+d*(f*x+e))/d-1/16*f*(4*Si(4*f*x+4*e+4*(c*f-d*e)/
d)*sin(4*(c*f-d*e)/d)/d+4*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*cos(4*(c*f-d*e)/d)/d)
+1/4*f*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2
*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d))
```

Maxima [A]
time = 0.36, size = 208, normalized size = 0.68

$$-\frac{f\cos\left(\frac{4(cf-de)}{d}\right)E_1\left(\frac{4(-i(fx+e)d-icf+ide)}{d}\right)-2f\cos\left(\frac{2(cf-de)}{d}\right)E_1\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)-ifE_1\left(\frac{4(-i(fx+e)d-icf+ide)}{d}\right)\sin\left(\frac{4(cf-de)}{d}\right)+2ifE_1\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\sin\left(\frac{2(cf-de)}{d}\right)-f\log\left((fx+e)d+cf-de\right)}{4a^2df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(f*cos(4*(c*f - d*e)/d)*exp_integral_e(1, 4*(-I*(f*x + e)*d - I*c*f +
I*d*e)/d) - 2*f*cos(2*(c*f - d*e)/d)*exp_integral_e(1, 2*(-I*(f*x + e)*d -
I*c*f + I*d*e)/d) - I*f*exp_integral_e(1, 4*(-I*(f*x + e)*d - I*c*f + I*d*e
)/d)*sin(4*(c*f - d*e)/d) + 2*I*f*exp_integral_e(1, 2*(-I*(f*x + e)*d - I*c
*f + I*d*e)/d)*sin(2*(c*f - d*e)/d) - f*log((f*x + e)*d + c*f - d*e))/(a^2*
d*f)
```

Fricas [A]
time = 3.27, size = 89, normalized size = 0.29

$$-\frac{2\,\mathrm{Ei}\left(-\frac{2\,(-i\,dfx-i\,cf)}{d}\right)e^{\left(-\frac{2\,(i\,cf-i\,de)}{d}\right)}-\mathrm{Ei}\left(-\frac{4\,(-i\,dfx-i\,cf)}{d}\right)e^{\left(-\frac{4\,(i\,cf-i\,de)}{d}\right)}-\log\left(\frac{dx+c}{d}\right)}{4\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*Ei(-2*(-I*d*f*x - I*c*f)/d)*e^(-2*(I*c*f - I*d*e)/d) - Ei(-4*(-I*d*
f*x - I*c*f)/d)*e^(-4*(I*c*f - I*d*e)/d) - log((d*x + c)/d))/(a^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{c\cot^2(e+fx)-2ic\cot(e+fx)-c+dx\cot^2(e+fx)-2idx\cot(e+fx)-dx}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))**2,x)
```

```
[Out] -Integral(1/(c*cot(e + f*x)**2 - 2*I*c*cot(e + f*x) - c + d*x*cot(e + f*x)*
*2 - 2*I*d*x*cot(e + f*x) - d*x), x)/a**2
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 939 vs. 2(279) = 558.
time = 0.46, size = 939, normalized size = 3.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/4*(cos(e)^4*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d) + 4*I*cos(e)^3*c
os(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e) - 6*cos(e)^2*cos(4*c*f/d
)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2 - 4*I*cos(e)*cos(4*c*f/d)*cos_in
tegral(4*(d*f*x + c*f)/d)*sin(e)^3 + cos(4*c*f/d)*cos_integral(4*(d*f*x + c
*f)/d)*sin(e)^4 - I*cos(e)^4*cos_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d) +
 4*cos(e)^3*cos_integral(4*(d*f*x + c*f)/d)*sin(e)*sin(4*c*f/d) + 6*I*cos(e
)^2*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2*sin(4*c*f/d) - 4*cos(e)*cos_in
tegral(4*(d*f*x + c*f)/d)*sin(e)^3*sin(4*c*f/d) - I*cos_integral(4*(d*f*x +
 c*f)/d)*sin(e)^4*sin(4*c*f/d) + I*cos(e)^4*cos(4*c*f/d)*sin_integral(4*(d*
f*x + c*f)/d) - 4*cos(e)^3*cos(4*c*f/d)*sin(e)*sin_integral(4*(d*f*x + c*f)
/d) - 6*I*cos(e)^2*cos(4*c*f/d)*sin(e)^2*sin_integral(4*(d*f*x + c*f)/d) +
4*cos(e)*cos(4*c*f/d)*sin(e)^3*sin_integral(4*(d*f*x + c*f)/d) + I*cos(4*c*
f/d)*sin(e)^4*sin_integral(4*(d*f*x + c*f)/d) + cos(e)^4*sin(4*c*f/d)*sin_i
ntegral(4*(d*f*x + c*f)/d) + 4*I*cos(e)^3*sin(e)*sin(4*c*f/d)*sin_integral(
4*(d*f*x + c*f)/d) - 6*cos(e)^2*sin(e)^2*sin(4*c*f/d)*sin_integral(4*(d*f*x
 + c*f)/d) - 4*I*cos(e)*sin(e)^3*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/
d) + sin(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 2*cos(e)^2*cos
(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d) - 4*I*cos(e)*cos(2*c*f/d)*cos_int
```

```
egral(2*(d*f*x + c*f)/d)*sin(e) + 2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*
f)/d)*sin(e)^2 + 2*I*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d)
- 4*cos(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)*sin(2*c*f/d) - 2*I*cos_in
tegral(2*(d*f*x + c*f)/d)*sin(e)^2*sin(2*c*f/d) - 2*I*cos(e)^2*cos(2*c*f/d)
*sin_integral(2*(d*f*x + c*f)/d) + 4*cos(e)*cos(2*c*f/d)*sin(e)*sin_integra
l(2*(d*f*x + c*f)/d) + 2*I*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*
f)/d) - 2*cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - 4*I*cos(e
)*sin(e)*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 2*sin(e)^2*sin(2*c*
f/d)*sin_integral(2*(d*f*x + c*f)/d) + log(d*x + c))/(a^2*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\cot\left(e+f\,x\right)\,1\mathrm{i}\right)^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cot(e + f*x)*1i)^2*(c + d*x)),x)
[Out] int(1/((a + a*cot(e + f*x)*1i)^2*(c + d*x)), x)
```

3.26 $\int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))^2}\,dx$

Optimal. Leaf size=434

$$-\frac{1}{4a^2d(c+dx)}+\frac{\cos(2e+2fx)}{2a^2d(c+dx)}-\frac{\cos^2(2e+2fx)}{4a^2d(c+dx)}-\frac{if\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{a^2d^2}+\frac{if\cos\left(4e-\right.}{}$$

```
[Out] -1/4/a^2/d/(d*x+c)+I*f*Ci(4*c*f/d+4*f*x)*cos(-4*e+4*c*f/d)/a^2/d^2-I*f*Ci(2
*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a^2/d^2+1/2*cos(2*f*x+2*e)/a^2/d/(d*x+c)-1/
4*cos(2*f*x+2*e)^2/a^2/d/(d*x+c)+f*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a^2/
d^2-f*cos(-4*e+4*c*f/d)*Si(4*c*f/d+4*f*x)/a^2/d^2+f*Ci(4*c*f/d+4*f*x)*sin(-
4*e+4*c*f/d)/a^2/d^2+I*f*Si(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^2/d^2-f*Ci(2
*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a^2/d^2-I*f*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*
f/d)/a^2/d^2+1/2*I*sin(2*f*x+2*e)/a^2/d/(d*x+c)+1/4*sin(2*f*x+2*e)^2/a^2/d/
(d*x+c)-1/4*I*sin(4*f*x+4*e)/a^2/d/(d*x+c)
```

Rubi [A]

time = 0.50, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3809, 3378, 3384, 3380, 3383, 3394, 12}

$$-\frac{f\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\sin\left(4e-\frac{4cf}{d}\right)}{a^2d^2}+\frac{f\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{a^2d^2}-\frac{if\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{a^2d^2}+\frac{if\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\cos\left(4e-\frac{4cf}{d}\right)}{a^2d^2}+\frac{if\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{a^2d^2}-\frac{if\sin\left(4e-\frac{4cf}{d}\right)\text{Si}\left(4xf+\frac{4cf}{d}\right)}{a^2d^2}+\frac{f\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{a^2d^2}-\frac{f\cos\left(4e-\frac{4cf}{d}\right)\text{Si}\left(4xf+\frac{4cf}{d}\right)}{a^2d^2}+\frac{\sin^2(2e+2fx)}{4a^2d(c+dx)}+\frac{i\sin(2e+2fx)}{2a^2d(c+dx)}-\frac{i\sin(4e+4fx)}{4a^2d(c+dx)}-\frac{\cos^2(2e+2fx)}{4a^2d(c+dx)}+\frac{\cos(2e+2fx)}{2a^2d(c+dx)}-\frac{1}{4a^2d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])^2),x]
```

```
[Out] -1/4*1/(a^2*d*(c + d*x)) + Cos[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - Cos[2*e +
 2*f*x]^2/(4*a^2*d*(c + d*x)) - (I*f*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*
f)/d + 2*f*x])/(a^2*d^2) + (I*f*Cos[4*e - (4*c*f)/d]*CosIntegral[(4*c*f)/d
+ 4*f*x])/(a^2*d^2) - (f*CosIntegral[(4*c*f)/d + 4*f*x]*Sin[4*e - (4*c*f)/d
])/(a^2*d^2) + (f*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a^2
*d^2) + ((I/2)*Sin[2*e + 2*f*x])/(a^2*d*(c + d*x)) + Sin[2*e + 2*f*x]^2/(4*
a^2*d*(c + d*x)) - ((I/4)*Sin[4*e + 4*f*x])/(a^2*d*(c + d*x)) + (f*Cos[2*e
- (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^2*d^2) + (I*f*Sin[2*e - (2*
c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^2*d^2) - (f*Cos[4*e - (4*c*f)/d]
*SinIntegral[(4*c*f)/d + 4*f*x])/(a^2*d^2) - (I*f*Sin[4*e - (4*c*f)/d]*SinI
ntegral[(4*c*f)/d + 4*f*x])/(a^2*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3809

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))^2}\,dx = \int \left(\frac{1}{4a^2(c+dx)^2} - \frac{\cos(2e+2fx)}{2a^2(c+dx)^2} + \frac{\cos^2(2e+2fx)}{4a^2(c+dx)^2} - \frac{i\sin(2e+2\cdots}{2a^2(c+d\cdots}\right.$$

$$= -\frac{1}{4a^2d(c+dx)} + \frac{i\int\frac{\sin(4e+4fx)}{(c+dx)^2}\,dx}{4a^2} - \frac{i\int\frac{\sin(2e+2fx)}{(c+dx)^2}\,dx}{2a^2} + \frac{\int\frac{\cos^2(2e+2f\cdots}{(c+dx)^2}}{4a^2}\cdots$$

$$= -\frac{1}{4a^2d(c+dx)} + \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} + \frac{i\sin(2e+2f\cdots}{2a^2d(c+dx\cdots}$$

$$= -\frac{1}{4a^2d(c+dx)} + \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} + \frac{i\sin(2e+2f\cdots}{2a^2d(c+dx\cdots}$$

$$= -\frac{1}{4a^2d(c+dx)} + \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} - \frac{if\cos\left(2e-\cdots\right.}{\cdots}$$

$$= -\frac{1}{4a^2d(c+dx)} + \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} - \frac{if\cos\left(2e-\cdots\right.}{\cdots}$$

<u>**Mathematica**</u> [A]
time = 0.52, size = 203, normalized size = 0.47

$$\frac{-d+2d(\cos(2(e+fx))+i\sin(2(e+fx)))-d(\cos(4(e+fx))+i\sin(4(e+fx)))+4f(c+dx)\left(-i\cos\left(2e-\frac{2cf}{d}\right)+\sin\left(2e-\frac{2cf}{d}\right)\right)\left(\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)+i\text{Si}\left(\frac{2f(c+dx)}{d}\right)\right)+(c+dx)\left(4if\cos\left(4e-\frac{4cf}{d}\right)-4f\sin\left(4e-\frac{4cf}{d}\right)\right)\left(\text{CosIntegral}\left(\frac{4f(c+dx)}{d}\right)+i\text{Si}\left(\frac{4f(c+dx)}{d}\right)\right)}{4a^2d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])^2),x]
```

```
[Out] (-d + 2*d*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]) - d*(Cos[4*(e + f*x)] + I
*Sin[4*(e + f*x)]) + 4*f*(c + d*x)*((-I)*Cos[2*e - (2*c*f)/d] + Sin[2*e - (
2*c*f)/d])*(CosIntegral[(2*f*(c + d*x))/d] + I*SinIntegral[(2*f*(c + d*x))/
d]) + (c + d*x)*((4*I)*f*Cos[4*e - (4*c*f)/d] - 4*f*Sin[4*e - (4*c*f)/d])*(
CosIntegral[(4*f*(c + d*x))/d] + I*SinIntegral[(4*f*(c + d*x))/d]))/(4*a^2*
d^2*(c + d*x))
```

<u>**Maple**</u> [A]
time = 0.78, size = 537, normalized size = 1.24

method	result
risch	$-\frac{1}{4a^2d(dx+c)} + \frac{if\,\mathrm{e}^{2i(fx+e)}}{2a^2d^2\left(ifx+\frac{icf}{d}\right)} + \frac{if\,\mathrm{e}^{-\frac{2i(cf-de)}{d}}\,\text{expIntegral}\left(1,-2ifx-2ie-\frac{2(icf-ide)}{d}\right)}{a^2d^2} - \frac{if\,\mathrm{e}^{4i(fx+e)}}{4a^2d^2\left(ifx+\frac{icf}{d}\right)} - \frac{if\,\mathrm{e}^{-\frac{4i(\cdots}{}}}{\cdots}$

default	$-\frac{\frac{i f^{2}\left(-\frac{2\sin(2fx+2e)}{(cf-de+d(fx+e))d}+\frac{\frac{4\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{4\,\mathrm{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}}{d}\right)}{4}-\frac{i f^{2}\Big(-}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/f*(1/4*I*f^2*(-2*sin(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/d+2*(2*Si(2*f*x+
2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2
*(c*f-d*e)/d)/d)/d)-1/16*I*f^2*(-4*sin(4*f*x+4*e)/(c*f-d*e+d*(f*x+e))/d+4*(
4*Si(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d)/d+4*Ci(4*f*x+4*e+4*(c*f-d*
e)/d)*cos(4*(c*f-d*e)/d)/d)/d)+1/4*f^2/(c*f-d*e+d*(f*x+e))/d-1/16*f^2*(-4*c
os(4*f*x+4*e)/(c*f-d*e+d*(f*x+e))/d-4*(4*Si(4*f*x+4*e+4*(c*f-d*e)/d)*cos(4*
(c*f-d*e)/d)/d-4*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d)/d)/d)+1/4*f
^2*(-2*cos(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/d-2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d
)*cos(2*(c*f-d*e)/d)/d-2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)/
d))
```

Maxima [A]
time = 0.43, size = 225, normalized size = 0.52

$$-\frac{f^{2}\cos\left(\frac{4(cf-de)}{d}\right)E_{2}\left(\frac{4(-i(fx+e)d-icf+ide)}{d}\right)-2f^{2}\cos\left(\frac{2(cf-de)}{d}\right)E_{2}\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)-if^{2}E_{2}\left(\frac{4(-i(fx+e)d-icf+ide)}{d}\right)\sin\left(\frac{4(cf-de)}{d}\right)+2if^{2}E_{2}\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\sin\left(\frac{2(cf-de)}{d}\right)+f^{2}}{4\left((fx+e)a^{2}d^{2}+a^{2}cdf-a^{2}d^{2}e\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(f^2*cos(4*(c*f - d*e)/d)*exp_integral_e(2, 4*(-I*(f*x + e)*d - I*c*f
+ I*d*e)/d) - 2*f^2*cos(2*(c*f - d*e)/d)*exp_integral_e(2, 2*(-I*(f*x + e)*
d - I*c*f + I*d*e)/d) - I*f^2*exp_integral_e(2, 4*(-I*(f*x + e)*d - I*c*f +
 I*d*e)/d)*sin(4*(c*f - d*e)/d) + 2*I*f^2*exp_integral_e(2, 2*(-I*(f*x + e)
*d - I*c*f + I*d*e)/d)*sin(2*(c*f - d*e)/d) + f^2)/(((f*x + e)*a^2*d^2 + a^
2*c*d*f - a^2*d^2*e)*f)
```

Fricas [A]
time = 2.02, size = 136, normalized size = 0.31

$$-\frac{4(idfx+icf)\mathrm{Ei}\left(-\frac{2(-idfx-icf)}{d}\right)e^{\left(-\frac{2(icf-ide)}{d}\right)}+4(-idfx-icf)\mathrm{Ei}\left(-\frac{4(-idfx-icf)}{d}\right)e^{\left(-\frac{4(icf-ide)}{d}\right)}+de^{(4ifx+4ie)}-2de^{(2ifx+2ie)}+d}{4(a^{2}d^{3}x+a^{2}cd^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*(I*d*f*x + I*c*f)*Ei(-2*(-I*d*f*x - I*c*f)/d)*e^(-2*(I*c*f - I*d*e)
/d) + 4*(-I*d*f*x - I*c*f)*Ei(-4*(-I*d*f*x - I*c*f)/d)*e^(-4*(I*c*f - I*d*e
```

```
)/d) + d*e^(4*I*f*x + 4*I*e) - 2*d*e^(2*I*f*x + 2*I*e) + d)/(a^2*d^3*x + a^
2*c*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{c^2\cot^2(e+fx)-2ic^2\cot(e+fx)-c^2+2cdx\cot^2(e+fx)-4icdx\cot(e+fx)-2cdx+d^2x^2\cot^2(e+fx)-2id^2x^2\cot(e+fx)-d^2x^2}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+I*a*cot(f*x+e))**2,x)
```

```
[Out] -Integral(1/(c**2*cot(e + f*x)**2 - 2*I*c**2*cot(e + f*x) - c**2 + 2*c*d*x*
cot(e + f*x)**2 - 4*I*c*d*x*cot(e + f*x) - 2*c*d*x + d**2*x**2*cot(e + f*x)
**2 - 2*I*d**2*x**2*cot(e + f*x) - d**2*x**2), x)/a**2
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2249 vs. 2(410) = 820.
time = 1.57, size = 2249, normalized size = 5.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/4*(4*I*d*f*x*cos(e)^4*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d) - 16*d
*f*x*cos(e)^3*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e) - 24*I*d*
f*x*cos(e)^2*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2 + 16*d*f
*x*cos(e)*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^3 + 4*I*d*f*x
*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^4 + 4*d*f*x*cos(e)^4*c
os_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d) + 16*I*d*f*x*cos(e)^3*cos_integ
ral(4*(d*f*x + c*f)/d)*sin(e)*sin(4*c*f/d) - 24*d*f*x*cos(e)^2*cos_integral
(4*(d*f*x + c*f)/d)*sin(e)^2*sin(4*c*f/d) - 16*I*d*f*x*cos(e)*cos_integral(
4*(d*f*x + c*f)/d)*sin(e)^3*sin(4*c*f/d) + 4*d*f*x*cos_integral(4*(d*f*x +
c*f)/d)*sin(e)^4*sin(4*c*f/d) - 4*d*f*x*cos(e)^4*cos(4*c*f/d)*sin_integral(
4*(d*f*x + c*f)/d) - 16*I*d*f*x*cos(e)^3*cos(4*c*f/d)*sin(e)*sin_integral(4
*(d*f*x + c*f)/d) + 24*d*f*x*cos(e)^2*cos(4*c*f/d)*sin(e)^2*sin_integral(4*
(d*f*x + c*f)/d) + 16*I*d*f*x*cos(e)*cos(4*c*f/d)*sin(e)^3*sin_integral(4*(
d*f*x + c*f)/d) - 4*d*f*x*cos(4*c*f/d)*sin(e)^4*sin_integral(4*(d*f*x + c*f
)/d) + 4*I*d*f*x*cos(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 16
*d*f*x*cos(e)^3*sin(e)*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 24*I*
d*f*x*cos(e)^2*sin(e)^2*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 16*d
*f*x*cos(e)*sin(e)^3*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 4*I*d*f
*x*sin(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 4*I*c*f*cos(e)^4
*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d) - 16*c*f*cos(e)^3*cos(4*c*f/d
)*cos_integral(4*(d*f*x + c*f)/d)*sin(e) - 24*I*c*f*cos(e)^2*cos(4*c*f/d)*c
```

```
os_integral(4*(d*f*x + c*f)/d)*sin(e)^2 + 16*c*f*cos(e)*cos(4*c*f/d)*cos_in
tegral(4*(d*f*x + c*f)/d)*sin(e)^3 + 4*I*c*f*cos(4*c*f/d)*cos_integral(4*(d
*f*x + c*f)/d)*sin(e)^4 + 4*c*f*cos(e)^4*cos_integral(4*(d*f*x + c*f)/d)*si
n(4*c*f/d) + 16*I*c*f*cos(e)^3*cos_integral(4*(d*f*x + c*f)/d)*sin(e)*sin(4
*c*f/d) - 24*c*f*cos(e)^2*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2*sin(4*c*
f/d) - 16*I*c*f*cos(e)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^3*sin(4*c*f/d
) + 4*c*f*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^4*sin(4*c*f/d) - 4*c*f*cos
(e)^4*cos(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 16*I*c*f*cos(e)^3*cos(
4*c*f/d)*sin(e)*sin_integral(4*(d*f*x + c*f)/d) + 24*c*f*cos(e)^2*cos(4*c*f
/d)*sin(e)^2*sin_integral(4*(d*f*x + c*f)/d) + 16*I*c*f*cos(e)*cos(4*c*f/d)
*sin(e)^3*sin_integral(4*(d*f*x + c*f)/d) - 4*c*f*cos(4*c*f/d)*sin(e)^4*sin
_integral(4*(d*f*x + c*f)/d) + 4*I*c*f*cos(e)^4*sin(4*c*f/d)*sin_integral(4
*(d*f*x + c*f)/d) - 16*c*f*cos(e)^3*sin(e)*sin(4*c*f/d)*sin_integral(4*(d*f
*x + c*f)/d) - 24*I*c*f*cos(e)^2*sin(e)^2*sin(4*c*f/d)*sin_integral(4*(d*f*
x + c*f)/d) + 16*c*f*cos(e)*sin(e)^3*sin(4*c*f/d)*sin_integral(4*(d*f*x + c
*f)/d) + 4*I*c*f*sin(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 4*
I*d*f*x*cos(e)^2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d) + 8*d*f*x*cos
(e)*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) + 4*I*d*f*x*cos(2*c
*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2 - 4*d*f*x*cos(e)^2*cos_integ
ral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) - 8*I*d*f*x*cos(e)*cos_integral(2*(d*f*
x + c*f)/d)*sin(e)*sin(2*c*f/d) + 4*d*f*x*cos_integral(2*(d*f*x + c*f)/d)*s
in(e)^2*sin(2*c*f/d) + 4*d*f*x*cos(e)^2*cos(2*c*f/d)*sin_integral(2*(d*f*x
+ c*f)/d) + 8*I*d*f*x*cos(e)*cos(2*c*f/d)*sin(e)*sin_integral(2*(d*f*x + c*
f)/d) - 4*d*f*x*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) - 4*I
*d*f*x*cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 8*d*f*x*cos(
e)*sin(e)*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 4*I*d*f*x*sin(e)^2
*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - d*cos(4*f*x)*cos(e)^4 - 4*I
*c*f*cos(e)^2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d) - I*d*cos(e)^4*s
in(4*f*x) - 4*I*d*cos(4*f*x)*cos(e)^3*sin(e) + 8*c*f*cos(e)*cos(2*c*f/d)*co
s_integral(2*(d*f*x + c*f)/d)*sin(e) + 4*d*cos(e)^3*sin(4*f*x)*sin(e) + 6*d
*cos(4*f*x)*cos(e)^2*sin(e)^2 + 4*I*c*f*cos(2*c*f/d)*cos_integral(2*(d*f*x
+ c*f)/d)*sin(e)^2 + 6*I*d*cos(e)^2*sin(4*f*x)*sin(e)^2 + 4*I*d*cos(4*f*x)*
cos(e)*sin(e)^3 - 4*d*cos(e)*sin(4*f*x)*sin(e)^3 - d*cos(4*f*x)*sin(e)^4 -
I*d*sin(4*f*x)*sin(e)^4 - 4*c*f*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d)*si
n(2*c*f/d) - 8*I*c*f*cos(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)*sin(2*c*
f/d) + 4*c*f*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2*sin(2*c*f/d) + 4*c*f*
cos(e)^2*cos(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 8*I*c*f*cos(e)*cos(
2*c*f/d)*sin(e)*sin_integral(2*(d*f*x + c*f)/d) - 4*c*f*cos(2*c*f/d)*sin(e)
^2*sin_integral(2*(d*f*x + c*f)/d) - 4*I*c*f*cos(e)^2*sin(2*c*f/d)*sin_inte
gral(2*(d*f*x + c*f)/d) + 8*c*f*cos(e)*sin(e)*sin(2*c*f/d)*sin_integral(2*(
d*f*x + c*f)/d) + 4*I*c*f*sin(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f
)/d) + 2*d*cos(2*f*x)*cos(e)^2 + 2*I*d*cos(e)^2*sin(2*f*x) + 4*I*d*cos(2*f*
x)*cos(e)*sin(e) - 4*d*cos(e)*sin(2*f*x)*sin(e) - 2*d*cos(2*f*x)*sin(e)^2 -
 2*I*d*sin(2*f*x)*sin(e)^2 - d)/(a^2*d^3*x + a^2*c*d^2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\cot\left(e+f\,x\right)\,1\mathrm{i}\right)^2\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cot(e + f*x)*1i)^2*(c + d*x)^2),x)
[Out] int(1/((a + a*cot(e + f*x)*1i)^2*(c + d*x)^2), x)
```

3.27 $\int \frac{(c+dx)^3}{(a+ia\cot(e+fx))^3}\,dx$

Optimal. Leaf size=396

$$\frac{9d^3e^{2ie+2ifx}}{64a^3f^4}-\frac{9d^3e^{4ie+4ifx}}{1024a^3f^4}+\frac{d^3e^{6ie+6ifx}}{1728a^3f^4}-\frac{9id^2e^{2ie+2ifx}(c+dx)}{32a^3f^3}+\frac{9id^2e^{4ie+4ifx}(c+dx)}{256a^3f^3}-\frac{id^2e^{6ie+6ifx}(c+dx)}{288a^3f^3}-$$

```
[Out] 9/64*d^3*exp(2*I*e+2*I*f*x)/a^3/f^4-9/1024*d^3*exp(4*I*e+4*I*f*x)/a^3/f^4+1
/1728*d^3*exp(6*I*e+6*I*f*x)/a^3/f^4-9/32*I*d^2*exp(2*I*e+2*I*f*x)*(d*x+c)/
a^3/f^3+9/256*I*d^2*exp(4*I*e+4*I*f*x)*(d*x+c)/a^3/f^3-1/288*I*d^2*exp(6*I*
e+6*I*f*x)*(d*x+c)/a^3/f^3-9/32*d*exp(2*I*e+2*I*f*x)*(d*x+c)^2/a^3/f^2+9/12
8*d*exp(4*I*e+4*I*f*x)*(d*x+c)^2/a^3/f^2-1/96*d*exp(6*I*e+6*I*f*x)*(d*x+c)^
2/a^3/f^2+3/16*I*exp(2*I*e+2*I*f*x)*(d*x+c)^3/a^3/f-3/32*I*exp(4*I*e+4*I*f*
x)*(d*x+c)^3/a^3/f+1/48*I*exp(6*I*e+6*I*f*x)*(d*x+c)^3/a^3/f+1/32*(d*x+c)^4
/a^3/d
```

Rubi [A]
time = 0.26, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3810, 2207, 2225}

$$-\frac{9id^2(c+dx)e^{2ie+2ifx}}{32a^3f^3}+\frac{9id^2(c+dx)e^{4ie+4ifx}}{256a^3f^3}-\frac{id^2(c+dx)e^{6ie+6ifx}}{288a^3f^3}-\frac{9d(c+dx)^2e^{2ie+2ifx}}{32a^3f^2}+\frac{9d(c+dx)^2e^{4ie+4ifx}}{128a^3f^2}-\frac{d(c+dx)^2e^{6ie+6ifx}}{96a^3f^2}+\frac{3i(c+dx)^3e^{2ie+2ifx}}{16a^3f}-\frac{3i(c+dx)^3e^{4ie+4ifx}}{32a^3f}+\frac{i(c+dx)^3e^{6ie+6ifx}}{48a^3f}+\frac{(c+dx)^4}{32a^3d}+\frac{9d^3e^{2ie+2ifx}}{64a^3f^4}-\frac{9d^3e^{4ie+4ifx}}{1024a^3f^4}+\frac{d^3e^{6ie+6ifx}}{1728a^3f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] (9*d^3*E^((2*I)*e + (2*I)*f*x))/(64*a^3*f^4) - (9*d^3*E^((4*I)*e + (4*I)*f*
x))/(1024*a^3*f^4) + (d^3*E^((6*I)*e + (6*I)*f*x))/(1728*a^3*f^4) - (((9*I)
/32)*d^2*E^((2*I)*e + (2*I)*f*x)*(c + d*x))/(a^3*f^3) + (((9*I)/256)*d^2*E^
((4*I)*e + (4*I)*f*x)*(c + d*x))/(a^3*f^3) - ((I/288)*d^2*E^((6*I)*e + (6*I
)*f*x)*(c + d*x))/(a^3*f^3) - (9*d*E^((2*I)*e + (2*I)*f*x)*(c + d*x)^2)/(32
*a^3*f^2) + (9*d*E^((4*I)*e + (4*I)*f*x)*(c + d*x)^2)/(128*a^3*f^2) - (d*E^
((6*I)*e + (6*I)*f*x)*(c + d*x)^2)/(96*a^3*f^2) + (((3*I)/16)*E^((2*I)*e +
(2*I)*f*x)*(c + d*x)^3)/(a^3*f) - (((3*I)/32)*E^((4*I)*e + (4*I)*f*x)*(c +
d*x)^3)/(a^3*f) + ((I/48)*E^((6*I)*e + (6*I)*f*x)*(c + d*x)^3)/(a^3*f) + (c
 + d*x)^4/(32*a^3*d)
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3810

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+ia\cot(e+fx))^3}\,dx &= \int \left(\frac{(c+dx)^3}{8a^3} - \frac{3e^{2ie+2ifx}(c+dx)^3}{8a^3} + \frac{3e^{4ie+4ifx}(c+dx)^3}{8a^3} - \frac{e^{6ie+6ifx}(c+dx}{8a^3}\right.\\
&= \frac{(c+dx)^4}{32a^3d} - \frac{\int e^{6ie+6ifx}(c+dx)^3\,dx}{8a^3} - \frac{3\int e^{2ie+2ifx}(c+dx)^3\,dx}{8a^3} + \frac{3\int e^{4ie+4if}}{8}\\
&= \frac{3ie^{2ie+2ifx}(c+dx)^3}{16a^3f} - \frac{3ie^{4ie+4ifx}(c+dx)^3}{32a^3f} + \frac{ie^{6ie+6ifx}(c+dx)^3}{48a^3f} + \frac{(c+dx)^4}{32a^3d}\\
&= -\frac{9de^{2ie+2ifx}(c+dx)^2}{32a^3f^2} + \frac{9de^{4ie+4ifx}(c+dx)^2}{128a^3f^2} - \frac{de^{6ie+6ifx}(c+dx)^2}{96a^3f^2} + \frac{3ie^{2ie+}}{}\\
&= -\frac{9id^2e^{2ie+2ifx}(c+dx)}{32a^3f^3} + \frac{9id^2e^{4ie+4ifx}(c+dx)}{256a^3f^3} - \frac{id^2e^{6ie+6ifx}(c+dx)}{288a^3f^3} - \frac{9de}{}\\
&= \frac{9d^3e^{2ie+2ifx}}{64a^3f^4} - \frac{9d^3e^{4ie+4ifx}}{1024a^3f^4} + \frac{d^3e^{6ie+6ifx}}{1728a^3f^4} - \frac{9id^2e^{2ie+2ifx}(c+dx)}{32a^3f^3} + \frac{9id^2e^{4ie+}}{256}\end{aligned}$$

Mathematica [A]
time = 2.74, size = 664, normalized size = 1.68

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] ((Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)])*((81*I)*(32*c^3*f^3 + 24*c^2*d*f^2
*(3*I + 4*f*x) + 12*c*d^2*f*(-7 + (12*I)*f*x + 8*f^2*x^2) + d^3*(-45*I - 84
*f*x + (72*I)*f^2*x^2 + 32*f^3*x^3))*Cos[e + f*x] + 16*(36*c^3*f^3*(I + 6*f
*x) + 18*c^2*d*f^2*(-1 + (6*I)*f*x + 18*f^2*x^2) + 6*c*d^2*f*(-I - 6*f*x +
(18*I)*f^2*x^2 + 36*f^3*x^3) + d^3*(1 - (6*I)*f*x - 18*f^2*x^2 + (36*I)*f^3
*x^3 + 54*f^4*x^4))*Cos[3*(e + f*x)] - (4131*I)*d^3*Sin[e + f*x] - 8748*c*d
```

^2*f*Sin[e + f*x] + (9720*I)*c^2*d*f^2*Sin[e + f*x] + 7776*c^3*f^3*Sin[e + f*x] - 8748*d^3*f*x*Sin[e + f*x] + (19440*I)*c*d^2*f^2*x*Sin[e + f*x] + 23328*c^2*d*f^3*x*Sin[e + f*x] + (9720*I)*d^3*f^2*x^2*Sin[e + f*x] + 23328*c*d^2*f^3*x^2*Sin[e + f*x] + 7776*d^3*f^3*x^3*Sin[e + f*x] + (16*I)*d^3*Sin[3*(e + f*x)] + 96*c*d^2*f*Sin[3*(e + f*x)] - (288*I)*c^2*d*f^2*Sin[3*(e + f*x)] - 576*c^3*f^3*Sin[3*(e + f*x)] + 96*d^3*f*x*Sin[3*(e + f*x)] - (576*I)*c*d^2*f^2*x*Sin[3*(e + f*x)] - 1728*c^2*d*f^3*x*Sin[3*(e + f*x)] - (3456*I)*c^3*f^4*x*Sin[3*(e + f*x)] - (288*I)*d^3*f^2*x^2*Sin[3*(e + f*x)] - 1728*c*d^2*f^3*x^2*Sin[3*(e + f*x)] - (5184*I)*c^2*d*f^4*x^2*Sin[3*(e + f*x)] - 576*d^3*f^3*x^3*Sin[3*(e + f*x)] - (3456*I)*c*d^2*f^4*x^3*Sin[3*(e + f*x)] - (864*I)*d^3*f^4*x^4*Sin[3*(e + f*x)]))/(27648*a^3*f^4)

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4078 vs. $2(328) = 656$.
time = 1.07, size = 4079, normalized size = 10.30

method	result
risch	$\frac{d^3x^4}{32a^3} + \frac{d^2c\,x^3}{8a^3} + \frac{3d\,c^2x^2}{16a^3} + \frac{c^3x}{8a^3} + \frac{c^4}{32a^3d} + \frac{i(36d^3x^3f^3+108c\,d^2f^3x^2+18id^3f^2x^2+108c^2d\,f^3x+36ic\,d^2f^2x+36c^3}{1728a^3f^4}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3/(a+I*a*cot(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] 1/f^4/a^3*(-12*c*d^2*f*((f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/48*(f*x+e)*sin(f*x+e)^4+1/192*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+47/1152*f*x+47/1152*e-1/16*(f*x+e)*cos(f*x+e)^2+1/32*cos(f*x+e)*sin(f*x+e)-1/24*(f*x+e)^3-(f*x+e)^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-1/18*(f*x+e)*sin(f*x+e)^6-1/108*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e))-4*I*d^3*e^3*(-1/6*sin(f*x+e)^2*cos(f*x+e)^4-1/12*cos(f*x+e)^4)+3/4*I*d^3*e^3*sin(f*x+e)^4+3*c^2*d*f^2*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2+1/64*(2*sin(f*x+e)^2+3)^2)+3*c*d^2*f*((f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/8*(f*x+e)*sin(f*x+e)^4+1/32*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+9/64*f*x+9/64*e-3/8*(f*x+e)*cos(f*x+e)^2+3/16*cos(f*x+e)*sin(f*x+e)-1/4*(f*x+e)^3)-3/4*I*c^3*f^3*sin(f*x+e)^4+4*I*c^3*f^3*(-1/6*sin(f*x+e)^2*cos(f*x+e)^4-1/12*cos(f*x+e)^4)-12*c^2*d*f^2*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/32*(f*x+e)^2+1/64*(2*sin(f*x+e)^2+3)^2-(f*x+e)*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-1/36*sin(f*x+e)^6-5/96*sin(f*x+e)^4-5/32*sin(f*x+e)^2)+18*I*c*d^2*e*f*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/32*f*x-3/32*e)-12*I*d^3*e*(1/4*(f*x+e)^2*sin(f*x+e)^4-1/2*(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/24*(f*x+e)^2-1/128*(2*sin(

f*x+e)^2+3)^2-1/6*(f*x+e)^2*sin(f*x+e)^6+1/3*(f*x+e)*(-1/6*(sin(f*x+e)^5+5/
4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+1/108*sin(f*x+e
)^6+5/288*sin(f*x+e)^4+5/96*sin(f*x+e)^2)-9*I*d^3*e^2*(1/4*(f*x+e)*sin(f*x+
e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/32*f*x-3/32*e)+9*I*d^3
e(1/4*(f*x+e)^2*sin(f*x+e)^4-1/2*(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+
e))*cos(f*x+e)+3/8*f*x+3/8*e)+3/32*(f*x+e)^2-1/128*(2*sin(f*x+e)^2+3)^2)+12
*I*d^3*e^2*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos
(f*x+e)-1/24*f*x-1/24*e-1/6*(f*x+e)*sin(f*x+e)^6-1/36*(sin(f*x+e)^5+5/4*sin
(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e))+12*I*c*d^2*e^2*f*(-1/6*sin(f*x+e)^2*
cos(f*x+e)^4-1/12*cos(f*x+e)^4)-3*e*d^3*((f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*
sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/8*(f*x+e)*sin(f*x+e)^4+1/32*(sin(f*
x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+9/64*f*x+9/64*e-3/8*(f*x+e)*cos(f*x+e)^2+
3/16*cos(f*x+e)*sin(f*x+e)-1/4*(f*x+e)^3)+4*I*d^3*(1/4*(f*x+e)^3*sin(f*x+e)
^4-3/4*(f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8
e)-1/24(f*x+e)*sin(f*x+e)^4-1/96*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)
-1/18*f*x-1/18*e+1/8*(f*x+e)*cos(f*x+e)^2-1/16*cos(f*x+e)*sin(f*x+e)+1/12*(
f*x+e)^3-1/6*(f*x+e)^3*sin(f*x+e)^6+1/2*(f*x+e)^2*(-1/6*(sin(f*x+e)^5+5/4*s
in(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+1/36*(f*x+e)*sin(f
x+e)^6+1/216(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e))-1
2*e^2*d^3*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3
/8*e)-1/32*(f*x+e)^2+1/64*(2*sin(f*x+e)^2+3)^2-(f*x+e)*(-1/6*(sin(f*x+e)^5+
5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-1/36*sin(f*x+
e)^6-5/96*sin(f*x+e)^4-5/32*sin(f*x+e)^2)+12*e*d^3*((f*x+e)^2*(-1/4*(sin(f*
x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/48*(f*x+e)*sin(f*x+e)^4+
1/192*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+47/1152*f*x+47/1152*e-1/16*(
f*x+e)*cos(f*x+e)^2+1/32*cos(f*x+e)*sin(f*x+e)-1/24*(f*x+e)^3-(f*x+e)^2*(-1
/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16
e)-1/18(f*x+e)*sin(f*x+e)^6-1/108*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin
(f*x+e))*cos(f*x+e))+3*e^2*d^3*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))
*cos(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2+1/64*(2*sin(f*x+e)^2+3)^2)-3*I*d^
3*(1/4*(f*x+e)^3*sin(f*x+e)^4-3/4*(f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x
+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/32*(f*x+e)*sin(f*x+e)^4-3/128*(sin(f*x+e)^
3+3/2*sin(f*x+e))*cos(f*x+e)-27/256*f*x-27/256*e+9/32*(f*x+e)*cos(f*x+e)^2-
9/64*cos(f*x+e)*sin(f*x+e)+3/16*(f*x+e)^3)-6*c*d^2*e*f*((f*x+e)*(-1/4*(sin(
f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2+1/64*(2*s
in(f*x+e)^2+3)^2)+12*c^2*d*e*f^2*(-1/6*cos(f*x+e)^3*sin(f*x+e)^3-1/8*cos(f*
x+e)^3*sin(f*x+e)+1/16*cos(f*x+e)*sin(f*x+e)+1/16*f*x+1/16*e)-12*c*d^2*e^2*
f*(-1/6*cos(f*x+e)^3*sin(f*x+e)^3-1/8*cos(f*x+e)^3*sin(f*x+e)+1/16*cos(f*x+
e)*sin(f*x+e)+1/16*f*x+1/16*e)+24*c*d^2*e*f*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/32*(f*x+e)^2+1/64*(2*sin(f*x+e)^2
+3)^2-(f*x+e)*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x
+e)+5/16*f*x+5/16*e)-1/36*sin(f*x+e)^6-5/96*sin(f*x+e)^4-5/32*sin(f*x+e)^2)
+12*I*c*d^2*f*(1/4*(f*x+e)^2*sin(f*x+e)^4-1/2*(f*x+e)*(-1/4*(sin(f*x+e)^3+3
/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/24*(f*x+e)^2-1/128*(2*sin(f*x+e)
^2+3)^2-1/6*(f*x+e)^2*sin(f*x+e)^6+1/3*(f*x+e)*(-1/6*(sin(f*x+e)^5+5/4*sin(

f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+1/108*sin(f*x+e)^6+5/288*sin(f*x+e)^4+5/96*sin(f*x+e)^2)-9*I*c^2*d*f...

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.18, size = 363, normalized size = 0.92

$$\frac{864\,d^3f^4x^4+3456\,cd^2f^4x^3+5184\,c^2df^4x^2+3456\,c^3f^4x-16\,(-36i\,d^3f^3x^3-36i\,c^3f^3+18\,c^2df^2+6i\,cd^2f-d^3+18\,(-6i\,cd^2f^3+d^3f^2)x^2+6\,(-18i\,c^2df^3+6\,cd^2f^2+i\,d^3f)x)e^{(6i\,fx+6i\,e)}-81\,(32i\,d^3f^3x^3+32i\,c^3f^3-24\,c^2df^2-12i\,cd^2f+3\,d^3+24\,(4i\,cd^2f^3-d^3f^2)x^2+12\,(8i\,c^2df^3-4\,cd^2f^2-i\,d^3f)x)e^{(4i\,fx+4i\,e)}-1296\,(-4i\,d^3f^3x^3-4i\,c^3f^3+6\,c^2df^2+6i\,cd^2f-3\,d^3+6\,(-2i\,cd^2f^3+d^3f^2)x^2+6\,(-2i\,c^2df^3+2\,cd^2f^2+i\,d^3f)x)e^{(2i\,fx+2i\,e)}}{27648\,a^3f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/27648*(864*d^3*f^4*x^4 + 3456*c*d^2*f^4*x^3 + 5184*c^2*d*f^4*x^2 + 3456*c
^3*f^4*x - 16*(-36*I*d^3*f^3*x^3 - 36*I*c^3*f^3 + 18*c^2*d*f^2 + 6*I*c*d^2*
f - d^3 + 18*(-6*I*c*d^2*f^3 + d^3*f^2)*x^2 + 6*(-18*I*c^2*d*f^3 + 6*c*d^2*
f^2 + I*d^3*f)*x)*e^(6*I*f*x + 6*I*e) - 81*(32*I*d^3*f^3*x^3 + 32*I*c^3*f^3
 - 24*c^2*d*f^2 - 12*I*c*d^2*f + 3*d^3 + 24*(4*I*c*d^2*f^3 - d^3*f^2)*x^2 +
 12*(8*I*c^2*d*f^3 - 4*c*d^2*f^2 - I*d^3*f)*x)*e^(4*I*f*x + 4*I*e) - 1296*(
-4*I*d^3*f^3*x^3 - 4*I*c^3*f^3 + 6*c^2*d*f^2 + 6*I*c*d^2*f - 3*d^3 + 6*(-2*
I*c*d^2*f^3 + d^3*f^2)*x^2 + 6*(-2*I*c^2*d*f^3 + 2*c*d^2*f^2 + I*d^3*f)*x)*
e^(2*I*f*x + 2*I*e))/(a^3*f^4)
```

Sympy [A]
time = 0.46, size = 932, normalized size = 2.35

$$\begin{cases} \frac{(21233664ia^6c^3f^{11}e^{2ie}+63700992ia^6c^2df^{11}xe^{2ie}-31850496a^6c^2df^{10}e^{2ie}+63700992ia^6cd^2f^{11}x^2e^{2ie}-63700992a^6cd^2f^{10}xe^{2ie}-31850496ia^6cd^2f^9e^{2ie}+21233664ia^6d^3f^{11}x^3e^{2ie}-31850496a^6d^3f^{10}x^2e^{2ie}-31850496ia^6d^3f^9xe^{2ie}+15925248a^6d^3f^8e^{2ie})e^{2ifx}+(-10616832ia^6c^3f^{11}e^{4ie}-31850496ia^6c^2df^{11}xe^{4ie}+7962624a^6c^2df^{10}e^{4ie}-31850496ia^6cd^2f^{11}x^2e^{4ie}+15925248a^6cd^2f^{10}xe^{4ie}+3981312ia^6cd^2f^9e^{4ie}-10616832ia^6d^3f^{11}x^3e^{4ie}+7962624a^6d^3f^{10}x^2e^{4ie}+3981312ia^6d^3f^9xe^{4ie}-995328a^6d^3f^8e^{4ie})e^{4ifx}+(2359296ia^6c^3f^{11}e^{6ie}+7077888ia^6c^2df^{11}xe^{6ie}-1179648a^6c^2df^{10}e^{6ie}+7077888ia^6cd^2f^{11}x^2e^{6ie}-2359296a^6cd^2f^{10}xe^{6ie}-393216ia^6cd^2f^9e^{6ie}+2359296ia^6d^3f^{11}x^3e^{6ie}-1179648a^6d^3f^{10}x^2e^{6ie}-393216ia^6d^3f^9xe^{6ie}+65536a^6d^3f^8e^{6ie})e^{6ifx}}{113246208a^9f^{12}} & \text{for } a^9f^{12}\neq 0 \\ \frac{x^4(-d^3e^{6ie}+3d^3e^{4ie}-3d^3e^{2ie})}{32a^3}+\frac{x^3(-cd^2e^{6ie}+3cd^2e^{4ie}-3cd^2e^{2ie})}{8a^3}+\frac{x^2(-3c^2de^{6ie}+9c^2de^{4ie}-9c^2de^{2ie})}{16a^3}+\frac{x(-c^3e^{6ie}+3c^3e^{4ie}-3c^3e^{2ie})}{8a^3} & \text{otherwise} \end{cases}+\frac{c^3x}{8a^3}+\frac{3c^2dx^2}{16a^3}+\frac{cd^2x^3}{8a^3}+\frac{d^3x^4}{32a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+I*a*cot(f*x+e))**3,x)
```

```
[Out] Piecewise((((21233664*I*a**6*c**3*f**11*exp(2*I*e) + 63700992*I*a**6*c**2*d
*f**11*x*exp(2*I*e) - 31850496*a**6*c**2*d*f**10*exp(2*I*e) + 63700992*I*a*
*6*c*d**2*f**11*x**2*exp(2*I*e) - 63700992*a**6*c*d**2*f**10*x*exp(2*I*e) -
 31850496*I*a**6*c*d**2*f**9*exp(2*I*e) + 21233664*I*a**6*d**3*f**11*x**3*e
xp(2*I*e) - 31850496*a**6*d**3*f**10*x**2*exp(2*I*e) - 31850496*I*a**6*d**3
*f**9*x*exp(2*I*e) + 15925248*a**6*d**3*f**8*exp(2*I*e))*exp(2*I*f*x) + (-1
```

```
0616832*I*a**6*c**3*f**11*exp(4*I*e) - 31850496*I*a**6*c**2*d*f**11*x*exp(4
*I*e) + 7962624*a**6*c**2*d*f**10*exp(4*I*e) - 31850496*I*a**6*c*d**2*f**11
*x**2*exp(4*I*e) + 15925248*a**6*c*d**2*f**10*x*exp(4*I*e) + 3981312*I*a**6
*c*d**2*f**9*exp(4*I*e) - 10616832*I*a**6*d**3*f**11*x**3*exp(4*I*e) + 7962
624*a**6*d**3*f**10*x**2*exp(4*I*e) + 3981312*I*a**6*d**3*f**9*x*exp(4*I*e)
 - 995328*a**6*d**3*f**8*exp(4*I*e))*exp(4*I*f*x) + (2359296*I*a**6*c**3*f*
*11*exp(6*I*e) + 7077888*I*a**6*c**2*d*f**11*x*exp(6*I*e) - 1179648*a**6*c*
*2*d*f**10*exp(6*I*e) + 7077888*I*a**6*c*d**2*f**11*x**2*exp(6*I*e) - 23592
96*a**6*c*d**2*f**10*x*exp(6*I*e) - 393216*I*a**6*c*d**2*f**9*exp(6*I*e) +
2359296*I*a**6*d**3*f**11*x**3*exp(6*I*e) - 1179648*a**6*d**3*f**10*x**2*ex
p(6*I*e) - 393216*I*a**6*d**3*f**9*x*exp(6*I*e) + 65536*a**6*d**3*f**8*exp(
6*I*e))*exp(6*I*f*x))/(113246208*a**9*f**12), Ne(a**9*f**12, 0)), (x**4*(-d
**3*exp(6*I*e) + 3*d**3*exp(4*I*e) - 3*d**3*exp(2*I*e))/(32*a**3) + x**3*(-
c*d**2*exp(6*I*e) + 3*c*d**2*exp(4*I*e) - 3*c*d**2*exp(2*I*e))/(8*a**3) + x
**2*(-3*c**2*d*exp(6*I*e) + 9*c**2*d*exp(4*I*e) - 9*c**2*d*exp(2*I*e))/(16*
a**3) + x*(-c**3*exp(6*I*e) + 3*c**3*exp(4*I*e) - 3*c**3*exp(2*I*e))/(8*a**
3), True)) + c**3*x/(8*a**3) + 3*c**2*d*x**2/(16*a**3) + c*d**2*x**3/(8*a**
3) + d**3*x**4/(32*a**3)
```

Giac [A]

time = 0.57, size = 593, normalized size = 1.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/27648*(864*d^3*f^4*x^4 + 3456*c*d^2*f^4*x^3 + 576*I*d^3*f^3*x^3*e^(6*I*f*
x + 6*I*e) - 2592*I*d^3*f^3*x^3*e^(4*I*f*x + 4*I*e) + 5184*I*d^3*f^3*x^3*e^
(2*I*f*x + 2*I*e) + 5184*c^2*d*f^4*x^2 + 1728*I*c*d^2*f^3*x^2*e^(6*I*f*x +
6*I*e) - 7776*I*c*d^2*f^3*x^2*e^(4*I*f*x + 4*I*e) + 15552*I*c*d^2*f^3*x^2*e
^(2*I*f*x + 2*I*e) + 3456*c^3*f^4*x + 1728*I*c^2*d*f^3*x*e^(6*I*f*x + 6*I*e
) - 288*d^3*f^2*x^2*e^(6*I*f*x + 6*I*e) - 7776*I*c^2*d*f^3*x*e^(4*I*f*x + 4
*I*e) + 1944*d^3*f^2*x^2*e^(4*I*f*x + 4*I*e) + 15552*I*c^2*d*f^3*x*e^(2*I*f
*x + 2*I*e) - 7776*d^3*f^2*x^2*e^(2*I*f*x + 2*I*e) + 576*I*c^3*f^3*e^(6*I*f
*x + 6*I*e) - 576*c*d^2*f^2*x*e^(6*I*f*x + 6*I*e) - 2592*I*c^3*f^3*e^(4*I*f
*x + 4*I*e) + 3888*c*d^2*f^2*x*e^(4*I*f*x + 4*I*e) + 5184*I*c^3*f^3*e^(2*I*
f*x + 2*I*e) - 15552*c*d^2*f^2*x*e^(2*I*f*x + 2*I*e) - 288*c^2*d*f^2*e^(6*I
*f*x + 6*I*e) - 96*I*d^3*f*x*e^(6*I*f*x + 6*I*e) + 1944*c^2*d*f^2*e^(4*I*f*
x + 4*I*e) + 972*I*d^3*f*x*e^(4*I*f*x + 4*I*e) - 7776*c^2*d*f^2*e^(2*I*f*x
+ 2*I*e) - 7776*I*d^3*f*x*e^(2*I*f*x + 2*I*e) - 96*I*c*d^2*f*e^(6*I*f*x + 6
*I*e) + 972*I*c*d^2*f*e^(4*I*f*x + 4*I*e) - 7776*I*c*d^2*f*e^(2*I*f*x + 2*I
*e) + 16*d^3*e^(6*I*f*x + 6*I*e) - 243*d^3*e^(4*I*f*x + 4*I*e) + 3888*d^3*e
^(2*I*f*x + 2*I*e))/(a^3*f^4)
```

Mupad [B]

time = 1.62, size = 418, normalized size = 1.06

$$\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\left(-\frac{\left(-12\,c^3\,f^3-c^2\,d\,f^2\,18\mathrm{i}+18\,c\,d^2\,f+d^3\,9\mathrm{i}\right)\,1\mathrm{i}}{64\,a^3\,f^4}+\frac{d^3\,x^3\,3\mathrm{i}}{16\,a^3\,f}+\frac{d\,x\,\left(2\,c^2\,f^2+c\,d\,f\,2\mathrm{i}-d^2\right)\,9\mathrm{i}}{32\,a^3\,f^3}+\frac{d^2\,x^2\,\left(2\,c\,f+d\,1\mathrm{i}\right)\,9\mathrm{i}}{32\,a^3\,f^2}\right)-\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\left(-\frac{\left(-96\,c^3\,f^3-c^2\,d\,f^2\,72\mathrm{i}+36\,c\,d^2\,f+d^3\,9\mathrm{i}\right)\,1\mathrm{i}}{1024\,a^3\,f^4}+\frac{d^3\,x^3\,3\mathrm{i}}{32\,a^3\,f}+\frac{d\,x\,\left(8\,c^2\,f^2+c\,d\,f\,4\mathrm{i}-d^2\right)\,9\mathrm{i}}{256\,a^3\,f^3}+\frac{d^2\,x^2\,\left(4\,c\,f+d\,1\mathrm{i}\right)\,9\mathrm{i}}{128\,a^3\,f^2}\right)+\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\left(-\frac{\left(-36\,c^3\,f^3-c^2\,d\,f^2\,18\mathrm{i}+6\,c\,d^2\,f+d^3\,1\mathrm{i}\right)\,1\mathrm{i}}{1728\,a^3\,f^4}+\frac{d^3\,x^3\,1\mathrm{i}}{48\,a^3\,f}+\frac{d\,x\,\left(18\,c^2\,f^2+c\,d\,f\,6\mathrm{i}-d^2\right)\,1\mathrm{i}}{288\,a^3\,f^3}+\frac{d^2\,x^2\,\left(6\,c\,f+d\,1\mathrm{i}\right)\,1\mathrm{i}}{96\,a^3\,f^2}\right)+\frac{c^3\,x}{8\,a^3}+\frac{d^3\,x^4}{32\,a^3}+\frac{3\,c^2\,d\,x^2}{16\,a^3}+\frac{c\,d^2\,x^3}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*cot(e + f*x)*1i)^3,x)
```

```
[Out] exp(e*2i + f*x*2i)*((d^3*x^3*3i)/(16*a^3*f) - ((d^3*9i - 12*c^3*f^3 - c^2*d
*f^2*18i + 18*c*d^2*f)*1i)/(64*a^3*f^4) + (d*x*(2*c^2*f^2 - d^2 + c*d*f*2i)
*9i)/(32*a^3*f^3) + (d^2*x^2*(d*1i + 2*c*f)*9i)/(32*a^3*f^2)) - exp(e*4i +
f*x*4i)*((d^3*x^3*3i)/(32*a^3*f) - ((d^3*9i - 96*c^3*f^3 - c^2*d*f^2*72i +
36*c*d^2*f)*1i)/(1024*a^3*f^4) + (d*x*(8*c^2*f^2 - d^2 + c*d*f*4i)*9i)/(256
*a^3*f^3) + (d^2*x^2*(d*1i + 4*c*f)*9i)/(128*a^3*f^2)) + exp(e*6i + f*x*6i)
*((d^3*x^3*1i)/(48*a^3*f) - ((d^3*1i - 36*c^3*f^3 - c^2*d*f^2*18i + 6*c*d^2
*f)*1i)/(1728*a^3*f^4) + (d*x*(18*c^2*f^2 - d^2 + c*d*f*6i)*1i)/(288*a^3*f^
3) + (d^2*x^2*(d*1i + 6*c*f)*1i)/(96*a^3*f^2)) + (c^3*x)/(8*a^3) + (d^3*x^4
)/(32*a^3) + (3*c^2*d*x^2)/(16*a^3) + (c*d^2*x^3)/(8*a^3)
```

3.28 $\int \frac{(c+dx)^2}{(a+ia\cot(e+fx))^3}\,dx$

Optimal. Leaf size=294

$$-\frac{3id^2e^{2ie+2ifx}}{32a^3f^3}+\frac{3id^2e^{4ie+4ifx}}{256a^3f^3}-\frac{id^2e^{6ie+6ifx}}{864a^3f^3}-\frac{3de^{2ie+2ifx}(c+dx)}{16a^3f^2}+\frac{3de^{4ie+4ifx}(c+dx)}{64a^3f^2}-\frac{de^{6ie+6ifx}(c+dx)}{144a^3f^2}+$$

```
[Out] -3/32*I*d^2*exp(2*I*e+2*I*f*x)/a^3/f^3+3/256*I*d^2*exp(4*I*e+4*I*f*x)/a^3/f
^3-1/864*I*d^2*exp(6*I*e+6*I*f*x)/a^3/f^3-3/16*d*exp(2*I*e+2*I*f*x)*(d*x+c)
/a^3/f^2+3/64*d*exp(4*I*e+4*I*f*x)*(d*x+c)/a^3/f^2-1/144*d*exp(6*I*e+6*I*f*
x)*(d*x+c)/a^3/f^2+3/16*I*exp(2*I*e+2*I*f*x)*(d*x+c)^2/a^3/f-3/32*I*exp(4*I
*e+4*I*f*x)*(d*x+c)^2/a^3/f+1/48*I*exp(6*I*e+6*I*f*x)*(d*x+c)^2/a^3/f+1/24*
(d*x+c)^3/a^3/d
```

Rubi [A]
time = 0.18, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3810, 2207, 2225}

$$-\frac{3d(c+dx)e^{2ie+2ifx}}{16a^3f^2}+\frac{3d(c+dx)e^{4ie+4ifx}}{64a^3f^2}-\frac{d(c+dx)e^{6ie+6ifx}}{144a^3f^2}+\frac{3i(c+dx)^2e^{2ie+2ifx}}{16a^3f}-\frac{3i(c+dx)^2e^{4ie+4ifx}}{32a^3f}+\frac{i(c+dx)^2e^{6ie+6ifx}}{48a^3f}+\frac{(c+dx)^3}{24a^3d}-\frac{3id^2e^{2ie+2ifx}}{32a^3f^3}+\frac{3id^2e^{4ie+4ifx}}{256a^3f^3}-\frac{id^2e^{6ie+6ifx}}{864a^3f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] (((-3*I)/32)*d^2*E^((2*I)*e + (2*I)*f*x))/(a^3*f^3) + (((3*I)/256)*d^2*E^((
4*I)*e + (4*I)*f*x))/(a^3*f^3) - ((I/864)*d^2*E^((6*I)*e + (6*I)*f*x))/(a^3
*f^3) - (3*d*E^((2*I)*e + (2*I)*f*x)*(c + d*x))/(16*a^3*f^2) + (3*d*E^((4*I
)*e + (4*I)*f*x)*(c + d*x))/(64*a^3*f^2) - (d*E^((6*I)*e + (6*I)*f*x)*(c +
d*x))/(144*a^3*f^2) + (((3*I)/16)*E^((2*I)*e + (2*I)*f*x)*(c + d*x)^2)/(a^3
*f) - (((3*I)/32)*E^((4*I)*e + (4*I)*f*x)*(c + d*x)^2)/(a^3*f) + ((I/48)*E^
((6*I)*e + (6*I)*f*x)*(c + d*x)^2)/(a^3*f) + (c + d*x)^3/(24*a^3*d)
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3810

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+ia\cot(e+fx))^3}\,dx = \int \left(\frac{(c+dx)^2}{8a^3} - \frac{3e^{2ie+2ifx}(c+dx)^2}{8a^3} + \frac{3e^{4ie+4ifx}(c+dx)^2}{8a^3} - \frac{e^{6ie+6ifx}(c+dx)^2}{8a^3}\right.$$

$$= \frac{(c+dx)^3}{24a^3d} - \frac{\int e^{6ie+6ifx}(c+dx)^2\,dx}{8a^3} - \frac{3\int e^{2ie+2ifx}(c+dx)^2\,dx}{8a^3} + \frac{3\int e^{4ie+4ifx}(}{8a}$$

$$= \frac{3ie^{2ie+2ifx}(c+dx)^2}{16a^3f} - \frac{3ie^{4ie+4ifx}(c+dx)^2}{32a^3f} + \frac{ie^{6ie+6ifx}(c+dx)^2}{48a^3f} + \frac{(c+dx)^3}{24a^3d} -$$

$$= -\frac{3de^{2ie+2ifx}(c+dx)}{16a^3f^2} + \frac{3de^{4ie+4ifx}(c+dx)}{64a^3f^2} - \frac{de^{6ie+6ifx}(c+dx)}{144a^3f^2} + \frac{3ie^{2ie+2ifx}(}{16a}$$

$$= -\frac{3id^2e^{2ie+2ifx}}{32a^3f^3} + \frac{3id^2e^{4ie+4ifx}}{256a^3f^3} - \frac{id^2e^{6ie+6ifx}}{864a^3f^3} - \frac{3de^{2ie+2ifx}(c+dx)}{16a^3f^2} + \frac{3de^{4ie+4}}{64}$$

Mathematica [A]
time = 0.68, size = 369, normalized size = 1.26

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] (288*f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 648*((1 + I)*c*f + d*(-1 + (1 + I)
*f*x))*((1 + I)*c*f + d*(I + (1 + I)*f*x))*Cos[2*f*x]*(Cos[2*e] + I*Sin[2*e
]) - 81*((2 + 2*I)*c*f + d*(-1 + (2 + 2*I)*f*x))*((2 + 2*I)*c*f + d*(I + (2
 + 2*I)*f*x))*Cos[4*f*x]*(Cos[4*e] + I*Sin[4*e]) + 8*((3 + 3*I)*c*f + d*(-1
 + (3 + 3*I)*f*x))*((3 + 3*I)*c*f + d*(I + (3 + 3*I)*f*x))*Cos[6*f*x]*(Cos[
6*e] + I*Sin[6*e]) + (648*I)*((1 + I)*c*f + d*(-1 + (1 + I)*f*x))*((1 + I)*
c*f + d*(I + (1 + I)*f*x))*(Cos[2*e] + I*Sin[2*e])*Sin[2*f*x] - 81*(d - (2
+ 2*I)*c*f - (2 + 2*I)*d*f*x)*(d + (2 - 2*I)*c*f + (2 - 2*I)*d*f*x)*(Cos[4*
e] + I*Sin[4*e])*Sin[4*f*x] + (8*I)*((3 + 3*I)*c*f + d*(-1 + (3 + 3*I)*f*x)
)*((3 + 3*I)*c*f + d*(I + (3 + 3*I)*f*x))*(Cos[6*e] + I*Sin[6*e])*Sin[6*f*x
])/(6912*a^3*f^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1878 vs. 2(241) = 482.
time = 0.88, size = 1879, normalized size = 6.39

method	result
risch	$\frac{d^2x^3}{24a^3}+\frac{dc\,x^2}{8a^3}+\frac{c^2x}{8a^3}+\frac{c^3}{24a^3d}+\frac{i(18d^2x^2f^2+36cd\,f^2x+6id^2fx+18c^2f^2+6icdf-d^2)\,\mathrm{e}^{6i(fx+e)}}{864a^3f^3}-\frac{3i(8d^2x^2f^2+1}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+I*a*cot(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^3/a^3*(-3/4*I*c^2*f^2*sin(f*x+e)^4+3/2*I*c*d*e*f*sin(f*x+e)^4-8*I*c*d*e
*f*(-1/6*sin(f*x+e)^2*cos(f*x+e)^4-1/12*cos(f*x+e)^4)+6*I*d^2*e*(1/4*(f*x+e
)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/32*f*x-3/32*
e)+4*I*d^2*(1/4*(f*x+e)^2*sin(f*x+e)^4-1/2*(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*
sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/24*(f*x+e)^2-1/128*(2*sin(f*x+e)^2+
3)^2-1/6*(f*x+e)^2*sin(f*x+e)^6+1/3*(f*x+e)*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x
+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+1/108*sin(f*x+e)^6+5/288
*sin(f*x+e)^4+5/96*sin(f*x+e)^2)+4*I*c^2*f^2*(-1/6*sin(f*x+e)^2*cos(f*x+e)^
4-1/12*cos(f*x+e)^4)-4*c^2*f^2*(-1/6*cos(f*x+e)^3*sin(f*x+e)^3-1/8*cos(f*x+
e)^3*sin(f*x+e)+1/16*cos(f*x+e)*sin(f*x+e)+1/16*f*x+1/16*e)+8*c*d*e*f*(-1/6
*cos(f*x+e)^3*sin(f*x+e)^3-1/8*cos(f*x+e)^3*sin(f*x+e)+1/16*cos(f*x+e)*sin(
f*x+e)+1/16*f*x+1/16*e)-8*c*d*f*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e)
)*cos(f*x+e)+3/8*f*x+3/8*e)-1/32*(f*x+e)^2+1/64*(2*sin(f*x+e)^2+3)^2-(f*x+e
)*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x
+5/16*e)-1/36*sin(f*x+e)^6-5/96*sin(f*x+e)^4-5/32*sin(f*x+e)^2)-4*d^2*e^2*(
-1/6*cos(f*x+e)^3*sin(f*x+e)^3-1/8*cos(f*x+e)^3*sin(f*x+e)+1/16*cos(f*x+e)*
sin(f*x+e)+1/16*f*x+1/16*e)+8*e*d^2*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*
x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/32*(f*x+e)^2+1/64*(2*sin(f*x+e)^2+3)^2-(f
*x+e)*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16
*f*x+5/16*e)-1/36*sin(f*x+e)^6-5/96*sin(f*x+e)^4-5/32*sin(f*x+e)^2)-4*d^2*(
(f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/4
8*(f*x+e)*sin(f*x+e)^4+1/192*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+47/11
52*f*x+47/1152*e-1/16*(f*x+e)*cos(f*x+e)^2+1/32*cos(f*x+e)*sin(f*x+e)-1/24*
(f*x+e)^3-(f*x+e)^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*c
os(f*x+e)+5/16*f*x+5/16*e)-1/18*(f*x+e)*sin(f*x+e)^6-1/108*(sin(f*x+e)^5+5/
4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e))-3/4*I*d^2*e^2*sin(f*x+e)^4-6*I*
c*d*f*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+
e)-3/32*f*x-3/32*e)-8*I*d^2*e*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+
3/2*sin(f*x+e))*cos(f*x+e)-1/24*f*x-1/24*e-1/6*(f*x+e)*sin(f*x+e)^6-1/36*(s
in(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e))+8*I*c*d*f*(1/4*(f
*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-1/24*f*x-1
/24*e-1/6*(f*x+e)*sin(f*x+e)^6-1/36*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin
(f*x+e))*cos(f*x+e))+4*I*d^2*e^2*(-1/6*sin(f*x+e)^2*cos(f*x+e)^4-1/12*cos(f
*x+e)^4)-3*I*d^2*(1/4*(f*x+e)^2*sin(f*x+e)^4-1/2*(f*x+e)*(-1/4*(sin(f*x+e)^
```

```
3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3/32*(f*x+e)^2-1/128*(2*sin(f*x
+e)^2+3)^2)+c^2*f^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+
3/8*e)-2*c*d*e*f*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8
*e)+2*c*d*f*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x
+3/8*e)-3/16*(f*x+e)^2+1/64*(2*sin(f*x+e)^2+3)^2)+d^2*e^2*(-1/4*(sin(f*x+e)
^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*e*d^2*((f*x+e)*(-1/4*(sin(f*
x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2+1/64*(2*sin
(f*x+e)^2+3)^2)+d^2*((f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+
e)+3/8*f*x+3/8*e)+1/8*(f*x+e)*sin(f*x+e)^4+1/32*(sin(f*x+e)^3+3/2*sin(f*x+e
))*cos(f*x+e)+9/64*f*x+9/64*e-3/8*(f*x+e)*cos(f*x+e)^2+3/16*cos(f*x+e)*sin(
f*x+e)-1/4*(f*x+e)^3))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 4.12, size = 214, normalized size = 0.73

$$\frac{288\,d^2f^3x^3+864\,cdf^3x^2+864\,c^2f^3x-8\,(-18i\,d^2f^2x^2-18i\,c^2f^2+6\,cdf+i\,d^2+6\,(-6i\,cdf^2+d^2f)x)e^{(6i\,fx+6i\,e)}-81\,(8i\,d^2f^2x^2+8i\,c^2f^2-4\,cdf-i\,d^2+4\,(4i\,cdf^2-d^2f)x)e^{(4i\,fx+4i\,e)}-648\,(-2i\,d^2f^2x^2-2i\,c^2f^2+2\,cdf+i\,d^2+2\,(-2i\,cdf^2+d^2f)x)e^{(2i\,fx+2i\,e)}}{6912\,a^3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6912*(288*d^2*f^3*x^3 + 864*c*d*f^3*x^2 + 864*c^2*f^3*x - 8*(-18*I*d^2*f^
2*x^2 - 18*I*c^2*f^2 + 6*c*d*f + I*d^2 + 6*(-6*I*c*d*f^2 + d^2*f)*x)*e^(6*I
*f*x + 6*I*e) - 81*(8*I*d^2*f^2*x^2 + 8*I*c^2*f^2 - 4*c*d*f - I*d^2 + 4*(4*
I*c*d*f^2 - d^2*f)*x)*e^(4*I*f*x + 4*I*e) - 648*(-2*I*d^2*f^2*x^2 - 2*I*c^2
*f^2 + 2*c*d*f + I*d^2 + 2*(-2*I*c*d*f^2 + d^2*f)*x)*e^(2*I*f*x + 2*I*e))/(
a^3*f^3)
```

Sympy [A]
time = 0.34, size = 575, normalized size = 1.96

$$\begin{cases} \frac{(1327104ia^6c^2f^8e^{2ie}+2654208ia^6cdf^8xe^{2ie}-1327104a^6cdf^7e^{2ie}+1327104ia^6d^2f^8x^2e^{2ie}-1327104a^6d^2f^7xe^{2ie}-663552ia^6d^2f^6e^{2ie})e^{2ifx}+(-663552ia^6c^2f^8e^{4ie}-1327104ia^6cdf^8xe^{4ie}+331776a^6cdf^7e^{4ie}-663552ia^6d^2f^8x^2e^{4ie}+331776a^6d^2f^7xe^{4ie}+82944ia^6d^2f^6e^{4ie})e^{4ifx}+(147456ia^6c^2f^8e^{6ie}+294912ia^6cdf^8xe^{6ie}-49152a^6cdf^7e^{6ie}+147456ia^6d^2f^8x^2e^{6ie}-49152a^6d^2f^7xe^{6ie}-8192ia^6d^2f^6e^{6ie})e^{6ifx}}{7077888a^9f^9} & \text{for}\: a^9f^9 \neq 0 \\ \frac{x^3(-d^2e^{6ie}+3d^2e^{4ie}-3d^2e^{2ie})}{24a^3}+\frac{x^2(-cde^{6ie}+3cde^{4ie}-3cde^{2ie})}{8a^3}+\frac{x(-c^2e^{6ie}+3c^2e^{4ie}-3c^2e^{2ie})}{8a^3} & \text{otherwise} \end{cases} + \frac{c^2x}{8a^3} + \frac{cdx^2}{8a^3} + \frac{d^2x^3}{24a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+I*a*cot(f*x+e))**3,x)
```

```
[Out] Piecewise((((1327104*I*a**6*c**2*f**8*exp(2*I*e) + 2654208*I*a**6*c*d*f**8*
x*exp(2*I*e) - 1327104*a**6*c*d*f**7*exp(2*I*e) + 1327104*I*a**6*d**2*f**8*
x**2*exp(2*I*e) - 1327104*a**6*d**2*f**7*x*exp(2*I*e) - 663552*I*a**6*d**2*
f**6*exp(2*I*e))*exp(2*I*f*x) + (-663552*I*a**6*c**2*f**8*exp(4*I*e) - 1327
104*I*a**6*c*d*f**8*x*exp(4*I*e) + 331776*a**6*c*d*f**7*exp(4*I*e) - 663552
*I*a**6*d**2*f**8*x**2*exp(4*I*e) + 331776*a**6*d**2*f**7*x*exp(4*I*e) + 82
944*I*a**6*d**2*f**6*exp(4*I*e))*exp(4*I*f*x) + (147456*I*a**6*c**2*f**8*ex
p(6*I*e) + 294912*I*a**6*c*d*f**8*x*exp(6*I*e) - 49152*a**6*c*d*f**7*exp(6*
I*e) + 147456*I*a**6*d**2*f**8*x**2*exp(6*I*e) - 49152*a**6*d**2*f**7*x*exp
(6*I*e) - 8192*I*a**6*d**2*f**6*exp(6*I*e))*exp(6*I*f*x))/(7077888*a**9*f**
9), Ne(a**9*f**9, 0)), (x**3*(-d**2*exp(6*I*e) + 3*d**2*exp(4*I*e) - 3*d**2
*exp(2*I*e))/(24*a**3) + x**2*(-c*d*exp(6*I*e) + 3*c*d*exp(4*I*e) - 3*c*d*e
xp(2*I*e))/(8*a**3) + x*(-c**2*exp(6*I*e) + 3*c**2*exp(4*I*e) - 3*c**2*exp(
2*I*e))/(8*a**3), True)) + c**2*x/(8*a**3) + c*d*x**2/(8*a**3) + d**2*x**3/
(24*a**3)
```

Giac [A]
time = 0.52, size = 333, normalized size = 1.13

$$\frac{288 d^2 f^3 x^3 + 864 cdf^3 x^2 + 144i d^2 f^2 x^2 e^{(6i fx+6i e)} - 648i d^2 f^2 x^2 e^{(4i fx+4i e)} + 1296i d^2 f^2 x^2 e^{(2i fx+2i e)} + 864 c^2 f^3 x + 288i cdf^2 x e^{(6i fx+6i e)} - 1296i cdf^2 x e^{(4i fx+4i e)} + 2592i cdf^2 x e^{(2i fx+2i e)} + 144i c^2 f^2 e^{(6i fx+6i e)} - 48 d^2 f x e^{(6i fx+6i e)} - 648i c^2 f^2 e^{(4i fx+4i e)} + 324 d^2 f x e^{(4i fx+4i e)} + 1296i c^2 f^2 e^{(2i fx+2i e)} - 1296 d^2 f x e^{(2i fx+2i e)} - 48 cdf e^{(6i fx+6i e)} + 324 cdf e^{(4i fx+4i e)} - 1296 cdf e^{(2i fx+2i e)} - 8i d^2 e^{(6i fx+6i e)} + 81i d^2 e^{(4i fx+4i e)} - 648i d^2 e^{(2i fx+2i e)}}{6912 a^3 f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/6912*(288*d^2*f^3*x^3 + 864*c*d*f^3*x^2 + 144*I*d^2*f^2*x^2*e^(6*I*f*x +
6*I*e) - 648*I*d^2*f^2*x^2*e^(4*I*f*x + 4*I*e) + 1296*I*d^2*f^2*x^2*e^(2*I*
f*x + 2*I*e) + 864*c^2*f^3*x + 288*I*c*d*f^2*x*e^(6*I*f*x + 6*I*e) - 1296*I
*c*d*f^2*x*e^(4*I*f*x + 4*I*e) + 2592*I*c*d*f^2*x*e^(2*I*f*x + 2*I*e) + 144
*I*c^2*f^2*e^(6*I*f*x + 6*I*e) - 48*d^2*f*x*e^(6*I*f*x + 6*I*e) - 648*I*c^2
*f^2*e^(4*I*f*x + 4*I*e) + 324*d^2*f*x*e^(4*I*f*x + 4*I*e) + 1296*I*c^2*f^2
*e^(2*I*f*x + 2*I*e) - 1296*d^2*f*x*e^(2*I*f*x + 2*I*e) - 48*c*d*f*e^(6*I*f
*x + 6*I*e) + 324*c*d*f*e^(4*I*f*x + 4*I*e) - 1296*c*d*f*e^(2*I*f*x + 2*I*e
) - 8*I*d^2*e^(6*I*f*x + 6*I*e) + 81*I*d^2*e^(4*I*f*x + 4*I*e) - 648*I*d^2*
e^(2*I*f*x + 2*I*e))/(a^3*f^3)
```

Mupad [B]
time = 1.00, size = 263, normalized size = 0.89

$$\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\left(\frac{\left(6\,c^2\,f^2+c\,d\,f\,6\mathrm{i}-3\,d^2\right)\,1\mathrm{i}}{32\,a^3\,f^3}+\frac{d^2\,x^2\,3\mathrm{i}}{16\,a^3\,f}+\frac{d\,x\,\left(2\,c\,f+d\,1\mathrm{i}\right)\,3\mathrm{i}}{16\,a^3\,f^2}\right)-\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\left(\frac{\left(24\,c^2\,f^2+c\,d\,f\,12\mathrm{i}-3\,d^2\right)\,1\mathrm{i}}{256\,a^3\,f^3}+\frac{d^2\,x^2\,3\mathrm{i}}{32\,a^3\,f}+\frac{d\,x\,\left(4\,c\,f+d\,1\mathrm{i}\right)\,3\mathrm{i}}{64\,a^3\,f^2}\right)+\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\left(\frac{\left(18\,c^2\,f^2+c\,d\,f\,6\mathrm{i}-d^2\right)\,1\mathrm{i}}{864\,a^3\,f^3}+\frac{d^2\,x^2\,1\mathrm{i}}{48\,a^3\,f}+\frac{d\,x\,\left(6\,c\,f+d\,1\mathrm{i}\right)\,1\mathrm{i}}{144\,a^3\,f^2}\right)+\frac{c^2\,x}{8\,a^3}+\frac{d^2\,x^3}{24\,a^3}+\frac{c\,d\,x^2}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*cot(e + f*x)*1i)^3,x)
```

```
[Out] exp(e*2i + f*x*2i)*(((6*c^2*f^2 - 3*d^2 + c*d*f*6i)*1i)/(32*a^3*f^3) + (d^2
*x^2*3i)/(16*a^3*f) + (d*x*(d*1i + 2*c*f)*3i)/(16*a^3*f^2)) - exp(e*4i + f*
x*4i)*(((24*c^2*f^2 - 3*d^2 + c*d*f*12i)*1i)/(256*a^3*f^3) + (d^2*x^2*3i)/(
```

```
32*a^3*f) + (d*x*(d*1i + 4*c*f)*3i)/(64*a^3*f^2)) + exp(e*6i + f*x*6i)*(((1
8*c^2*f^2 - d^2 + c*d*f*6i)*1i)/(864*a^3*f^3) + (d^2*x^2*1i)/(48*a^3*f) + (
d*x*(d*1i + 6*c*f)*1i)/(144*a^3*f^2)) + (c^2*x)/(8*a^3) + (d^2*x^3)/(24*a^3
) + (c*d*x^2)/(8*a^3)
```

3.29 $\int \frac{c+dx}{(a+ia\cot(e+fx))^3}\,dx$

Optimal. Leaf size=209

$$\frac{11idx}{96a^3f}-\frac{dx^2}{16a^3}+\frac{x(c+dx)}{8a^3}+\frac{d}{36f^2(a+ia\cot(e+fx))^3}-\frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3}+\frac{5d}{96af^2(a+ia\cot(e+fx))}$$

```
[Out] 11/96*I*d*x/a^3/f-1/16*d*x^2/a^3+1/8*x*(d*x+c)/a^3+1/36*d/f^2/(a+I*a*cot(f*
x+e))^3-1/6*I*(d*x+c)/f/(a+I*a*cot(f*x+e))^3+5/96*d/a/f^2/(a+I*a*cot(f*x+e)
)^2-1/8*I*(d*x+c)/a/f/(a+I*a*cot(f*x+e))^2+11/96*d/f^2/(a^3+I*a^3*cot(f*x+e
))-1/8*I*(d*x+c)/f/(a^3+I*a^3*cot(f*x+e))
```

Rubi [A]
time = 0.16, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3560, 8, 3811}

$$-\frac{i(c+dx)}{8f\left(a^3+ia^3\cot(e+fx)\right)}+\frac{x(c+dx)}{8a^3}+\frac{11d}{96f^2\left(a^3+ia^3\cot(e+fx)\right)}+\frac{11idx}{96a^3f}-\frac{dx^2}{16a^3}-\frac{i(c+dx)}{8af(a+ia\cot(e+fx))^2}-\frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3}+\frac{5d}{96af^2(a+ia\cot(e+fx))^2}+\frac{d}{36f^2(a+ia\cot(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] (((11*I)/96)*d*x)/(a^3*f) - (d*x^2)/(16*a^3) + (x*(c + d*x))/(8*a^3) + d/(3
6*f^2*(a + I*a*Cot[e + f*x])^3) - ((I/6)*(c + d*x))/(f*(a + I*a*Cot[e + f*x
])^3) + (5*d)/(96*a*f^2*(a + I*a*Cot[e + f*x])^2) - ((I/8)*(c + d*x))/(a*f*
(a + I*a*Cot[e + f*x])^2) + (11*d)/(96*f^2*(a^3 + I*a^3*Cot[e + f*x])) - ((
I/8)*(c + d*x))/(f*(a^3 + I*a^3*Cot[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3811

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Dist[(c + d*x)
^m, u, x] - Dist[d*m, Int[Dist[(c + d*x)^(m - 1), u, x], x], x]] /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+ia\cot(e+fx))^3}\,dx &= \frac{x(c+dx)}{8a^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{8af(a+ia\cot(e+fx))^2} - \frac{i(}{8f\,(a^3+i} \\ &= -\frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{8af(a+ia\cot(e+fx))^2} - \\ &= -\frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3} + \\ &= \frac{idx}{16a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+} \\ &= \frac{3idx}{32a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+} \\ &= \frac{11idx}{96a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+}\end{aligned}$$

Mathematica [**A**]
time = 0.56, size = 244, normalized size = 1.17

$$\frac{-72de^2+144cef+144cf^2x+72df^2x^2+108i(2cf+d(i+2fx))\cos(2(e+fx))+27(d-4icf-4idfx)\cos(4(e+fx))-4d\cos(6(e+fx))+24icf\cos(6(e+fx))+24idfx\cos(6(e+fx))-108id\sin(2(e+fx))-216cf\sin(2(e+fx))-216dfx\sin(2(e+fx))+27id\sin(4(e+fx))+108cf\sin(4(e+fx))+108dfx\sin(4(e+fx))-4id\sin(6(e+fx))-24cf\sin(6(e+fx))-24dfx\sin(6(e+fx))}{1152a^3f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] (-72*d*e^2 + 144*c*e*f + 144*c*f^2*x + 72*d*f^2*x^2 + (108*I)*(2*c*f + d*(I
 + 2*f*x))*Cos[2*(e + f*x)] + 27*(d - (4*I)*c*f - (4*I)*d*f*x)*Cos[4*(e + f
*x)] - 4*d*Cos[6*(e + f*x)] + (24*I)*c*f*Cos[6*(e + f*x)] + (24*I)*d*f*x*Co
s[6*(e + f*x)] - (108*I)*d*Sin[2*(e + f*x)] - 216*c*f*Sin[2*(e + f*x)] - 21
6*d*f*x*Sin[2*(e + f*x)] + (27*I)*d*Sin[4*(e + f*x)] + 108*c*f*Sin[4*(e + f
*x)] + 108*d*f*x*Sin[4*(e + f*x)] - (4*I)*d*Sin[6*(e + f*x)] - 24*c*f*Sin[6
*(e + f*x)] - 24*d*f*x*Sin[6*(e + f*x)])/(1152*a^3*f^2)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 664 vs. $2(181) = 362$.
time = 0.63, size = 665, normalized size = 3.18 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+I*a*cot(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^2/a^3*(4*I*c*f*(-1/6*sin(f*x+e)^2*cos(f*x+e)^4-1/12*cos(f*x+e)^4)-4*I*d
*e*(-1/6*sin(f*x+e)^2*cos(f*x+e)^4-1/12*cos(f*x+e)^4)+4*I*d*(1/4*(f*x+e)*si
n(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-1/24*f*x-1/24*e-1/
6*(f*x+e)*sin(f*x+e)^6-1/36*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))
*cos(f*x+e))-4*c*f*(-1/6*cos(f*x+e)^3*sin(f*x+e)^3-1/8*cos(f*x+e)^3*sin(f*x
```

```
+e)+1/16*cos(f*x+e)*sin(f*x+e)+1/16*f*x+1/16*e)+4*d*e*(-1/6*cos(f*x+e)^3*si
n(f*x+e)^3-1/8*cos(f*x+e)^3*sin(f*x+e)+1/16*cos(f*x+e)*sin(f*x+e)+1/16*f*x+
1/16*e)-4*d*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x
+3/8*e)-1/32*(f*x+e)^2+1/64*(2*sin(f*x+e)^2+3)^2-(f*x+e)*(-1/6*(sin(f*x+e)^
5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-1/36*sin(f*
x+e)^6-5/96*sin(f*x+e)^4-5/32*sin(f*x+e)^2)-3/4*I*c*f*sin(f*x+e)^4+3/4*I*d*
e*sin(f*x+e)^4-3*I*d*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f
*x+e))*cos(f*x+e)-3/32*f*x-3/32*e)+c*f*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*
cos(f*x+e)+3/8*f*x+3/8*e)-d*e*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e
)+3/8*f*x+3/8*e)+d*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+
3/8*f*x+3/8*e)-3/16*(f*x+e)^2+1/64*(2*sin(f*x+e)^2+3)^2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 3.40, size = 96, normalized size = 0.46

$$\frac{72\,df^2x^2 + 144\,cf^2x - 4\,(-6i\,dfx - 6i\,cf + d)e^{(6i\,fx+6i\,e)} - 27\,(4i\,dfx + 4i\,cf - d)e^{(4i\,fx+4i\,e)} - 108\,(-2i\,dfx - 2i\,cf + d)e^{(2i\,fx+2i\,e)}}{1152\,a^3f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/1152*(72*d*f^2*x^2 + 144*c*f^2*x - 4*(-6*I*d*f*x - 6*I*c*f + d)*e^(6*I*f*
x + 6*I*e) - 27*(4*I*d*f*x + 4*I*c*f - d)*e^(4*I*f*x + 4*I*e) - 108*(-2*I*d
*f*x - 2*I*c*f + d)*e^(2*I*f*x + 2*I*e))/(a^3*f^2)
```

Sympy [A]
time = 0.25, size = 298, normalized size = 1.43

$$\begin{cases} \frac{\left(221184ia^6cf^5e^{2ie}+221184ia^6df^5xe^{2ie}-110592a^6df^4e^{2ie}\right)e^{2ifx}+\left(-110592ia^6cf^5e^{4ie}-110592ia^6df^5xe^{4ie}+27648a^6df^4e^{4ie}\right)e^{4ifx}+\left(24576ia^6cf^5e^{6ie}+24576ia^6df^5xe^{6ie}-4096a^6df^4e^{6ie}\right)e^{6ifx}}{1179648a^9f^6} & \text{for}\ a^9f^6\neq 0 \\ \frac{x^2\left(-de^{6ie}+3de^{4ie}-3de^{2ie}\right)}{16a^3}+\frac{x\left(-ce^{6ie}+3ce^{4ie}-3ce^{2ie}\right)}{8a^3} & \text{otherwise} \end{cases} + \frac{cx}{8a^3} + \frac{dx^2}{16a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))**3,x)
```

```
[Out] Piecewise((((221184*I*a**6*c*f**5*exp(2*I*e) + 221184*I*a**6*d*f**5*x*exp(2
*I*e) - 110592*a**6*d*f**4*exp(2*I*e))*exp(2*I*f*x) + (-110592*I*a**6*c*f**
```

```
5*exp(4*I*e) - 110592*I*a**6*d*f**5*x*exp(4*I*e) + 27648*a**6*d*f**4*exp(4*
I*e))*exp(4*I*f*x) + (24576*I*a**6*c*f**5*exp(6*I*e) + 24576*I*a**6*d*f**5*
x*exp(6*I*e) - 4096*a**6*d*f**4*exp(6*I*e))*exp(6*I*f*x))/(1179648*a**9*f**
6), Ne(a**9*f**6, 0)), (x**2*(-d*exp(6*I*e) + 3*d*exp(4*I*e) - 3*d*exp(2*I*
e))/(16*a**3) + x*(-c*exp(6*I*e) + 3*c*exp(4*I*e) - 3*c*exp(2*I*e))/(8*a**3
), True)) + c*x/(8*a**3) + d*x**2/(16*a**3)
```

Giac [A]
time = 0.52, size = 142, normalized size = 0.68

$$\frac{72\,df^2x^2+144\,cf^2x+24i\,dfxe^{(6i\,fx+6i\,e)}-108i\,dfxe^{(4i\,fx+4i\,e)}+216i\,dfxe^{(2i\,fx+2i\,e)}+24i\,cfe^{(6i\,fx+6i\,e)}-108i\,cfe^{(4i\,fx+4i\,e)}+216i\,cfe^{(2i\,fx+2i\,e)}-4\,de^{(6i\,fx+6i\,e)}+27\,de^{(4i\,fx+4i\,e)}-108\,de^{(2i\,fx+2i\,e)}}{1152\,a^3f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/1152*(72*d*f^2*x^2 + 144*c*f^2*x + 24*I*d*f*x*e^(6*I*f*x + 6*I*e) - 108*I
*d*f*x*e^(4*I*f*x + 4*I*e) + 216*I*d*f*x*e^(2*I*f*x + 2*I*e) + 24*I*c*f*e^(
6*I*f*x + 6*I*e) - 108*I*c*f*e^(4*I*f*x + 4*I*e) + 216*I*c*f*e^(2*I*f*x + 2
*I*e) - 4*d*e^(6*I*f*x + 6*I*e) + 27*d*e^(4*I*f*x + 4*I*e) - 108*d*e^(2*I*f
*x + 2*I*e))/(a^3*f^2)
```

Mupad [B]
time = 0.62, size = 144, normalized size = 0.69

$$\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\left(\frac{\left(6\,c\,f+d\,3\mathrm{i}\right)\,1\mathrm{i}}{32\,a^3\,f^2}+\frac{d\,x\,3\mathrm{i}}{16\,a^3\,f}\right)-\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\left(\frac{\left(12\,c\,f+d\,3\mathrm{i}\right)\,1\mathrm{i}}{128\,a^3\,f^2}+\frac{d\,x\,3\mathrm{i}}{32\,a^3\,f}\right)+\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\left(\frac{\left(6\,c\,f+d\,1\mathrm{i}\right)\,1\mathrm{i}}{288\,a^3\,f^2}+\frac{d\,x\,1\mathrm{i}}{48\,a^3\,f}\right)+\frac{d\,x^2}{16\,a^3}+\frac{c\,x}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*cot(e + f*x)*1i)^3,x)
```

```
[Out] exp(e*2i + f*x*2i)*(((d*3i + 6*c*f)*1i)/(32*a^3*f^2) + (d*x*3i)/(16*a^3*f))
 - exp(e*4i + f*x*4i)*(((d*3i + 12*c*f)*1i)/(128*a^3*f^2) + (d*x*3i)/(32*a^
3*f)) + exp(e*6i + f*x*6i)*(((d*1i + 6*c*f)*1i)/(288*a^3*f^2) + (d*x*1i)/(4
8*a^3*f)) + (d*x^2)/(16*a^3) + (c*x)/(8*a^3)
```

3.30 $\int \frac{1}{(c+dx)(a+ia\cot(e+fx))^3}\,dx$

Optimal. Leaf size=449

$$-\frac{3\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{8a^3d}+\frac{3\cos\left(4e-\frac{4cf}{d}\right)\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)}{8a^3d}-\frac{\cos\left(6e-\frac{6cf}{d}\right)\text{CosInt}}{8a^3}$$

```
[Out] -1/8*Ci(6*c*f/d+6*f*x)*cos(-6*e+6*c*f/d)/a^3/d+3/8*Ci(4*c*f/d+4*f*x)*cos(-4
*e+4*c*f/d)/a^3/d-3/8*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a^3/d+1/8*ln(d*x+
c)/a^3/d-3/8*I*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a^3/d+3/8*I*cos(-4*e+4*c
*f/d)*Si(4*c*f/d+4*f*x)/a^3/d-1/8*I*cos(-6*e+6*c*f/d)*Si(6*c*f/d+6*f*x)/a^3
/d+1/8*I*Ci(6*c*f/d+6*f*x)*sin(-6*e+6*c*f/d)/a^3/d-1/8*Si(6*c*f/d+6*f*x)*si
n(-6*e+6*c*f/d)/a^3/d-3/8*I*Ci(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^3/d+3/8*S
i(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^3/d+3/8*I*Ci(2*c*f/d+2*f*x)*sin(-2*e+2
*c*f/d)/a^3/d-3/8*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a^3/d
```

Rubi [A]

time = 1.25, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 53, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3809, 3384, 3380, 3383, 3393, 4491, 4513}

$$-\frac{3i\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{8a^3d}-\frac{i\text{CosIntegral}\left(\frac{6cf}{d}+6fx\right)\sin\left(6e-\frac{6cf}{d}\right)}{8a^3d}+\frac{3i\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\sin\left(4e-\frac{4cf}{d}\right)}{8a^3d}-\frac{3\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{8a^3d}+\frac{3\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\cos\left(4e-\frac{4cf}{d}\right)}{8a^3d}-\frac{\text{CosIntegral}\left(\frac{6cf}{d}+6fx\right)\cos\left(6e-\frac{6cf}{d}\right)}{8a^3d}+\frac{3\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{8a^3d}-\frac{3\sin\left(4e-\frac{4cf}{d}\right)\text{Si}\left(4xf+\frac{4cf}{d}\right)}{8a^3d}+\frac{\sin\left(6e-\frac{6cf}{d}\right)\text{Si}\left(6xf+\frac{6cf}{d}\right)}{8a^3d}-\frac{3i\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{8a^3d}+\frac{3i\cos\left(4e-\frac{4cf}{d}\right)\text{Si}\left(4xf+\frac{4cf}{d}\right)}{8a^3d}-\frac{i\cos\left(6e-\frac{6cf}{d}\right)\text{Si}\left(6xf+\frac{6cf}{d}\right)}{8a^3d}+\frac{\log(c+dx)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + I*a*Cot[e + f*x])^3),x]
```

```
[Out] (-3*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) + (3*Cos
[4*e - (4*c*f)/d]*CosIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) - (Cos[6*e - (6
*c*f)/d]*CosIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d) + Log[c + d*x]/(8*a^3*d)
 - ((I/8)*CosIntegral[(6*c*f)/d + 6*f*x]*Sin[6*e - (6*c*f)/d])/(a^3*d) + ((
(3*I)/8)*CosIntegral[(4*c*f)/d + 4*f*x]*Sin[4*e - (4*c*f)/d])/(a^3*d) - (((
3*I)/8)*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a^3*d) - (((3
*I)/8)*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^3*d) + (3*Si
n[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) + (((3*I)/8)*C
os[4*e - (4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(a^3*d) - (3*Sin[4*e -
(4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) - ((I/8)*Cos[6*e - (6*
c*f)/d]*SinIntegral[(6*c*f)/d + 6*f*x])/(a^3*d) + (Sin[6*e - (6*c*f)/d]*Sin
Integral[(6*c*f)/d + 6*f*x])/(8*a^3*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3809

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4513

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d
_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sin[a + b*x
]^p*Sin[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] &&
 IGtQ[q, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+ia\cot(e+fx))^3}\,dx = \int \left(\frac{1}{8a^3(c+dx)} - \frac{3\cos(2e+2fx)}{8a^3(c+dx)} + \frac{3\cos^2(2e+2fx)}{8a^3(c+dx)} - \frac{\cos^3(2e+\cdots}{8a^3(c+\cdots}\right.$$

$$= \frac{\log(c+dx)}{8a^3d} + \frac{i\int \frac{\sin^3(2e+2fx)}{c+dx}\,dx}{8a^3} - \frac{(3i)\int \frac{\sin(2e+2fx)}{c+dx}\,dx}{8a^3} - \frac{(3i)\int \frac{\cos^2(2\cdots}{\cdots}}{\cdots}$$

$$= \frac{\log(c+dx)}{8a^3d} + \frac{i\int \left(\frac{3\sin(2e+2fx)}{4(c+dx)} - \frac{\sin(6e+6fx)}{4(c+dx)}\right)dx}{8a^3} - \frac{(3i)\int \left(\frac{\sin(2e+2fx)}{4(c+dx)}\cdots\right.}{8\cdots}$$

$$= -\frac{3\cos\left(2e-\frac{2cf}{d}\right)\text{Ci}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} + \frac{\log(c+dx)}{8a^3d} + \frac{3i\text{Ci}\left(\frac{4cf}{d}+4fx\right)\text{s}\cdots}{8a^3d}$$

$$= -\frac{3\cos\left(2e-\frac{2cf}{d}\right)\text{Ci}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} + \frac{\log(c+dx)}{8a^3d} + \frac{3i\text{Ci}\left(\frac{4cf}{d}+4fx\right)\text{s}\cdots}{8a^3d}$$

$$= -\frac{3\cos\left(2e-\frac{2cf}{d}\right)\text{Ci}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} - \frac{\cos\left(6e-\frac{6cf}{d}\right)\text{Ci}\left(\frac{6cf}{d}+6fx\right)}{8a^3d} + \cdots$$

Mathematica [A]
time = 0.54, size = 197, normalized size = 0.44

$$\frac{\log(c+dx) - 3\left(\cos\left(2e-\frac{2cf}{d}\right)+i\sin\left(2e-\frac{2cf}{d}\right)\right)\left(\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)+i\text{Si}\left(\frac{2f(c+dx)}{d}\right)\right) + 3\left(\cos\left(4e-\frac{4cf}{d}\right)+i\sin\left(4e-\frac{4cf}{d}\right)\right)\left(\text{CosIntegral}\left(\frac{4f(c+dx)}{d}\right)+i\text{Si}\left(\frac{4f(c+dx)}{d}\right)\right) - \left(\cos\left(6e-\frac{6cf}{d}\right)+i\sin\left(6e-\frac{6cf}{d}\right)\right)\left(\text{CosIntegral}\left(\frac{6f(c+dx)}{d}\right)+i\text{Si}\left(\frac{6f(c+dx)}{d}\right)\right)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + I*a*Cot[e + f*x])^3),x]
```

```
[Out] (Log[c + d*x] - 3*(Cos[2*e - (2*c*f)/d] + I*Sin[2*e - (2*c*f)/d])*(CosInteg
ral[(2*f*(c + d*x))/d] + I*SinIntegral[(2*f*(c + d*x))/d]) + 3*(Cos[4*e - (
4*c*f)/d] + I*Sin[4*e - (4*c*f)/d])*(CosIntegral[(4*f*(c + d*x))/d] + I*Sin
Integral[(4*f*(c + d*x))/d]) - (Cos[6*e - (6*c*f)/d] + I*Sin[6*e - (6*c*f)/
d])*(CosIntegral[(6*f*(c + d*x))/d] + I*SinIntegral[(6*f*(c + d*x))/d]))/(8
*a^3*d)
```

Maple [A]
time = 0.59, size = 531, normalized size = 1.18

method	result
risch	$\frac{\ln(dx+c)}{8a^3d} + \frac{3\,\mathrm{e}^{-\frac{2i(cf-de)}{d}}\,\text{expIntegral}\left(1,-2ifx-2ie-\frac{2(icf-ide)}{d}\right)}{8a^3d} - \frac{3\,\mathrm{e}^{-\frac{4i(cf-de)}{d}}\,\text{expIntegral}\left(1,-4ifx-4ie-\frac{4(i\cdots}{\cdots}\right.}{8a^3d}$
derivativedivides	$-\frac{3i\left(\frac{2\,\text{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d} - \frac{2\,\text{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}\right)}{16} + \frac{3i\left(\frac{4\,\text{sinIntegral}\left(\cdots\right.}{\cdots}\right.}{\cdots}$

default	$-\frac{3i\left(\frac{2\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\,\mathrm{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}\right)}{16}+\frac{3i\left(\frac{4\,\mathrm{sinIntegral}\left(4f\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+I*a*cot(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-3/16*I*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2*Ci(2*f
*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)+3/32*I*(4*Si(4*f*x+4*e+4*(c*f-d
*e)/d)*cos(4*(c*f-d*e)/d)/d-4*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d
)/d)+3/8*Si(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d)/d+3/8*Ci(4*f*x+4*e+
4*(c*f-d*e)/d)*cos(4*(c*f-d*e)/d)/d+1/8*ln(c*f-d*e+d*(f*x+e))/d-3/8*Si(2*f*
x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d-3/8*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*c
os(2*(c*f-d*e)/d)/d-1/8*Si(6*f*x+6*e+6*(c*f-d*e)/d)*sin(6*(c*f-d*e)/d)/d-1/
8*Ci(6*f*x+6*e+6*(c*f-d*e)/d)*cos(6*(c*f-d*e)/d)/d-1/48*I*(6*Si(6*f*x+6*e+6
*(c*f-d*e)/d)*cos(6*(c*f-d*e)/d)/d-6*Ci(6*f*x+6*e+6*(c*f-d*e)/d)*sin(6*(c*f
-d*e)/d)/d))
```

Maxima [A]

time = 0.36, size = 295, normalized size = 0.66

$$\frac{f\cos\left(\frac{6(cf-de)}{d}\right)E_1\left(\frac{6(-i(fx+e)d-icf+ide)}{d}\right)-3f\cos\left(\frac{4(cf-de)}{d}\right)E_1\left(\frac{4(-i(fx+e)d-icf+ide)}{d}\right)+3f\cos\left(\frac{2(cf-de)}{d}\right)E_1\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)-ifE_1\left(\frac{6(-i(fx+e)d-icf+ide)}{d}\right)\sin\left(\frac{6(cf-de)}{d}\right)+3ifE_1\left(\frac{4(-i(fx+e)d-icf+ide)}{d}\right)\sin\left(\frac{4(cf-de)}{d}\right)-3ifE_1\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\sin\left(\frac{2(cf-de)}{d}\right)+f\log\left((fx+e)d+cf-de\right)}{8a^3df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(f*cos(6*(c*f - d*e)/d)*exp_integral_e(1, 6*(-I*(f*x + e)*d - I*c*f + I
*d*e)/d) - 3*f*cos(4*(c*f - d*e)/d)*exp_integral_e(1, 4*(-I*(f*x + e)*d - I
*c*f + I*d*e)/d) + 3*f*cos(2*(c*f - d*e)/d)*exp_integral_e(1, 2*(-I*(f*x +
e)*d - I*c*f + I*d*e)/d) - I*f*exp_integral_e(1, 6*(-I*(f*x + e)*d - I*c*f
+ I*d*e)/d)*sin(6*(c*f - d*e)/d) + 3*I*f*exp_integral_e(1, 4*(-I*(f*x + e)*
d - I*c*f + I*d*e)/d)*sin(4*(c*f - d*e)/d) - 3*I*f*exp_integral_e(1, 2*(-I*
(f*x + e)*d - I*c*f + I*d*e)/d)*sin(2*(c*f - d*e)/d) + f*log((f*x + e)*d +
c*f - d*e))/(a^3*d*f)
```

Fricas [A]

time = 4.03, size = 122, normalized size = 0.27

$$-\frac{3\,\mathrm{Ei}\left(-\frac{2(-idfx-icf)}{d}\right)e^{\left(-\frac{2(icf-ide)}{d}\right)}-3\,\mathrm{Ei}\left(-\frac{4(-idfx-icf)}{d}\right)e^{\left(-\frac{4(icf-ide)}{d}\right)}+\mathrm{Ei}\left(-\frac{6(-idfx-icf)}{d}\right)e^{\left(-\frac{6(icf-ide)}{d}\right)}-\log\left(\frac{dx+c}{d}\right)}{8a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*Ei(-2*(-I*d*f*x - I*c*f)/d)*e^(-2*(I*c*f - I*d*e)/d) - 3*Ei(-4*(-I*
d*f*x - I*c*f)/d)*e^(-4*(I*c*f - I*d*e)/d) + Ei(-6*(-I*d*f*x - I*c*f)/d)*e^
(-6*(I*c*f - I*d*e)/d) - log((d*x + c)/d))/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int \frac{1}{c\cot^3(e+fx)-3ic\cot^2(e+fx)-3c\cot(e+fx)+ic+dx\cot^3(e+fx)-3idx\cot^2(e+fx)-3dx\cot(e+fx)+idx}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))**3,x)
```

```
[Out] I*Integral(1/(c*cot(e + f*x)**3 - 3*I*c*cot(e + f*x)**2 - 3*c*cot(e + f*x)
+ I*c + d*x*cot(e + f*x)**3 - 3*I*d*x*cot(e + f*x)**2 - 3*d*x*cot(e + f*x)
+ I*d*x), x)/a**3
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1791 vs. 2(411) = 822.
time = 0.54, size = 1791, normalized size = 3.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/8*(cos(e)^6*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d) + 6*I*cos(e)^5*
cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e) - 15*cos(e)^4*cos(6*c*f
/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^2 - 20*I*cos(e)^3*cos(6*c*f/d)*c
os_integral(6*(d*f*x + c*f)/d)*sin(e)^3 + 15*cos(e)^2*cos(6*c*f/d)*cos_inte
gral(6*(d*f*x + c*f)/d)*sin(e)^4 + 6*I*cos(e)*cos(6*c*f/d)*cos_integral(6*(
d*f*x + c*f)/d)*sin(e)^5 - cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin
(e)^6 - I*cos(e)^6*cos_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/d) + 6*cos(e)^
5*cos_integral(6*(d*f*x + c*f)/d)*sin(e)*sin(6*c*f/d) + 15*I*cos(e)^4*cos_i
ntegral(6*(d*f*x + c*f)/d)*sin(e)^2*sin(6*c*f/d) - 20*cos(e)^3*cos_integral
(6*(d*f*x + c*f)/d)*sin(e)^3*sin(6*c*f/d) - 15*I*cos(e)^2*cos_integral(6*(d
*f*x + c*f)/d)*sin(e)^4*sin(6*c*f/d) + 6*cos(e)*cos_integral(6*(d*f*x + c*f
)/d)*sin(e)^5*sin(6*c*f/d) + I*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^6*sin
(6*c*f/d) + I*cos(e)^6*cos(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) - 6*cos
(e)^5*cos(6*c*f/d)*sin(e)*sin_integral(6*(d*f*x + c*f)/d) - 15*I*cos(e)^4*c
os(6*c*f/d)*sin(e)^2*sin_integral(6*(d*f*x + c*f)/d) + 20*cos(e)^3*cos(6*c*
f/d)*sin(e)^3*sin_integral(6*(d*f*x + c*f)/d) + 15*I*cos(e)^2*cos(6*c*f/d)*
sin(e)^4*sin_integral(6*(d*f*x + c*f)/d) - 6*cos(e)*cos(6*c*f/d)*sin(e)^5*s
in_integral(6*(d*f*x + c*f)/d) - I*cos(6*c*f/d)*sin(e)^6*sin_integral(6*(d*
f*x + c*f)/d) + cos(e)^6*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) + 6*I
*cos(e)^5*sin(e)*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) - 15*cos(e)^4
*sin(e)^2*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) - 20*I*cos(e)^3*sin(
e)^3*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) + 15*cos(e)^2*sin(e)^4*si
n(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) + 6*I*cos(e)*sin(e)^5*sin(6*c*f/
```

```
d)*sin_integral(6*(d*f*x + c*f)/d) - sin(e)^6*sin(6*c*f/d)*sin_integral(6*(
d*f*x + c*f)/d) - 3*cos(e)^4*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d) -
 12*I*cos(e)^3*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e) + 18*cos
(e)^2*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2 + 12*I*cos(e)*c
os(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^3 - 3*cos(4*c*f/d)*cos_i
ntegral(4*(d*f*x + c*f)/d)*sin(e)^4 + 3*I*cos(e)^4*cos_integral(4*(d*f*x +
c*f)/d)*sin(4*c*f/d) - 12*cos(e)^3*cos_integral(4*(d*f*x + c*f)/d)*sin(e)*s
in(4*c*f/d) - 18*I*cos(e)^2*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2*sin(4*
c*f/d) + 12*cos(e)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^3*sin(4*c*f/d) +
3*I*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^4*sin(4*c*f/d) - 3*I*cos(e)^4*co
s(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 12*cos(e)^3*cos(4*c*f/d)*sin(e
)*sin_integral(4*(d*f*x + c*f)/d) + 18*I*cos(e)^2*cos(4*c*f/d)*sin(e)^2*sin
_integral(4*(d*f*x + c*f)/d) - 12*cos(e)*cos(4*c*f/d)*sin(e)^3*sin_integral
(4*(d*f*x + c*f)/d) - 3*I*cos(4*c*f/d)*sin(e)^4*sin_integral(4*(d*f*x + c*f
)/d) - 3*cos(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 12*I*cos(e
)^3*sin(e)*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 18*cos(e)^2*sin(e
)^2*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 12*I*cos(e)*sin(e)^3*sin
(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 3*sin(e)^4*sin(4*c*f/d)*sin_int
egral(4*(d*f*x + c*f)/d) + 3*cos(e)^2*cos(2*c*f/d)*cos_integral(2*(d*f*x +
c*f)/d) + 6*I*cos(e)*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) -
3*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2 - 3*I*cos(e)^2*cos_
integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) + 6*cos(e)*cos_integral(2*(d*f*x +
 c*f)/d)*sin(e)*sin(2*c*f/d) + 3*I*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2
*sin(2*c*f/d) + 3*I*cos(e)^2*cos(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) -
 6*cos(e)*cos(2*c*f/d)*sin(e)*sin_integral(2*(d*f*x + c*f)/d) - 3*I*cos(2*c
*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) + 3*cos(e)^2*sin(2*c*f/d)*si
n_integral(2*(d*f*x + c*f)/d) + 6*I*cos(e)*sin(e)*sin(2*c*f/d)*sin_integral
(2*(d*f*x + c*f)/d) - 3*sin(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/
d) - log(d*x + c))/(a^3*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + a\cot\left(e + f\,x\right)\,1\mathrm{i}\right)^3\,\left(c + d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cot(e + f*x)*1i)^3*(c + d*x)),x)
[Out] int(1/((a + a*cot(e + f*x)*1i)^3*(c + d*x)), x)
```

3.31 $\int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))^3}\,dx$

Optimal. Leaf size=712

$$-\frac{1}{8a^3d(c+dx)}+\frac{9\cos(2e+2fx)}{32a^3d(c+dx)}-\frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)}+\frac{\cos^3(2e+2fx)}{8a^3d(c+dx)}+\frac{3\cos(6e+6fx)}{32a^3d(c+dx)}-\frac{3if\cos\left(2e-\frac{2cf}{d}\right)}{}$$

```
[Out] -1/8/a^3/d/(d*x+c)+3/2*I*f*Ci(4*c*f/d+4*f*x)*cos(-4*e+4*c*f/d)/a^3/d^2+15/3
2*I*sin(2*f*x+2*e)/a^3/d/(d*x+c)+3/32*I*sin(6*f*x+6*e)/a^3/d/(d*x+c)+9/32*c
os(2*f*x+2*e)/a^3/d/(d*x+c)-3/8*cos(2*f*x+2*e)^2/a^3/d/(d*x+c)+1/8*cos(2*f*
x+2*e)^3/a^3/d/(d*x+c)+3/32*cos(6*f*x+6*e)/a^3/d/(d*x+c)+3/4*f*cos(-2*e+2*c
*f/d)*Si(2*c*f/d+2*f*x)/a^3/d^2-3/2*f*cos(-4*e+4*c*f/d)*Si(4*c*f/d+4*f*x)/a
^3/d^2+3/4*f*cos(-6*e+6*c*f/d)*Si(6*c*f/d+6*f*x)/a^3/d^2-3/4*f*Ci(6*c*f/d+6
*f*x)*sin(-6*e+6*c*f/d)/a^3/d^2-1/8*I*sin(2*f*x+2*e)^3/a^3/d/(d*x+c)+3/2*f*
Ci(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^3/d^2-3/4*I*f*Si(6*c*f/d+6*f*x)*sin(-
6*e+6*c*f/d)/a^3/d^2-3/4*f*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a^3/d^2-3/4*
I*f*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a^3/d^2+3/2*I*f*Si(4*c*f/d+4*f*x)*s
in(-4*e+4*c*f/d)/a^3/d^2+3/8*sin(2*f*x+2*e)^2/a^3/d/(d*x+c)-3/4*I*f*Ci(6*c*
f/d+6*f*x)*cos(-6*e+6*c*f/d)/a^3/d^2-3/8*I*sin(4*f*x+4*e)/a^3/d/(d*x+c)-3/4
*I*f*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a^3/d^2
```

Rubi [A]
time = 1.20, antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 60, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3809, 3378, 3384, 3380, 3383, 3394, 12, 4491, 4513}

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])^3),x]
```

```
[Out] -1/8*1/(a^3*d*(c + d*x)) + (9*Cos[2*e + 2*f*x])/(32*a^3*d*(c + d*x)) - (3*C
os[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) + Cos[2*e + 2*f*x]^3/(8*a^3*d*(c + d
*x)) + (3*Cos[6*e + 6*f*x])/(32*a^3*d*(c + d*x)) - (((3*I)/4)*f*Cos[2*e - (
2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(a^3*d^2) + (((3*I)/2)*f*Cos[4*e
- (4*c*f)/d]*CosIntegral[(4*c*f)/d + 4*f*x])/(a^3*d^2) - (((3*I)/4)*f*Cos[6
*e - (6*c*f)/d]*CosIntegral[(6*c*f)/d + 6*f*x])/(a^3*d^2) + (3*f*CosIntegra
l[(6*c*f)/d + 6*f*x]*Sin[6*e - (6*c*f)/d])/(4*a^3*d^2) - (3*f*CosIntegral[(
4*c*f)/d + 4*f*x]*Sin[4*e - (4*c*f)/d])/(2*a^3*d^2) + (3*f*CosIntegral[(2*c
*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(4*a^3*d^2) + (((15*I)/32)*Sin[2*e + 2
*f*x])/(a^3*d*(c + d*x)) + (3*Sin[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) - ((I
/8)*Sin[2*e + 2*f*x]^3)/(a^3*d*(c + d*x)) - (((3*I)/8)*Sin[4*e + 4*f*x])/(a
^3*d*(c + d*x)) + (((3*I)/32)*Sin[6*e + 6*f*x])/(a^3*d*(c + d*x)) + (3*f*Co
s[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(4*a^3*d^2) + (((3*I)/4)
*f*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^3*d^2) - (3*f*Co
```

```
s[4*e - (4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(2*a^3*d^2) - (((3*I)/2)
*f*Sin[4*e - (4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(a^3*d^2) + (3*f*Co
s[6*e - (6*c*f)/d]*SinIntegral[(6*c*f)/d + 6*f*x])/(4*a^3*d^2) + (((3*I)/4)
*f*Sin[6*e - (6*c*f)/d]*SinIntegral[(6*c*f)/d + 6*f*x])/(a^3*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3809

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
```

```
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4513

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d
_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sin[a + b*x
]^p*Sin[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] &&
 IGtQ[q, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))^3}\,dx = \int \left(\frac{1}{8a^3(c+dx)^2} - \frac{3\cos(2e+2fx)}{8a^3(c+dx)^2} + \frac{3\cos^2(2e+2fx)}{8a^3(c+dx)^2} - \frac{\cos^3(2e}{8a^3(c}\right.$$

$$= -\frac{1}{8a^3d(c+dx)} + \frac{i\int\frac{\sin^3(2e+2fx)}{(c+dx)^2}\,dx}{8a^3} - \frac{(3i)\int\frac{\sin(2e+2fx)}{(c+dx)^2}\,dx}{8a^3} - (3i)\int$$

$$= -\frac{1}{8a^3d(c+dx)} + \frac{3\cos(2e+2fx)}{8a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cos^3(2e+}{8a^3d(c+}$$

$$= -\frac{1}{8a^3d(c+dx)} + \frac{3\cos(2e+2fx)}{8a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cos^3(2e+}{8a^3d(c+}$$

$$= -\frac{1}{8a^3d(c+dx)} + \frac{9\cos(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cos^3(2e+}{8a^3d(c+}$$

$$= -\frac{1}{8a^3d(c+dx)} + \frac{9\cos(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cos^3(2e+}{8a^3d(c+}$$

$$= -\frac{1}{8a^3d(c+dx)} + \frac{9\cos(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cos^3(2e+}{8a^3d(c+}$$

Mathematica [A]
time = 0.62, size = 292, normalized size = 0.41

$$\frac{-d+3d(\cos(2(e+fx))+i\sin(2(e+fx)))-3d(\cos(4(e+fx))+i\sin(4(e+fx)))+d(\cos(6(e+fx))+i\sin(6(e+fx)))+6f(c+dx)\left(-i\cos\left(2e-\frac{2cf}{d}\right)+\sin\left(2e-\frac{2cf}{d}\right)\right)\left(\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)+i\text{Si}\left(\frac{2f(c+dx)}{d}\right)\right)+12if(c+dx)\left(\cos\left(4e-\frac{4cf}{d}\right)+i\sin\left(4e-\frac{4cf}{d}\right)\right)\left(\text{CosIntegral}\left(\frac{4f(c+dx)}{d}\right)+i\text{Si}\left(\frac{4f(c+dx)}{d}\right)\right)+6f(c+dx)\left(-i\cos\left(6e-\frac{6cf}{d}\right)+\sin\left(6e-\frac{6cf}{d}\right)\right)\left(\text{CosIntegral}\left(\frac{6f(c+dx)}{d}\right)+i\text{Si}\left(\frac{6f(c+dx)}{d}\right)\right)}{8a^3d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])^3),x]
```

```
[Out] (-d + 3*d*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]) - 3*d*(Cos[4*(e + f*x)] +
 I*Sin[4*(e + f*x)]) + d*(Cos[6*(e + f*x)] + I*Sin[6*(e + f*x)]) + 6*f*(c +
 d*x)*((-I)*Cos[2*e - (2*c*f)/d] + Sin[2*e - (2*c*f)/d])*(CosIntegral[(2*f*
(c + d*x))/d] + I*SinIntegral[(2*f*(c + d*x))/d]) + (12*I)*f*(c + d*x)*(Cos
[4*e - (4*c*f)/d] + I*Sin[4*e - (4*c*f)/d])*(CosIntegral[(4*f*(c + d*x))/d]
 + I*SinIntegral[(4*f*(c + d*x))/d]) + 6*f*(c + d*x)*((-I)*Cos[6*e - (6*c*f
)/d] + Sin[6*e - (6*c*f)/d])*(CosIntegral[(6*f*(c + d*x))/d] + I*SinIntegra
l[(6*f*(c + d*x))/d]))/(8*a^3*d^2*(c + d*x))
```

Maple [A]
time = 0.63, size = 755, normalized size = 1.06 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] f/a^3*(-3/16*I*(-2*sin(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/d+2*(2*Si(2*f*x+2*e+2
*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f
-d*e)/d)/d)/d)+3/32*I*(-4*sin(4*f*x+4*e)/(c*f-d*e+d*(f*x+e))/d+4*(4*Si(4*f*
x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d)/d+4*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*cos
(4*(c*f-d*e)/d)/d)/d)-3/8*cos(4*f*x+4*e)/(c*f-d*e+d*(f*x+e))/d-3/8*(4*Si(4*
f*x+4*e+4*(c*f-d*e)/d)*cos(4*(c*f-d*e)/d)/d-4*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*s
in(4*(c*f-d*e)/d)/d)/d-1/8/(c*f-d*e+d*(f*x+e))/d+3/8*cos(2*f*x+2*e)/(c*f-d*
e+d*(f*x+e))/d+3/8*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2*Ci
(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)/d+1/8*cos(6*f*x+6*e)/(c*f-d
*e+d*(f*x+e))/d+1/8*(6*Si(6*f*x+6*e+6*(c*f-d*e)/d)*cos(6*(c*f-d*e)/d)/d-6*C
i(6*f*x+6*e+6*(c*f-d*e)/d)*sin(6*(c*f-d*e)/d)/d)/d-1/48*I*(-6*sin(6*f*x+6*e
)/(c*f-d*e+d*(f*x+e))/d+6*(6*Si(6*f*x+6*e+6*(c*f-d*e)/d)*sin(6*(c*f-d*e)/d)
/d+6*Ci(6*f*x+6*e+6*(c*f-d*e)/d)*cos(6*(c*f-d*e)/d)/d)/d))
```

Maxima [A]
time = 0.44, size = 319, normalized size = 0.45

$$\frac{f^2 \cos\left(\frac{6(cf-de)}{d}\right) E_2\left(\frac{6(-i(fx+e)d-icf+ide)}{d}\right) - 3f^2 \cos\left(\frac{4(cf-de)}{d}\right) E_2\left(\frac{4(-i(fx+e)d-icf+ide)}{d}\right) + 3f^2 \cos\left(\frac{2(cf-de)}{d}\right) E_2\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right) - i f^2 E_2\left(\frac{6(-i(fx+e)d-icf+ide)}{d}\right) \sin\left(\frac{6(cf-de)}{d}\right) + 3i f^2 E_2\left(\frac{4(-i(fx+e)d-icf+ide)}{d}\right) \sin\left(\frac{4(cf-de)}{d}\right) - 3i f^2 E_2\left(\frac{2(-i(fx+e)d-icf+ide)}{d}\right) \sin\left(\frac{2(cf-de)}{d}\right) - f^2}{8\left((fx+e)a^3d^2 + a^3cdf - a^3d^2e\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(f^2*cos(6*(c*f - d*e)/d)*exp_integral_e(2, 6*(-I*(f*x + e)*d - I*c*f +
 I*d*e)/d) - 3*f^2*cos(4*(c*f - d*e)/d)*exp_integral_e(2, 4*(-I*(f*x + e)*d
 - I*c*f + I*d*e)/d) + 3*f^2*cos(2*(c*f - d*e)/d)*exp_integral_e(2, 2*(-I*(
f*x + e)*d - I*c*f + I*d*e)/d) - I*f^2*exp_integral_e(2, 6*(-I*(f*x + e)*d
- I*c*f + I*d*e)/d)*sin(6*(c*f - d*e)/d) + 3*I*f^2*exp_integral_e(2, 4*(-I*
(f*x + e)*d - I*c*f + I*d*e)/d)*sin(4*(c*f - d*e)/d) - 3*I*f^2*exp_integral
```

```
_e(2, 2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d)*sin(2*(c*f - d*e)/d) - f^2)/(((
f*x + e)*a^3*d^2 + a^3*c*d*f - a^3*d^2*e)*f)
```

Fricas [A]
time = 3.18, size = 194, normalized size = 0.27

$$-\frac{6\,(i\,dfx+i\,cf)\mathrm{Ei}\left(-\frac{2\,(-i\,dfx-i\,cf)}{d}\right)e^{\left(-\frac{2\,(i\,cf-i\,de)}{d}\right)}+12\,(-i\,dfx-i\,cf)\mathrm{Ei}\left(-\frac{4\,(-i\,dfx-i\,cf)}{d}\right)e^{\left(-\frac{4\,(i\,cf-i\,de)}{d}\right)}+6\,(i\,dfx+i\,cf)\mathrm{Ei}\left(-\frac{6\,(-i\,dfx-i\,cf)}{d}\right)e^{\left(-\frac{6\,(i\,cf-i\,de)}{d}\right)}-de^{(6i\,fx+6i\,e)}+3\,de^{(4i\,fx+4i\,e)}-3\,de^{(2i\,fx+2i\,e)}+d}{8\,(a^3d^3x+a^3cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*(I*d*f*x + I*c*f)*Ei(-2*(-I*d*f*x - I*c*f)/d)*e^(-2*(I*c*f - I*d*e)
/d) + 12*(-I*d*f*x - I*c*f)*Ei(-4*(-I*d*f*x - I*c*f)/d)*e^(-4*(I*c*f - I*d*
e)/d) + 6*(I*d*f*x + I*c*f)*Ei(-6*(-I*d*f*x - I*c*f)/d)*e^(-6*(I*c*f - I*d*
e)/d) - d*e^(6*I*f*x + 6*I*e) + 3*d*e^(4*I*f*x + 4*I*e) - 3*d*e^(2*I*f*x +
2*I*e) + d)/(a^3*d^3*x + a^3*c*d^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+I*a*cot(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4284 vs. 2(648) = 1296.
time = 1.67, size = 4284, normalized size = 6.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/8*(-6*I*d*f*x*cos(e)^6*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d) + 36*
d*f*x*cos(e)^5*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e) + 90*I*d
*f*x*cos(e)^4*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^2 - 120*d
*f*x*cos(e)^3*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^3 - 90*I*
d*f*x*cos(e)^2*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^4 + 36*d
*f*x*cos(e)*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^5 + 6*I*d*f
*x*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^6 - 6*d*f*x*cos(e)^6
*cos_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/d) - 36*I*d*f*x*cos(e)^5*cos_int
egral(6*(d*f*x + c*f)/d)*sin(e)*sin(6*c*f/d) + 90*d*f*x*cos(e)^4*cos_integr
al(6*(d*f*x + c*f)/d)*sin(e)^2*sin(6*c*f/d) + 120*I*d*f*x*cos(e)^3*cos_inte
```

gral(6*(d*f*x + c*f)/d)*sin(e)^3*sin(6*c*f/d) - 90*d*f*x*cos(e)^2*cos_integ
ral(6*(d*f*x + c*f)/d)*sin(e)^4*sin(6*c*f/d) - 36*I*d*f*x*cos(e)*cos_integr
al(6*(d*f*x + c*f)/d)*sin(e)^5*sin(6*c*f/d) + 6*d*f*x*cos_integral(6*(d*f*x
 + c*f)/d)*sin(e)^6*sin(6*c*f/d) + 6*d*f*x*cos(e)^6*cos(6*c*f/d)*sin_integr
al(6*(d*f*x + c*f)/d) + 36*I*d*f*x*cos(e)^5*cos(6*c*f/d)*sin(e)*sin_integra
l(6*(d*f*x + c*f)/d) - 90*d*f*x*cos(e)^4*cos(6*c*f/d)*sin(e)^2*sin_integral
(6*(d*f*x + c*f)/d) - 120*I*d*f*x*cos(e)^3*cos(6*c*f/d)*sin(e)^3*sin_integr
al(6*(d*f*x + c*f)/d) + 90*d*f*x*cos(e)^2*cos(6*c*f/d)*sin(e)^4*sin_integra
l(6*(d*f*x + c*f)/d) + 36*I*d*f*x*cos(e)*cos(6*c*f/d)*sin(e)^5*sin_integral
(6*(d*f*x + c*f)/d) - 6*d*f*x*cos(6*c*f/d)*sin(e)^6*sin_integral(6*(d*f*x +
 c*f)/d) - 6*I*d*f*x*cos(e)^6*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d)
+ 36*d*f*x*cos(e)^5*sin(e)*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) + 9
0*I*d*f*x*cos(e)^4*sin(e)^2*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) -
120*d*f*x*cos(e)^3*sin(e)^3*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) -
90*I*d*f*x*cos(e)^2*sin(e)^4*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) +
 36*d*f*x*cos(e)*sin(e)^5*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) + 6*
I*d*f*x*sin(e)^6*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) - 6*I*c*f*cos
(e)^6*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d) + 36*c*f*cos(e)^5*cos(6*
c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e) + 90*I*c*f*cos(e)^4*cos(6*c*f
/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^2 - 120*c*f*cos(e)^3*cos(6*c*f/d
)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^3 - 90*I*c*f*cos(e)^2*cos(6*c*f/d)
cos_integral(6(d*f*x + c*f)/d)*sin(e)^4 + 36*c*f*cos(e)*cos(6*c*f/d)*cos_
integral(6*(d*f*x + c*f)/d)*sin(e)^5 + 6*I*c*f*cos(6*c*f/d)*cos_integral(6*
(d*f*x + c*f)/d)*sin(e)^6 - 6*c*f*cos(e)^6*cos_integral(6*(d*f*x + c*f)/d)*
sin(6*c*f/d) - 36*I*c*f*cos(e)^5*cos_integral(6*(d*f*x + c*f)/d)*sin(e)*sin
(6*c*f/d) + 90*c*f*cos(e)^4*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^2*sin(6*
c*f/d) + 120*I*c*f*cos(e)^3*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^3*sin(6*
c*f/d) - 90*c*f*cos(e)^2*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^4*sin(6*c*f
/d) - 36*I*c*f*cos(e)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^5*sin(6*c*f/d)
 + 6*c*f*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^6*sin(6*c*f/d) + 6*c*f*cos(
e)^6*cos(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) + 36*I*c*f*cos(e)^5*cos(6
*c*f/d)*sin(e)*sin_integral(6*(d*f*x + c*f)/d) - 90*c*f*cos(e)^4*cos(6*c*f/
d)*sin(e)^2*sin_integral(6*(d*f*x + c*f)/d) - 120*I*c*f*cos(e)^3*cos(6*c*f/
d)*sin(e)^3*sin_integral(6*(d*f*x + c*f)/d) + 90*c*f*cos(e)^2*cos(6*c*f/d)*
sin(e)^4*sin_integral(6*(d*f*x + c*f)/d) + 36*I*c*f*cos(e)*cos(6*c*f/d)*sin
(e)^5*sin_integral(6*(d*f*x + c*f)/d) - 6*c*f*cos(6*c*f/d)*sin(e)^6*sin_int
egral(6*(d*f*x + c*f)/d) - 6*I*c*f*cos(e)^6*sin(6*c*f/d)*sin_integral(6*(d*
f*x + c*f)/d) + 36*c*f*cos(e)^5*sin(e)*sin(6*c*f/d)*sin_integral(6*(d*f*x +
 c*f)/d) + 90*I*c*f*cos(e)^4*sin(e)^2*sin(6*c*f/d)*sin_integral(6*(d*f*x +
c*f)/d) - 120*c*f*cos(e)^3*sin(e)^3*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*
f)/d) - 90*I*c*f*cos(e)^2*sin(e)^4*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f
)/d) + 36*c*f*cos(e)*sin(e)^5*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d)
+ 6*I*c*f*sin(e)^6*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) + 12*I*d*f*
x*cos(e)^4*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d) - 48*d*f*x*cos(e)^3
*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e) - 72*I*d*f*x*cos(e)^2*

```
cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2 + 48*d*f*x*cos(e)*cos
(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^3 + 12*I*d*f*x*cos(4*c*f/d
)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^4 + 12*d*f*x*cos(e)^4*cos_integral
(4*(d*f*x + c*f)/d)*sin(4*c*f/d) + 48*I*d*f*x*cos(e)^3*cos_integral(4*(d*f*
x + c*f)/d)*sin(e)*sin(4*c*f/d) - 72*d*f*x*cos(e)^2*cos_integral(4*(d*f*x +
 c*f)/d)*sin(e)^2*sin(4*c*f/d) - 48*I*d*f*x*cos(e)*cos_integral(4*(d*f*x +
c*f)/d)*sin(e)^3*sin(4*c*f/d) + 12*d*f*x*cos_integral(4*(d*f*x + c*f)/d)*si
n(e)^4*sin(4*c*f/d) - 12*d*f*x*cos(e)^4*cos(4*c*f/d)*sin_integral(4*(d*f*x
+ c*f)/d) - 48*I*d*f*x*cos(e)^3*cos(4*c*f/d)*sin(e)*sin_integral(4*(d*f*x +
 c*f)/d) + 72*d*f*x*cos(e)^2*cos(4*c*f/d)*sin(e)^2*sin_integral(4*(d*f*x +
c*f)/d) + 48*I*d*f*x*cos(e)*cos(4*c*f/d)*sin(e)^3*sin_integral(4*(d*f*x + c
*f)/d) - 12*d*f*x*cos(4*c*f/d)*sin(e)^4*sin_int...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+a\cot(e+f\,x)\ 1\mathrm{i})^3\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cot(e + f*x)*1i)^3*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*cot(e + f*x)*1i)^3*(c + d*x)^2), x)
```

3.32 $\int (c+dx)^m(a+ia\cot(e+fx))^2\,dx$

Optimal. Leaf size=26

$$\text{Int}\left((c+dx)^m(a+ia\cot(e+fx))^2,x\right)$$

```
[Out] Unintegrable((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m(a+ia\cot(e+fx))^2\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(a + I*a*Cot[e + f*x])^2, x]
```

Rubi steps

$$\int (c+dx)^m(a+ia\cot(e+fx))^2\,dx = \int (c+dx)^m(a+ia\cot(e+fx))^2\,dx$$

Mathematica [A]
time = 8.84, size = 0, normalized size = 0.00

$$\int (c+dx)^m(a+ia\cot(e+fx))^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*x)^m*(a + I*a*Cot[e + f*x])^2, x]
```

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int (dx+c)^m\,(a+ia\cot(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (d*x + c)^(m + 1)*a^2/(d*(m + 1)) - integrate(-((d*x + c)^m*a^2*cos(4*f*x +
 4*e)^2 + 4*(d*x + c)^m*a^2*cos(2*f*x + 2*e)^2 + (d*x + c)^m*a^2*sin(4*f*x
+ 4*e)^2 - 4*(d*x + c)^m*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(d*x + c
)^m*a^2*sin(2*f*x + 2*e)^2 + 4*(d*x + c)^m*a^2*cos(2*f*x + 2*e) - 3*(d*x +
c)^m*a^2 - 2*(2*(d*x + c)^m*a^2*cos(2*f*x + 2*e) + (d*x + c)^m*a^2)*cos(4*f
*x + 4*e))/(2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^
2 - 4*cos(2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*
x + 2*e) - 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1), x) - I*integrate
(-4*((d*x + c)^m*a^2*sin(4*f*x + 4*e) - 2*(d*x + c)^m*a^2*sin(2*f*x + 2*e))
/(2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - 4*cos(
2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) -
 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (2*I*(d*x + c)^m*a^2 + (f*e^(2*I*f*x + 2*I*e) - f)*integral(2*(2*a^2*d*f*x
+ 2*a^2*c*f + I*a^2*d*m)*(d*x + c)^m/(d*f*x + c*f - (d*f*x + c*f)*e^(2*I*f*
x + 2*I*e)), x))/(f*e^(2*I*f*x + 2*I*e) - f)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-a^{2}\left(\int (c+dx)^{m}\cot^{2}(e+fx)\,dx+\int \left(-2i(c+dx)^{m}\cot(e+fx)\right)dx+\int \left(-(c+dx)^{m}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+I*a*cot(f*x+e))**2,x)
```

```
[Out] -a**2*(Integral((c + d*x)**m*cot(e + f*x)**2, x) + Integral(-2*I*(c + d*x)*
*m*cot(e + f*x), x) + Integral(-(c + d*x)**m, x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*cot(f*x + e) + a)^2*(d*x + c)^m, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int (a + a\cot(e + f\,x)\;1\mathrm{i})^2\,(c + d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(e + f*x)*1i)^2*(c + d*x)^m,x)
```

```
[Out] int((a + a*cot(e + f*x)*1i)^2*(c + d*x)^m, x)
```

3.33 $\int (c+dx)^m(a+ia\cot(e+fx))\,dx$

Optimal. Leaf size=24

$$\mathrm{Int}((c+dx)^m(a+ia\cot(e+fx)),x)$$

```
[Out] Unintegrable((d*x+c)^m*(a+I*a*cot(f*x+e)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m(a+ia\cot(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + I*a*Cot[e + f*x]),x]
[Out] Defer[Int][(c + d*x)^m*(a + I*a*Cot[e + f*x]), x]
```

Rubi steps

$$\int (c+dx)^m(a+ia\cot(e+fx))\,dx = \int (c+dx)^m(a+ia\cot(e+fx))\,dx$$

Mathematica [A]
time = 5.71, size = 0, normalized size = 0.00

$$\int (c+dx)^m(a+ia\cot(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + I*a*Cot[e + f*x]),x]
[Out] Integrate[(c + d*x)^m*(a + I*a*Cot[e + f*x]), x]
```

Maple [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int (dx+c)^m\left(a+ia\cot\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+I*a*cot(f*x+e)),x)
```

```
[Out] int((d*x+c)^m*(a+I*a*cot(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*I*a*integrate((d*x + c)^m*sin(2*f*x + 2*e)/(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 - 2*cos(2*f*x + 2*e) + 1), x) + (d*x + c)^(m + 1)*a/(d*(m + 1))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(-2*(d*x + c)^m*a/(e^(2*I*f*x + 2*I*e) - 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i(c+dx)^m\right)\,dx+\int(c+dx)^m\cot\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+I*a*cot(f*x+e)),x)
```

```
[Out] I*a*(Integral(-I*(c + d*x)**m, x) + Integral((c + d*x)**m*cot(e + f*x), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((I*a*cot(f*x + e) + a)*(d*x + c)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (a + a\cot(e + f\,x)\ 1\mathrm{i})\ (c + d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(e + f*x)*1i)*(c + d*x)^m,x)
[Out] int((a + a*cot(e + f*x)*1i)*(c + d*x)^m, x)
```

3.34 $\int \frac{(c+dx)^m}{a+ia\cot(e+fx)}\,dx$

Optimal. Leaf size=98

$$\frac{(c+dx)^{1+m}}{2ad(1+m)}+\frac{i2^{-2-m}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{2if(c+dx)}{d}\right)}{af}$$

```
[Out] 1/2*(d*x+c)^(1+m)/a/d/(1+m)+I*2^(-2-m)*exp(2*I*(e-c*f/d))*(d*x+c)^m*GAMMA(1
+m,-2*I*f*(d*x+c)/d)/a/f/((-I*f*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3808, 2212}

$$\frac{(c+dx)^{m+1}}{2ad(m+1)}+\frac{i2^{-m-2}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2if(c+dx)}{d}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] (c + d*x)^(1 + m)/(2*a*d*(1 + m)) + (I*2^(-2 - m)*E^((2*I)*(e - (c*f)/d))*(
c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(a*f*(((-I)*f*(c + d*x))/d
)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3808

```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sym
bol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + Dist[1/(2*a), Int[(c +
 d*x)^m*E^(2*(a/b)*(e + f*x)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[a^2 + b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(c+dx)^m}{a+ia\cot(e+fx)}\,dx = \frac{(c+dx)^{1+m}}{2ad(1+m)} + \frac{\int e^{2i\left(e+\frac{\pi}{2}+fx\right)}(c+dx)^m\,dx}{2a}$$
$$= \frac{(c+dx)^{1+m}}{2ad(1+m)} + \frac{i2^{-2-m}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+dx)}{d}\right)}{af}$$

Mathematica [A]
time = 1.25, size = 190, normalized size = 1.94

$$\frac{2^{-2-m}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^m\left(\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(2^{1+m}f(c+dx)\left(-\frac{if(c+dx)}{d}\right)^m\left(\cos\left(e-\frac{cf}{d}\right)-i\sin\left(e-\frac{cf}{d}\right)\right)+id(1+m)\text{Gamma}\left(1+m,-\frac{2if(c+dx)}{d}\right)\left(\cos\left(e-\frac{cf}{d}\right)+i\sin\left(e-\frac{cf}{d}\right)\right)\right)\left(\cos\left(e-\frac{cf}{d}\right)+i\sin\left(e-\frac{cf}{d}\right)\right)}{adf(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] (2^(-2 - m)*(c + d*x)^m*((I*f*(c + d*x))/d)^m*(2^(1 + m)*f*(c + d*x)*(((-I)
*f*(c + d*x))/d)^m*(Cos[e - (c*f)/d] - I*Sin[e - (c*f)/d]) + I*d*(1 + m)*Ga
mma[1 + m, ((-2*I)*f*(c + d*x))/d]*(Cos[e - (c*f)/d] + I*Sin[e - (c*f)/d]))
*(Cos[e - (c*f)/d] + I*Sin[e - (c*f)/d]))/(a*d*f*(1 + m)*((f^2*(c + d*x)^2)
/d^2)^m)
```

Maple [F]
time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{a+ia\cot(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+I*a*cot(f*x+e)),x)
```

```
[Out] int((d*x+c)^m/(a+I*a*cot(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*((d*m + d)*integrate((d*x + c)^m*cos(2*f*x + 2*e), x) + (I*d*m + I*d)*
integrate((d*x + c)^m*sin(2*f*x + 2*e), x) - e^(m*log(d*x + c) + log(d*x +
c)))/(a*d*m + a*d)
```

Fricas [A]
time = 0.50, size = 86, normalized size = 0.88

$$\frac{(i\,dm+i\,d)e^{\left(-\frac{dm\log\left(-\frac{2i\,f}{d}\right)+2i\,cf-2i\,de}{d}\right)}\Gamma\left(m+1,-\frac{2\,(i\,dfx+i\,cf)}{d}\right)+2\,(dfx+cf)(dx+c)^m}{4\,(adfm+adf)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*((I*d*m + I*d)*e^(-(d*m*log(-2*I*f/d) + 2*I*c*f - 2*I*d*e)/d)*gamma(m +
 1, -2*(I*d*f*x + I*c*f)/d) + 2*(d*f*x + c*f)*(d*x + c)^m)/(a*d*f*m + a*d*f
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{(c+dx)^m}{\cot(e+fx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+I*a*cot(f*x+e)),x)
```

```
[Out] -I*Integral((c + d*x)**m/(cot(e + f*x) - I), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(I*a*cot(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(c+d\,x)^m}{a+a\cot(e+f\,x)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*cot(e + f*x)*1i),x)
```

```
[Out] int((c + d*x)^m/(a + a*cot(e + f*x)*1i), x)
```

3.35 $\int \frac{(c+dx)^m}{(a+ia\cot(e+fx))^2}\,dx$

Optimal. Leaf size=171

$$\frac{(c+dx)^{1+m}}{4a^2d(1+m)}+\frac{i2^{-2-m}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{2if(c+dx)}{d}\right)}{a^2f}-\frac{i4^{-2-m}e^{4i\left(e-\frac{cf}{d}\right)}(c+$$

```
[Out] 1/4*(d*x+c)^(1+m)/a^2/d/(1+m)+I*2^(-2-m)*exp(2*I*(e-c*f/d))*(d*x+c)^m*GAMMA
(1+m,-2*I*f*(d*x+c)/d)/a^2/f/((-I*f*(d*x+c)/d)^m)-I*4^(-2-m)*exp(4*I*(e-c*f
/d))*(d*x+c)^m*GAMMA(1+m,-4*I*f*(d*x+c)/d)/a^2/f/((-I*f*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.13, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3810, 2212}

$$\frac{i2^{-m-2}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2if(c+dx)}{d}\right)}{a^2f}-\frac{i4^{-m-2}e^{4i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{4if(c+dx)}{d}\right)}{a^2f}+\frac{(c+dx)^{m+1}}{4a^2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] (c + d*x)^(1 + m)/(4*a^2*d*(1 + m)) + (I*2^(-2 - m)*E^((2*I)*(e - (c*f)/d))
*(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(a^2*f*(((-I)*f*(c + d*x
))/d)^m) - (I*4^(-2 - m)*E^((4*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, (
(-4*I)*f*(c + d*x))/d])/(a^2*f*(((-I)*f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3810

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+dx)^m}{(a+ia\cot(e+fx))^2}\,dx = \int \left(\frac{(c+dx)^m}{4a^2} - \frac{e^{2ie+2ifx}(c+dx)^m}{2a^2} + \frac{e^{4ie+4ifx}(c+dx)^m}{4a^2}\right) dx$$
$$= \frac{(c+dx)^{1+m}}{4a^2d(1+m)} + \frac{\int e^{4ie+4ifx}(c+dx)^m\,dx}{4a^2} - \frac{\int e^{2ie+2ifx}(c+dx)^m\,dx}{2a^2}$$
$$= \frac{(c+dx)^{1+m}}{4a^2d(1+m)} + \frac{i2^{-2-m}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+dx)}{d}\right.}{a^2f}$$

Mathematica [A]
time = 9.65, size = 155, normalized size = 0.91

$$\frac{(c+dx)^m\left(\frac{4c+4dx}{d+dm}+\frac{i2^{2-m}e^{2i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{2if(c+dx)}{d}\right)}{f}-\frac{i4^{-m}e^{4i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{4if(c+dx)}{d}\right)}{f}\right)}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] ((c + d*x)^m*((4*c + 4*d*x)/(d + d*m) + (I*2^(2 - m)*E^((2*I)*(e - (c*f)/d)
)*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(f*(((-I)*f*(c + d*x))/d)^m) - (I*E
^((4*I)*(e - (c*f)/d))*Gamma[1 + m, ((-4*I)*f*(c + d*x))/d])/(4^m*f*(((-I)*
f*(c + d*x))/d)^m)))/(16*a^2)
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{(a+ia\cot(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+I*a*cot(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m/(a+I*a*cot(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((d*m + d)*integrate((d*x + c)^m*cos(4*f*x + 4*e), x) - 2*(d*m + d)*int
egrate((d*x + c)^m*cos(2*f*x + 2*e), x) + (I*d*m + I*d)*integrate((d*x + c)
^m*sin(4*f*x + 4*e), x) - 2*(I*d*m + I*d)*integrate((d*x + c)^m*sin(2*f*x +
 2*e), x) + e^(m*log(d*x + c) + log(d*x + c)))/(a^2*d*m + a^2*d)
```

Fricas [A]
time = 0.72, size = 146, normalized size = 0.85

$$-\frac{4\left(-i\,dm-i\,d\right)e^{\left(-\frac{dm\log\left(-\frac{2i\,f}{d}\right)+2i\,cf-2i\,de}{d}\right)}\Gamma\left(m+1,-\frac{2\,(i\,dfx+i\,cf)}{d}\right)-\left(-i\,dm-i\,d\right)e^{\left(-\frac{dm\log\left(-\frac{4i\,f}{d}\right)+4i\,cf-4i\,de}{d}\right)}\Gamma\left(m+1,-\frac{4\,(i\,dfx+i\,cf)}{d}\right)-4\,(dfx+cf)(dx+c)^{m}}{16\,(a^2dfm+a^2df)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*(-I*d*m - I*d)*e^(-(d*m*log(-2*I*f/d) + 2*I*c*f - 2*I*d*e)/d)*gamm
a(m + 1, -2*(I*d*f*x + I*c*f)/d) - (-I*d*m - I*d)*e^(-(d*m*log(-4*I*f/d) +
4*I*c*f - 4*I*d*e)/d)*gamma(m + 1, -4*(I*d*f*x + I*c*f)/d) - 4*(d*f*x + c*f
)*(d*x + c)^m)/(a^2*d*f*m + a^2*d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(c+dx)^m}{\cot^2(e+fx)-2i\cot(e+fx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+I*a*cot(f*x+e))**2,x)
```

```
[Out] -Integral((c + d*x)**m/(cot(e + f*x)**2 - 2*I*cot(e + f*x) - 1), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(I*a*cot(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^m}{\left(a+a\cot\left(e+f\,x\right)\,1{}\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*cot(e + f*x)*1i)^2,x)
```

```
[Out] int((c + d*x)^m/(a + a*cot(e + f*x)*1i)^2, x)
```

3.36 $\int \frac{(c+dx)^m}{(a+ia\cot(e+fx))^3}\,dx$

Optimal. Leaf size=251

$$\frac{(c+dx)^{1+m}}{8a^3d(1+m)}+\frac{3i2^{-4-m}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,-\frac{2if(c+dx)}{d}\right)}{a^3f}-\frac{3i2^{-5-2m}e^{4i\left(e-\frac{cf}{d}\right)}(c$$

```
[Out] 1/8*(d*x+c)^(1+m)/a^3/d/(1+m)+3*I*2^(-4-m)*exp(2*I*(e-c*f/d))*(d*x+c)^m*GAM
MA(1+m,-2*I*f*(d*x+c)/d)/a^3/f/((-I*f*(d*x+c)/d)^m)-3*I*2^(-5-2*m)*exp(4*I*
(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-4*I*f*(d*x+c)/d)/a^3/f/((-I*f*(d*x+c)/d)^m)
+I*2^(-4-m)*3^(-1-m)*exp(6*I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-6*I*f*(d*x+c)/
d)/a^3/f/((-I*f*(d*x+c)/d)^m)
```

Rubi [A]

time = 0.17, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3810, 2212}

$$\frac{3i2^{-m-4}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2if(c+dx)}{d}\right)}{a^3f}-\frac{3i2^{-2m-5}e^{4i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{4if(c+dx)}{d}\right)}{a^3f}+\frac{i2^{-m-4}3^{-m-1}e^{6i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{6if(c+dx)}{d}\right)}{a^3f}+\frac{(c+dx)^{m+1}}{8a^3d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] (c + d*x)^(1 + m)/(8*a^3*d*(1 + m)) + ((3*I)*2^(-4 - m)*E^((2*I)*(e - (c*f)
/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(a^3*f*(((-I)*f*(c +
 d*x))/d)^m) - ((3*I)*2^(-5 - 2*m)*E^((4*I)*(e - (c*f)/d))*(c + d*x)^m*Gamm
a[1 + m, ((-4*I)*f*(c + d*x))/d])/(a^3*f*(((-I)*f*(c + d*x))/d)^m) + (I*2^(
-4 - m)*3^(-1 - m)*E^((6*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-6*I)
*f*(c + d*x))/d])/(a^3*f*(((-I)*f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3810

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+dx)^m}{(a+ia\cot(e+fx))^3}\,dx = \int \left(\frac{(c+dx)^m}{8a^3} - \frac{3e^{2ie+2ifx}(c+dx)^m}{8a^3} + \frac{3e^{4ie+4ifx}(c+dx)^m}{8a^3} - \frac{e^{6ie+6ifx}(c+\ldots}{8a^3}\right.$$
$$= \frac{(c+dx)^{1+m}}{8a^3d(1+m)} - \frac{\int e^{6ie+6ifx}(c+dx)^m\,dx}{8a^3} - \frac{3\int e^{2ie+2ifx}(c+dx)^m\,dx}{8a^3} + \frac{3\int e^{4i\ldots}}{}$$
$$= \frac{(c+dx)^{1+m}}{8a^3d(1+m)} + \frac{3i2^{-4-m}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+\ldots}{d}\right.}{a^3f}$$

Mathematica [A]
time = 31.60, size = 238, normalized size = 0.95

$$\frac{2^{-5-2m}3^{-1-m}e^{-\frac{6icf}{d}}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\left(12^{1+m}e^{\frac{6icf}{d}}f(c+dx)\left(-\frac{if(c+dx)}{d}\right)^m+i2^{1+m}3^{2+m}de^{2ie+\frac{4icf}{d}}(1+m)\text{Gamma}\left(1+m,-\frac{2if(c+dx)}{d}\right)-i3^{2+m}de^{2i\left(2e+\frac{cf}{d}\right)}(1+m)\text{Gamma}\left(1+m,-\frac{4if(c+dx)}{d}\right)+i2^{1+m}de^{6ie}(1+m)\text{Gamma}\left(1+m,-\frac{6if(c+dx)}{d}\right)\right)}{a^3df(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] (2^(-5 - 2*m)*3^(-1 - m)*(c + d*x)^m*(12^(1 + m)*E^(((6*I)*c*f)/d)*f*(c + d
*x)*(((-I)*f*(c + d*x))/d)^m + I*2^(1 + m)*3^(2 + m)*d*E^((2*I)*e + ((4*I)*
c*f)/d)*(1 + m)*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d] - I*3^(2 + m)*d*E^((2*
I)*(2*e + (c*f)/d))*(1 + m)*Gamma[1 + m, ((-4*I)*f*(c + d*x))/d] + I*2^(1 +
 m)*d*E^((6*I)*e)*(1 + m)*Gamma[1 + m, ((-6*I)*f*(c + d*x))/d]))/(a^3*d*E^(
((6*I)*c*f)/d)*f*(1 + m)*(((-I)*f*(c + d*x))/d)^m)
```

Maple [F]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{(a+ia\cot(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+I*a*cot(f*x+e))^3,x)
```

```
[Out] int((d*x+c)^m/(a+I*a*cot(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((d*m + d)*integrate((d*x + c)^m*cos(6*f*x + 6*e), x) - 3*(d*m + d)*in
tegrate((d*x + c)^m*cos(4*f*x + 4*e), x) + 3*(d*m + d)*integrate((d*x + c)^
m*cos(2*f*x + 2*e), x) + (I*d*m + I*d)*integrate((d*x + c)^m*sin(6*f*x + 6*
e), x) - 3*(I*d*m + I*d)*integrate((d*x + c)^m*sin(4*f*x + 4*e), x) - 3*(-I
*d*m - I*d)*integrate((d*x + c)^m*sin(2*f*x + 2*e), x) - e^(m*log(d*x + c)
+ log(d*x + c)))/(a^3*d*m + a^3*d)
```

Fricas [**A**]
time = 0.46, size = 201, normalized size = 0.80

$$-\frac{18\,(-i\,dm-i\,d)e^{\left(-\frac{dm\log\left(-\frac{2i\,f}{d}\right)+2i\,cf-2i\,de}{d}\right)}\Gamma\left(m+1,-\frac{2\,(i\,dfx+i\,cf)}{d}\right)+9\,(i\,dm+i\,d)e^{\left(-\frac{dm\log\left(-\frac{4i\,f}{d}\right)+4i\,cf-4i\,de}{d}\right)}\Gamma\left(m+1,-\frac{4\,(i\,dfx+i\,cf)}{d}\right)+2\,(-i\,dm-i\,d)e^{\left(-\frac{dm\log\left(-\frac{6i\,f}{d}\right)+6i\,cf-6i\,de}{d}\right)}\Gamma\left(m+1,-\frac{6\,(i\,dfx+i\,cf)}{d}\right)-12\,(dfx+cf)(dx+c)^{m}}{96\,(a^{3}dfm+a^{3}df)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/96*(18*(-I*d*m - I*d)*e^(-(d*m*log(-2*I*f/d) + 2*I*c*f - 2*I*d*e)/d)*gam
ma(m + 1, -2*(I*d*f*x + I*c*f)/d) + 9*(I*d*m + I*d)*e^(-(d*m*log(-4*I*f/d)
+ 4*I*c*f - 4*I*d*e)/d)*gamma(m + 1, -4*(I*d*f*x + I*c*f)/d) + 2*(-I*d*m -
I*d)*e^(-(d*m*log(-6*I*f/d) + 6*I*c*f - 6*I*d*e)/d)*gamma(m + 1, -6*(I*d*f*
x + I*c*f)/d) - 12*(d*f*x + c*f)*(d*x + c)^m)/(a^3*d*f*m + a^3*d*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+I*a*cot(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(I*a*cot(f*x + e) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,x\right)}^m}{{\left(a+a\,\mathrm{cot}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*cot(e + f*x)*1i)^3,x)
[Out] int((c + d*x)^m/(a + a*cot(e + f*x)*1i)^3, x)
```

3.37 $\int (c+dx)^3(a+b\cot(e+fx))\,dx$

Optimal. Leaf size=147

$$\frac{a(c+dx)^4}{4d}-\frac{ib(c+dx)^4}{4d}+\frac{b(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{3ibd(c+dx)^2\text{PolyLog}\left(2,e^{2i(e+fx)}\right)}{2f^2}+\frac{3bd^2(c+dx)\text{Po}}{}$$

```
[Out] 1/4*a*(d*x+c)^4/d-1/4*I*b*(d*x+c)^4/d+b*(d*x+c)^3*ln(1-exp(2*I*(f*x+e)))/f-
3/2*I*b*d*(d*x+c)^2*polylog(2,exp(2*I*(f*x+e)))/f^2+3/2*b*d^2*(d*x+c)*polyl
og(3,exp(2*I*(f*x+e)))/f^3+3/4*I*b*d^3*polylog(4,exp(2*I*(f*x+e)))/f^4
```

Rubi [A]
time = 0.18, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3803, 3798, 2221, 2611, 6744, 2320, 6724}

$$\frac{a(c+dx)^4}{4d}+\frac{3bd^2(c+dx)\text{Li}_3\left(e^{2i(e+fx)}\right)}{2f^3}-\frac{3ibd(c+dx)^2\text{Li}_2\left(e^{2i(e+fx)}\right)}{2f^2}+\frac{b(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ib(c+dx)^4}{4d}+\frac{3ibd^3\text{Li}_4\left(e^{2i(e+fx)}\right)}{4f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Cot[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) - ((I/4)*b*(c + d*x)^4)/d + (b*(c + d*x)^3*Log[1 - E^
((2*I)*(e + f*x))])/f - (((3*I)/2)*b*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(e +
 f*x))])/f^2 + (3*b*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(e + f*x))])/(2*f^3)
+ (((3*I)/4)*b*d^3*PolyLog[4, E^((2*I)*(e + f*x))])/f^4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^3(a+b\cot(e+fx))\,dx &= \int \left(a(c+dx)^3+b(c+dx)^3\cot(e+fx)\right)\,dx \\
&= \frac{a(c+dx)^4}{4d}+b\int (c+dx)^3\cot(e+fx)\,dx \\
&= \frac{a(c+dx)^4}{4d}-\frac{ib(c+dx)^4}{4d}-(2ib)\int \frac{e^{2i(e+fx)}(c+dx)^3}{1-e^{2i(e+fx)}}\,dx \\
&= \frac{a(c+dx)^4}{4d}-\frac{ib(c+dx)^4}{4d}+\frac{b(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{(3bd)\int(c+ \\
&= \frac{a(c+dx)^4}{4d}-\frac{ib(c+dx)^4}{4d}+\frac{b(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{3ibd(c+dx \\
&= \frac{a(c+dx)^4}{4d}-\frac{ib(c+dx)^4}{4d}+\frac{b(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{3ibd(c+dx \\
&= \frac{a(c+dx)^4}{4d}-\frac{ib(c+dx)^4}{4d}+\frac{b(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{3ibd(c+dx \\
&= \frac{a(c+dx)^4}{4d}-\frac{ib(c+dx)^4}{4d}+\frac{b(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{3ibd(c+dx
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 524 vs. $2(147) = 294$.
time = 7.14, size = 524, normalized size = 3.56

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Cot[e + f*x]),x]
```

```
[Out] -1/4*(b*c*d^2*Csc[e]*(2*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*(-1 + E^((2*I)*e
))*Log[1 - E^((2*I)*(e + f*x))]) + 6*(-1 + E^((2*I)*e))*f*x*PolyLog[2, E^((
2*I)*(e + f*x))] + (3*I)*(-1 + E^((2*I)*e))*PolyLog[3, E^((2*I)*(e + f*x))]
))/(E^(I*e)*f^3) - (b*d^3*E^(I*e)*Csc[e]*(x^4 + (-1 + E^((-2*I)*e))*x^4 + (
(-1 + E^((2*I)*e))*(2*f^4*x^4 + (4*I)*f^3*x^3*Log[1 - E^((2*I)*(e + f*x))]
+ 6*f^2*x^2*PolyLog[2, E^((2*I)*(e + f*x))] + (6*I)*f*x*PolyLog[3, E^((2*I)
*(e + f*x))] - 3*PolyLog[4, E^((2*I)*(e + f*x))]))/(2*E^((2*I)*e)*f^4)))/4
+ (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Csc[e]*(b*Cos[e] + a*Sin[e
]))/4 + (b*c^3*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x
]]*Sin[e]))/(f*(Cos[e]^2 + Sin[e]^2)) - (3*b*c^2*d*Csc[e]*Sec[e]*(E^(I*ArcT
an[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2*
I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]])
)] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]] + I
```

```
*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[e])/Sqrt[1 + Tan[e]^2]))
/(2*f^2*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(126) = 252$.
time = 0.54, size = 876, normalized size = 5.96

method	result
risch	$d^2 ac\, x^3 + \frac{3da\, c^2 x^2}{2} + \frac{6bc\, d^2 \operatorname{polylog}(3, e^{i(fx+e)})}{f^3} - \frac{3ib\, d^3 e^4}{2f^4} + \frac{b\, d^3 \ln(e^{i(fx+e)}+1)x^3}{f} + \frac{b\, d^3 \ln(1-e^{i(fx+e)})x^3}{f} + \frac{b\, d^3 \ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*cot(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*a*c*x^3+3/2*d*a*c^2*x^2+6*I/f^2*b*d^2*c*e^2*x+6/f^3*b*c*d^2*polylog(3,e
xp(I*(f*x+e)))+6/f^3*b*c*d^2*polylog(3,-exp(I*(f*x+e)))+a*c^3*x+I*b*c^3*x+1
/4*I/d*b*c^4-6*I/f^2*b*c*d^2*polylog(2,-exp(I*(f*x+e)))*x-1/4*I*d^3*b*x^4-3
*I/f^2*b*d^3*polylog(2,exp(I*(f*x+e)))*x^2-3*I/f^2*b*d^3*polylog(2,-exp(I*(
f*x+e)))*x^2+1/4*d^3*a*x^4+1/4/d*a*c^4+6*I/f^4*b*d^3*polylog(4,-exp(I*(f*x+
e)))+6*I/f^4*b*d^3*polylog(4,exp(I*(f*x+e)))-3/2*I/f^4*b*d^3*e^4-3*I/f^2*b*
c^2*d*e^2-3*I/f^2*b*c^2*d*polylog(2,exp(I*(f*x+e)))-3*I/f^2*b*c^2*d*polylog
(2,-exp(I*(f*x+e)))-3/f^2*b*c^2*d*e*ln(exp(I*(f*x+e))-1)-6/f^3*b*c*d^2*e^2*
ln(exp(I*(f*x+e)))+3/f^3*b*c*d^2*e^2*ln(exp(I*(f*x+e))-1)+1/f*b*d^3*ln(exp(
I*(f*x+e))+1)*x^3+1/f*b*d^3*ln(1-exp(I*(f*x+e)))*x^3+1/f^4*b*d^3*ln(1-exp(I
*(f*x+e)))*e^3+4*I/f^3*b*d^2*c*e^3-2*I/f^3*b*d^3*e^3*x-I*d^2*b*c*x^3-3/2*I*
d*b*c^2*x^2+1/f*b*c^3*ln(exp(I*(f*x+e))+1)-2/f*b*c^3*ln(exp(I*(f*x+e)))+1/f
*b*c^3*ln(exp(I*(f*x+e))-1)-6*I/f*b*c^2*d*e*x+2/f^4*b*d^3*e^3*ln(exp(I*(f*x
+e)))-1/f^4*b*d^3*e^3*ln(exp(I*(f*x+e))-1)+6/f^3*b*d^3*polylog(3,-exp(I*(f*
x+e)))*x+6/f^3*b*d^3*polylog(3,exp(I*(f*x+e)))*x-6*I/f^2*b*c*d^2*polylog(2,
exp(I*(f*x+e)))*x+3/f*b*c*d^2*ln(exp(I*(f*x+e))+1)*x^2+3/f*b*c^2*d*ln(exp(I
*(f*x+e))+1)*x+3/f*b*c^2*d*ln(1-exp(I*(f*x+e)))*x+3/f^2*b*c^2*d*ln(1-exp(I*
(f*x+e)))*e+3/f*b*c*d^2*ln(1-exp(I*(f*x+e)))*x^2-3/f^3*b*c*d^2*ln(1-exp(I*(
f*x+e)))*e^2+6/f^2*b*c^2*d*e*ln(exp(I*(f*x+e)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1037 vs. $2(126) = 252$.
time = 0.39, size = 1037, normalized size = 7.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c^3 + (f*x + e)^4*a*d^3/f^3 + 4*(f*x + e)^3*a*c*d^2/f^2
+ 6*(f*x + e)^2*a*c^2*d/f - 4*(f*x + e)^3*a*d^3*e/f^3 - 12*(f*x + e)^2*a*c*
d^2*e/f^2 - 12*(f*x + e)*a*c^2*d*e/f + 4*b*c^3*log(sin(f*x + e)) - 12*b*c^2
```

```
*d*e*log(sin(f*x + e))/f + 6*(f*x + e)^2*a*d^3*e^2/f^3 + 12*(f*x + e)*a*c*d
^2*e^2/f^2 + 12*b*c*d^2*e^2*log(sin(f*x + e))/f^2 - 4*(f*x + e)*a*d^3*e^3/f
^3 - 4*b*d^3*e^3*log(sin(f*x + e))/f^3 + (-I*(f*x + e)^4*b*d^3 + 24*I*b*d^3
*polylog(4, -e^(I*f*x + I*e)) + 24*I*b*d^3*polylog(4, e^(I*f*x + I*e)) - 4*
(I*b*c*d^2*f - I*b*d^3*e)*(f*x + e)^3 - 6*(I*b*c^2*d*f^2 - 2*I*b*c*d^2*f*e
+ I*b*d^3*e^2)*(f*x + e)^2 - 4*(-I*(f*x + e)^3*b*d^3 + 3*(-I*b*c*d^2*f + I*
b*d^3*e)*(f*x + e)^2 + 3*(-I*b*c^2*d*f^2 + 2*I*b*c*d^2*f*e - I*b*d^3*e^2)*(
f*x + e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) - 4*(I*(f*x + e)^3*b*d^3
+ 3*(I*b*c*d^2*f - I*b*d^3*e)*(f*x + e)^2 + 3*(I*b*c^2*d*f^2 - 2*I*b*c*d^2*
f*e + I*b*d^3*e^2)*(f*x + e))*arctan2(sin(f*x + e), -cos(f*x + e) + 1) - 12
*(I*(f*x + e)^2*b*d^3 + I*b*c^2*d*f^2 - 2*I*b*c*d^2*f*e + I*b*d^3*e^2 + 2*(
I*b*c*d^2*f - I*b*d^3*e)*(f*x + e))*dilog(-e^(I*f*x + I*e)) - 12*(I*(f*x +
e)^2*b*d^3 + I*b*c^2*d*f^2 - 2*I*b*c*d^2*f*e + I*b*d^3*e^2 + 2*(I*b*c*d^2*f
 - I*b*d^3*e)*(f*x + e))*dilog(e^(I*f*x + I*e)) + 2*((f*x + e)^3*b*d^3 + 3*
(b*c*d^2*f - b*d^3*e)*(f*x + e)^2 + 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*e + b*d^3*
e^2)*(f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) +
 2*((f*x + e)^3*b*d^3 + 3*(b*c*d^2*f - b*d^3*e)*(f*x + e)^2 + 3*(b*c^2*d*f^
2 - 2*b*c*d^2*f*e + b*d^3*e^2)*(f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)
^2 - 2*cos(f*x + e) + 1) + 24*((f*x + e)*b*d^3 + b*c*d^2*f - b*d^3*e)*polyl
og(3, -e^(I*f*x + I*e)) + 24*((f*x + e)*b*d^3 + b*c*d^2*f - b*d^3*e)*polylo
g(3, e^(I*f*x + I*e)))/f^3)/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(126) = 252$.
time = 4.35, size = 644, normalized size = 4.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*d^3*f^4*x^4 + 8*a*c*d^2*f^4*x^3 + 12*a*c^2*d*f^4*x^2 + 8*a*c^3*f^4
*x + 3*I*b*d^3*polylog(4, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) - 3*I*b*d^
3*polylog(4, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) - 6*(I*b*d^3*f^2*x^2 +
2*I*b*c*d^2*f^2*x + I*b*c^2*d*f^2)*dilog(cos(2*f*x + 2*e) + I*sin(2*f*x + 2
*e)) - 6*(-I*b*d^3*f^2*x^2 - 2*I*b*c*d^2*f^2*x - I*b*c^2*d*f^2)*dilog(cos(2
*f*x + 2*e) - I*sin(2*f*x + 2*e)) + 4*(b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*
d^2*f*e^2 - b*d^3*e^3)*log(-1/2*cos(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) +
 1/2) + 4*(b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^2 - b*d^3*e^3)*log(-
1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2) + 4*(b*d^3*f^3*x^3 + 3
*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3*b*c^2*d*f^2*e - 3*b*c*d^2*f*e^2 + b*
d^3*e^3)*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1) + 4*(b*d^3*f^3*x^3
 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3*b*c^2*d*f^2*e - 3*b*c*d^2*f*e^2
+ b*d^3*e^3)*log(-cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1) + 6*(b*d^3*f*x
 + b*c*d^2*f)*polylog(3, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) + 6*(b*d^3*
f*x + b*c*d^2*f)*polylog(3, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)))/f^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \cot (e + fx)) (c + dx)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*cot(f*x+e)),x)
[Out] Integral((a + b*cot(e + f*x))*(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e)),x, algorithm="giac")
[Out] integrate((d*x + c)^3*(b*cot(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \cot(e + f x)) \, (c + d x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))*(c + d*x)^3,x)
[Out] int((a + b*cot(e + f*x))*(c + d*x)^3, x)
```

3.38 $\int (c+dx)^2(a+b\cot(e+fx))\,dx$

Optimal. Leaf size=112

$$\frac{a(c+dx)^3}{3d}-\frac{ib(c+dx)^3}{3d}+\frac{b(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ibd(c+dx)\text{PolyLog}\left(2,e^{2i(e+fx)}\right)}{f^2}+\frac{bd^2\text{PolyLog}\left(3,e^{2\cdots}\right.}{2f^3}$$

```
[Out] 1/3*a*(d*x+c)^3/d-1/3*I*b*(d*x+c)^3/d+b*(d*x+c)^2*ln(1-exp(2*I*(f*x+e)))/f-
I*b*d*(d*x+c)*polylog(2,exp(2*I*(f*x+e)))/f^2+1/2*b*d^2*polylog(3,exp(2*I*(
f*x+e)))/f^3
```

Rubi [A]
time = 0.14, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3803, 3798, 2221, 2611, 2320, 6724}

$$\frac{a(c+dx)^3}{3d}-\frac{ibd(c+dx)\text{Li}_2\left(e^{2i(e+fx)}\right)}{f^2}+\frac{b(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ib(c+dx)^3}{3d}+\frac{bd^2\text{Li}_3\left(e^{2i(e+fx)}\right)}{2f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Cot[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) - ((I/3)*b*(c + d*x)^3)/d + (b*(c + d*x)^2*Log[1 - E^
((2*I)*(e + f*x))])/f - (I*b*d*(c + d*x)*PolyLog[2, E^((2*I)*(e + f*x))])/f
^2 + (b*d^2*PolyLog[3, E^((2*I)*(e + f*x))])/(2*f^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^2(a+b\cot(e+fx))\,dx &= \int \left(a(c+dx)^2+b(c+dx)^2\cot(e+fx)\right)\,dx \\
&= \frac{a(c+dx)^3}{3d}+b\int (c+dx)^2\cot(e+fx)\,dx \\
&= \frac{a(c+dx)^3}{3d}-\frac{ib(c+dx)^3}{3d}-(2ib)\int \frac{e^{2i(e+fx)}(c+dx)^2}{1-e^{2i(e+fx)}}\,dx \\
&= \frac{a(c+dx)^3}{3d}-\frac{ib(c+dx)^3}{3d}+\frac{b(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{(2bd)\int(c
\\
&= \frac{a(c+dx)^3}{3d}-\frac{ib(c+dx)^3}{3d}+\frac{b(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ibd(c+dx
\\
&= \frac{a(c+dx)^3}{3d}-\frac{ib(c+dx)^3}{3d}+\frac{b(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ibd(c+dx
\\
&= \frac{a(c+dx)^3}{3d}-\frac{ib(c+dx)^3}{3d}+\frac{b(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ibd(c+dx
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 361 vs. 2(112) = 224.

time = 6.44, size = 361, normalized size = 3.22

$$-\frac{bd^2e^{-ie}\csc(e)\left(2f^2x^2\left(2e^{2ie}fx+3i(-1+e^{2ie})\log\left(1-e^{2i(e+fx)}\right)\right)+6(-1+e^{2ie})\,fx\text{PolyLog}\left(2,e^{2i(e+fx)}\right)+3i(-1+e^{2ie})\,\text{PolyLog}\left(3,e^{2i(e+fx)}\right)\right)}{12f^3}+\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right)\csc(e)(b\cos(e)+a\sin(e))+\frac{bc^2\csc(e)(-fx\cos(e)+\log(\cos(fx)\sin(e)+\cos(e)\sin(fx))\sin(e))}{f\left(\cos^2(e)+\sin^2(e)\right)}-\frac{bcd\csc(e)\sec(e)\left(e^{i\text{ArcTan}(\tan(e))}f^2x^2+\frac{\left(ifx(-\pi+2\text{ArcTan}(\tan(e)))-\pi\log\left(1+e^{-2ifx}\right)-2(fx+\text{ArcTan}(\tan(e)))\log\left(1-e^{2i(fx+\text{ArcTan}(\tan(e)))}\right)+\pi\log(\cos(fx))+2\text{ArcTan}(\tan(e))\log(\sin(fx+\text{ArcTan}(\tan(e))))+i\text{PolyLog}\left(2,e^{2i(fx+\text{ArcTan}(\tan(e)))}\right)\right)\tan(e)}{\sqrt{1+\tan^2(e)}}\right)}{f^2\sqrt{\sec^2(e)\left(\cos^2(e)+\sin^2(e)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Cot[e + f*x]),x]
```

```
[Out] -1/12*(b*d^2*Csc[e]*(2*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*(-1 + E^((2*I)*e)
)*Log[1 - E^((2*I)*(e + f*x))]) + 6*(-1 + E^((2*I)*e))*f*x*PolyLog[2, E^((2
*I)*(e + f*x))] + (3*I)*(-1 + E^((2*I)*e))*PolyLog[3, E^((2*I)*(e + f*x))])
)/(E^(I*e)*f^3) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Csc[e]*(b*Cos[e] + a*Sin[e
]))/3 + (b*c^2*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x
]]*Sin[e]))/(f*(Cos[e]^2 + Sin[e]^2)) - (b*c*d*Csc[e]*Sec[e]*(E^(I*ArcTan[T
an[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f
*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]]))] +
 Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]] + I*Pol
yLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[e])/Sqrt[1 + Tan[e]^2]))/(f^
2*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 534 vs. 2(98) = 196.
time = 0.50, size = 535, normalized size = 4.78

method	result
risch	$-\frac{2ib\,d^2\,\text{polylog}\left(2,e^{i(fx+e)}\right)x}{f^2}-\frac{2ibcd\,\text{polylog}\left(2,e^{i(fx+e)}\right)}{f^2}-\frac{2ibcd\,\text{polylog}\left(2,-e^{i(fx+e)}\right)}{f^2}-\frac{2bcde\ln\left(e^{i(fx+e)}-1\right)}{f^2}+\frac{2ib\,d^2e^2x}{f^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*cot(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -4*I/f*b*c*d*e*x+2/f*b*c*d*ln(exp(I*(f*x+e))+1)*x+2/f*b*c*d*ln(1-exp(I*(f*x
+e)))*x+2/f^2*b*c*d*ln(1-exp(I*(f*x+e)))*e+4/f^2*b*c*d*e*ln(exp(I*(f*x+e)))
-1/3*I*d^2*b*x^3+1/f*b*c^2*ln(exp(I*(f*x+e))-1)+1/f*b*c^2*ln(exp(I*(f*x+e))
+1)-2/f*b*c^2*ln(exp(I*(f*x+e)))+2/f^3*b*d^2*polylog(3,exp(I*(f*x+e)))+1/3*
d^2*a*x^3+1/3/d*a*c^3-2/f^2*b*c*d*e*ln(exp(I*(f*x+e))-1)-2*I/f^2*b*d^2*poly
log(2,-exp(I*(f*x+e)))*x+2*I/f^2*b*d^2*e^2*x-2*I/f^2*b*c*d*e^2-2*I/f^2*b*d^
2*polylog(2,exp(I*(f*x+e)))*x-2*I/f^2*b*c*d*polylog(2,exp(I*(f*x+e)))-2*I/f
^2*b*c*d*polylog(2,-exp(I*(f*x+e)))-I*d*b*c*x^2+4/3*I/f^3*b*d^2*e^3+1/f*b*d
^2*ln(1-exp(I*(f*x+e)))*x^2-1/f^3*b*d^2*ln(1-exp(I*(f*x+e)))*e^2+1/f*b*d^2*
ln(exp(I*(f*x+e))+1)*x^2+1/f^3*b*d^2*e^2*ln(exp(I*(f*x+e))-1)-2/f^3*b*d^2*e
^2*ln(exp(I*(f*x+e)))+d*a*c*x^2+a*c^2*x+I*b*c^2*x+1/3*I/d*b*c^3+2/f^3*b*d^2
*polylog(3,-exp(I*(f*x+e)))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 570 vs. 2(98) = 196.

time = 0.37, size = 570, normalized size = 5.09

$$\frac{6\,(fx+e)ac^2 + \frac{2\,(fx+e)^3ad^2}{f^2} + \frac{6\,(fx+e)^2acd}{f} - \frac{6\,(fx+e)^2ad^2e}{f^2} - \frac{12\,(fx+e)acde}{f} + 6\,bc^2\log\left(\sin\left(fx+e\right)\right) - \frac{12\,bcde\log(\sin(fx+e))}{f} + \frac{6\,(fx+e)ad^2e^2}{f^2} + \frac{6\,bd^2e^2\log(\sin(fx+e))}{f^2} + \frac{-2i\,(fx+e)^3bd^2 + 12\,bd^2\mathrm{Li}_3(-e^{(i\,fx+i\,e)}) + 12\,bd^2\mathrm{Li}_3(e^{(i\,fx+i\,e)}) - 6\,(i\,bcdf - i\,bd^2e)(fx+e)^2 - 6\left(-i\,(fx+e)^2bd^2 + 2\,(-i\,bcdf + i\,bd^2e)(fx+e)\right)\arctan(\sin(fx+e),\cos(fx+e)+1) - 6\left(i\,(fx+e)^2bd^2 + 2\,(i\,bcdf - i\,bd^2e)(fx+e)\right)\arctan(\sin(fx+e),-\cos(fx+e)+1) - 12\,(i\,(fx+e)bd^2 + i\,bcdf - i\,bd^2e)\mathrm{Li}_2(-e^{(i\,fx+i\,e)}) - 12\,(i\,(fx+e)bd^2 + i\,bcdf - i\,bd^2e)\mathrm{Li}_2(e^{(i\,fx+i\,e)}) + 3\left((fx+e)^2bd^2 + 2\,(bcdf - bd^2e)(fx+e)\right)\log\left(\cos(fx+e)^2 + \sin(fx+e)^2 + 2\,\cos(fx+e) + 1\right) + 3\left((fx+e)^2bd^2 + 2\,(bcdf - bd^2e)(fx+e)\right)\log\left(\cos(fx+e)^2 + \sin(fx+e)^2 - 2\,\cos(fx+e) + 1\right)}{f^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(f*x + e)*a*c^2 + 2*(f*x + e)^3*a*d^2/f^2 + 6*(f*x + e)^2*a*c*d/f -
6*(f*x + e)^2*a*d^2*e/f^2 - 12*(f*x + e)*a*c*d*e/f + 6*b*c^2*log(sin(f*x +
e)) - 12*b*c*d*e*log(sin(f*x + e))/f + 6*(f*x + e)*a*d^2*e^2/f^2 + 6*b*d^2*
e^2*log(sin(f*x + e))/f^2 + (-2*I*(f*x + e)^3*b*d^2 + 12*b*d^2*polylog(3, -
e^(I*f*x + I*e)) + 12*b*d^2*polylog(3, e^(I*f*x + I*e)) - 6*(I*b*c*d*f - I*
b*d^2*e)*(f*x + e)^2 - 6*(-I*(f*x + e)^2*b*d^2 + 2*(-I*b*c*d*f + I*b*d^2*e)
*(f*x + e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) - 6*(I*(f*x + e)^2*b*d^
2 + 2*(I*b*c*d*f - I*b*d^2*e)*(f*x + e))*arctan2(sin(f*x + e), -cos(f*x + e
) + 1) - 12*(I*(f*x + e)*b*d^2 + I*b*c*d*f - I*b*d^2*e)*dilog(-e^(I*f*x + I
*e)) - 12*(I*(f*x + e)*b*d^2 + I*b*c*d*f - I*b*d^2*e)*dilog(e^(I*f*x + I*e)
) + 3*((f*x + e)^2*b*d^2 + 2*(b*c*d*f - b*d^2*e)*(f*x + e))*log(cos(f*x + e
)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 3*((f*x + e)^2*b*d^2 + 2*(b*c*
d*f - b*d^2*e)*(f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x +
 e) + 1))/f^2)/f
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(98) = 196$.
time = 3.41, size = 421, normalized size = 3.76

$$\frac{4\,ad^2f^3x^3 + 12\,acdf^3x^2 + 12\,ac^2f^3x + 3\,bd^2\mathrm{polylog}(3, \cos(2\,fx+2\,e) + i\,\sin(2\,fx+2\,e)) + 3\,bd^2\mathrm{polylog}(3, \cos(2\,fx+2\,e) - i\,\sin(2\,fx+2\,e)) - 6\,(i\,bd^2fx + i\,bcdf)\mathrm{Li}_2(\cos(2\,fx+2\,e) + i\,\sin(2\,fx+2\,e)) - 6\,(-i\,bd^2fx - i\,bcdf)\mathrm{Li}_2(\cos(2\,fx+2\,e) - i\,\sin(2\,fx+2\,e)) + 6\,(bc^2f^2 - 2\,bcdfe + bd^2e^2)\log\left(-\frac{1}{2}\,\cos(2\,fx+2\,e) + \frac{1}{2}i\,\sin(2\,fx+2\,e) + \frac{1}{2}\right) + 6\,(bc^2f^2 - 2\,bcdfe + bd^2e^2)\log\left(-\frac{1}{2}\,\cos(2\,fx+2\,e) - \frac{1}{2}i\,\sin(2\,fx+2\,e) + \frac{1}{2}\right) + 6\,(bd^2f^2x^2 + 2\,bcdf^2x + 2\,bcdfe - bd^2e^2)\log(-\cos(2\,fx+2\,e) + i\,\sin(2\,fx+2\,e) + 1) + 6\,(bd^2f^2x^2 + 2\,bcdf^2x + 2\,bcdfe - bd^2e^2)\log(-\cos(2\,fx+2\,e) - i\,\sin(2\,fx+2\,e) + 1)}{12\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*d^2*f^3*x^3 + 12*a*c*d*f^3*x^2 + 12*a*c^2*f^3*x + 3*b*d^2*polylog
(3, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) + 3*b*d^2*polylog(3, cos(2*f*x +
 2*e) - I*sin(2*f*x + 2*e)) - 6*(I*b*d^2*f*x + I*b*c*d*f)*dilog(cos(2*f*x +
 2*e) + I*sin(2*f*x + 2*e)) - 6*(-I*b*d^2*f*x - I*b*c*d*f)*dilog(cos(2*f*x
+ 2*e) - I*sin(2*f*x + 2*e)) + 6*(b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)*log(
-1/2*cos(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2) + 6*(b*c^2*f^2 - 2*b*
c*d*f*e + b*d^2*e^2)*log(-1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1
/2) + 6*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + 2*b*c*d*f*e - b*d^2*e^2)*log(-cos(
2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1) + 6*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x +
 2*b*c*d*f*e - b*d^2*e^2)*log(-cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1))/
f^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot\left(e + fx\right)\right)\left(c + dx\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*cot(f*x+e)),x)
```

```
[Out] Integral((a + b*cot(e + f*x))*(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*(b*cot(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\cot(e+f\,x))\,\left(c+d\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))*(c + d*x)^2,x)
```

```
[Out] int((a + b*cot(e + f*x))*(c + d*x)^2, x)
```

3.39 $\int (c+dx)(a+b\cot(e+fx))\,dx$

Optimal. Leaf size=83

$$\frac{a(c+dx)^2}{2d}-\frac{ib(c+dx)^2}{2d}+\frac{b(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ibd\text{PolyLog}\left(2,e^{2i(e+fx)}\right)}{2f^2}$$

```
[Out] 1/2*a*(d*x+c)^2/d-1/2*I*b*(d*x+c)^2/d+b*(d*x+c)*ln(1-exp(2*I*(f*x+e)))/f-1/
2*I*b*d*polylog(2,exp(2*I*(f*x+e)))/f^2
```

Rubi [A]
time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3803, 3798, 2221, 2317, 2438}

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ib(c+dx)^2}{2d}-\frac{ibd\text{Li}_2\left(e^{2i(e+fx)}\right)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Cot[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) - ((I/2)*b*(c + d*x)^2)/d + (b*(c + d*x)*Log[1 - E^((
2*I)*(e + f*x))])/f - ((I/2)*b*d*PolyLog[2, E^((2*I)*(e + f*x))])/f^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
```

```
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)(a+b\cot(e+fx))\,dx &= \int (a(c+dx)+b(c+dx)\cot(e+fx))\,dx \\
&= \frac{a(c+dx)^2}{2d}+b\int (c+dx)\cot(e+fx)\,dx \\
&= \frac{a(c+dx)^2}{2d}-\frac{ib(c+dx)^2}{2d}-(2ib)\int \frac{e^{2i(e+fx)}(c+dx)}{1-e^{2i(e+fx)}}\,dx \\
&= \frac{a(c+dx)^2}{2d}-\frac{ib(c+dx)^2}{2d}+\frac{b(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{(bd)\int \log(1-
\end{aligned}$$

$$\begin{aligned}
&= \frac{a(c+dx)^2}{2d}-\frac{ib(c+dx)^2}{2d}+\frac{b(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f}+\frac{(ibd)\mathrm{Subst}\Big(\int}{} \\
&= \frac{a(c+dx)^2}{2d}-\frac{ib(c+dx)^2}{2d}+\frac{b(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ibd\mathrm{Li}_2\left(e^{2i(e+f}\right.}{2f^2}
\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 204 vs. $2(83) = 166$.
time = 5.34, size = 204, normalized size = 2.46

$$acx+\frac{1}{2}adx^2+\frac{1}{2}bdx^2\cot(e)+\frac{bc(\log(\cos(e+fx))+\log(\tan(e+fx)))}{f}-\frac{bd\csc(e)\sec(e)\left(e^{i\text{ArcTan}(\tan(e))}f^2x^2+\frac{\left(ifx(-\pi+2\text{ArcTan}(\tan(e)))-\pi\log\left(1+e^{-2ifx}\right)-2(fx+\text{ArcTan}(\tan(e)))\log\left(1-e^{2i(fx+\text{ArcTan}(\tan(e)))}\right)+\pi\log(\cos(fx))+2\text{ArcTan}(\tan(e))\log(\sin(fx+\text{ArcTan}(\tan(e))))+i\text{PolyLog}\left(2,e^{2i(fx+\text{ArcTan}(\tan(e)))}\right)\right)\tan(e)}{\sqrt{1+\tan^2(e)}}\right)}{2f^2\sqrt{\sec^2(e)\left(\cos^2(e)+\sin^2(e)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Cot[e + f*x]),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*d*x^2*Cot[e])/2 + (b*c*(Log[Cos[e + f*x]] + Log[Ta
n[e + f*x]]))/f - (b*d*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*
x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[T
an[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*Ar
cTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x +
ArcTan[Tan[e]]))])*Tan[e])/Sqrt[1 + Tan[e]^2]))/(2*f^2*Sqrt[Sec[e]^2*(Cos[e
]^2 + Sin[e]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(71) = 142.
time = 0.39, size = 240, normalized size = 2.89

method	result
risch	$-\frac{ibd\,x^{2}}{2}+\frac{ad\,x^{2}}{2}-\frac{ibd\,e^{2}}{f^{2}}+acx+\frac{bc\ln\left(\mathrm{e}^{i(fx+e)}-1\right)}{f}+\frac{bc\ln\left(\mathrm{e}^{i(fx+e)}+1\right)}{f}-\frac{2bc\ln\left(\mathrm{e}^{i(fx+e)}\right)}{f}-\frac{ibd\,\mathrm{polylog}\left(2,\mathrm{e}^{i(fx+e)}\right)}{f^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*cot(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] I*b*c*x+1/2*a*d*x^2-I/f^2*b*d*polylog(2,-exp(I*(f*x+e)))+a*c*x+1/f*b*c*ln(e
xp(I*(f*x+e))-1)+1/f*b*c*ln(exp(I*(f*x+e))+1)-2/f*b*c*ln(exp(I*(f*x+e)))-1/
2*I*b*d*x^2-I/f^2*b*d*polylog(2,exp(I*(f*x+e)))-I/f^2*b*d*e^2+1/f*b*d*ln(1-
exp(I*(f*x+e)))*x+1/f^2*b*d*ln(1-exp(I*(f*x+e)))*e-2*I/f*b*d*e*x+1/f*b*d*ln
(exp(I*(f*x+e))+1)*x-1/f^2*b*d*e*ln(exp(I*(f*x+e))-1)+2/f^2*b*d*e*ln(exp(I*
(f*x+e)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(70) = 140.
time = 0.36, size = 223, normalized size = 2.69

$$\frac{(a-ib)df^{2}x^{2}+2(a-ib)cf^{2}x-2ibdfx\arctan\left(\sin\left(fx+e\right),-\cos\left(fx+e\right)+1\right)+2ibcf\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)-1\right)-2ibd\mathrm{Li}_2\left(-e^{(ifx+ie)}\right)-2ibd\mathrm{Li}_2\left(e^{(ifx+ie)}\right)-2\left(-ibdfx-ibcf\right)\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)+1\right)+\left(bdfx+bcf\right)\log\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}+2\cos\left(fx+e\right)+1\right)+\left(bdfx+bcf\right)\log\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}-2\cos\left(fx+e\right)+1\right)}{2f^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*((a - I*b)*d*f^2*x^2 + 2*(a - I*b)*c*f^2*x - 2*I*b*d*f*x*arctan2(sin(f*
x + e), -cos(f*x + e) + 1) + 2*I*b*c*f*arctan2(sin(f*x + e), cos(f*x + e) -
 1) - 2*I*b*d*dilog(-e^(I*f*x + I*e)) - 2*I*b*d*dilog(e^(I*f*x + I*e)) - 2*
(-I*b*d*f*x - I*b*c*f)*arctan2(sin(f*x + e), cos(f*x + e) + 1) + (b*d*f*x +
 b*c*f)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + (b*d*f*
x + b*c*f)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1))/f^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(70) = 140.
time = 3.55, size = 240, normalized size = 2.89

$$\frac{2adf^{2}x^{2}+4acf^{2}x-ibd\mathrm{Li}_2(\cos\left(2fx+2e\right)+i\sin\left(2fx+2e\right))+ibd\mathrm{Li}_2(\cos\left(2fx+2e\right)-i\sin\left(2fx+2e\right))+2\left(bcf-bde\right)\log\left(-\frac{1}{2}\cos\left(2fx+2e\right)+\frac{1}{2}i\sin\left(2fx+2e\right)+\frac{1}{2}\right)+2\left(bcf-bde\right)\log\left(-\frac{1}{2}\cos\left(2fx+2e\right)-\frac{1}{2}i\sin\left(2fx+2e\right)+\frac{1}{2}\right)+2\left(bdfx+bde\right)\log\left(-\cos\left(2fx+2e\right)+i\sin\left(2fx+2e\right)+1\right)+2\left(bdfx+bde\right)\log\left(-\cos\left(2fx+2e\right)-i\sin\left(2fx+2e\right)+1\right)}{4f^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d*f^2*x^2 + 4*a*c*f^2*x - I*b*d*dilog(cos(2*f*x + 2*e) + I*sin(2*f
*x + 2*e)) + I*b*d*dilog(cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) + 2*(b*c*f
```

```
- b*d*e)*log(-1/2*cos(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2) + 2*(b*c
*f - b*d*e)*log(-1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2) + 2*(
b*d*f*x + b*d*e)*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1) + 2*(b*d*f
*x + b*d*e)*log(-cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1))/f^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \cot (e + fx)) (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e)),x)
```

```
[Out] Integral((a + b*cot(e + f*x))*(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*cot(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \cot(e + f x)) \, (c + d x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))*(c + d*x),x)
```

```
[Out] int((a + b*cot(e + f*x))*(c + d*x), x)
```

3.40 $\int \frac{a+b\cot(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{a+b\cot(e+fx)}{c+dx},x\right)$$

```
[Out] Unintegrable((a+b*cot(f*x+e))/(d*x+c),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\cot(e+fx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Cot[e + f*x])/(c + d*x),x]
[Out] Defer[Int][(a + b*Cot[e + f*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{a+b\cot(e+fx)}{c+dx}\,dx = \int \frac{a+b\cot(e+fx)}{c+dx}\,dx$$

Mathematica [A]
time = 2.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\cot(e+fx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Cot[e + f*x])/(c + d*x),x]
[Out] Integrate[(a + b*Cot[e + f*x])/(c + d*x), x]
```

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{a+b\cot(fx+e)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(f*x+e))/(d*x+c),x)
```

```
[Out] int((a+b*cot(f*x+e))/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] -(b*d*integrate(sin(f*x + e)/((d*x + c)*cos(f*x + e)^2 + (d*x + c)*sin(f*x
+ e)^2 + d*x + 2*(d*x + c)*cos(f*x + e) + c), x) - b*d*integrate(sin(f*x +
e)/((d*x + c)*cos(f*x + e)^2 + (d*x + c)*sin(f*x + e)^2 + d*x - 2*(d*x + c)
*cos(f*x + e) + c), x) - a*log(d*x + c))/d
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*cot(f*x + e) + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\cot{\left(e + fx \right)}}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c),x)
```

```
[Out] Integral((a + b*cot(e + f*x))/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cot(f*x + e) + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a+b\cot(e+f\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))/(c + d*x),x)
```

```
[Out] int((a + b*cot(e + f*x))/(c + d*x), x)
```

3.41 $\int \frac{a+b\cot(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{a+b\cot(e+fx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable((a+b*cot(f*x+e))/(d*x+c)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\cot(e+fx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Cot[e + f*x])/(c + d*x)^2,x]
[Out] Defer[Int][(a + b*Cot[e + f*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{a+b\cot(e+fx)}{(c+dx)^2}\,dx = \int \frac{a+b\cot(e+fx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 8.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\cot(e+fx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Cot[e + f*x])/(c + d*x)^2,x]
[Out] Integrate[(a + b*Cot[e + f*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{a+b\cot(fx+e)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(f*x+e))/(d*x+c)^2,x)
```

```
[Out] int((a+b*cot(f*x+e))/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((b*d^2*x + b*c*d)*integrate(sin(f*x + e)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 +
2*c*d*x + c^2)*cos(f*x + e)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(f*x + e)^2 +
c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(f*x + e)), x) - (b*d^2*x + b*c*d)*int
egrate(sin(f*x + e)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(f*x
+ e)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(f*x + e)^2 + c^2 - 2*(d^2*x^2 + 2*c*
d*x + c^2)*cos(f*x + e)), x) + a)/(d^2*x + c*d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*cot(f*x + e) + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\cot{\left(e + fx \right)}}{\left(c + dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*cot(e + f*x))/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cot(f*x + e) + a)/(d*x + c)^2, x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a+b\cot(e+f\,x)}{{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))/(c + d*x)^2,x)
[Out] int((a + b*cot(e + f*x))/(c + d*x)^2, x)
```

3.42 $\int (c+dx)^3(a+b\cot(e+fx))^2\,dx$

Optimal. Leaf size=295

$$-\frac{ib^2(c+dx)^3}{f}+\frac{a^2(c+dx)^4}{4d}-\frac{iab(c+dx)^4}{2d}-\frac{b^2(c+dx)^4}{4d}-\frac{b^2(c+dx)^3\cot(e+fx)}{f}+\frac{3b^2d(c+dx)^2\log\left(1-e\right.}{f^2}$$

```
[Out] -I*b^2*(d*x+c)^3/f+1/4*a^2*(d*x+c)^4/d-1/2*I*a*b*(d*x+c)^4/d-1/4*b^2*(d*x+c
)^4/d-b^2*(d*x+c)^3*cot(f*x+e)/f+3*b^2*d*(d*x+c)^2*ln(1-exp(2*I*(f*x+e)))/f
^2+2*a*b*(d*x+c)^3*ln(1-exp(2*I*(f*x+e)))/f-3*I*b^2*d^2*(d*x+c)*polylog(2,e
xp(2*I*(f*x+e)))/f^3-3*I*a*b*d*(d*x+c)^2*polylog(2,exp(2*I*(f*x+e)))/f^2+3/
2*b^2*d^3*polylog(3,exp(2*I*(f*x+e)))/f^4+3*a*b*d^2*(d*x+c)*polylog(3,exp(2
*I*(f*x+e)))/f^3+3/2*I*a*b*d^3*polylog(4,exp(2*I*(f*x+e)))/f^4
```

Rubi [A]
time = 0.38, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3803, 3798, 2221, 2611, 6744, 2320, 6724, 3801, 32}

$$\frac{a^2(c+dx)^4}{4d}+\frac{3abd^2(c+dx)\text{Li}_3\left(e^{2i(e+fx)}\right)}{f^3}-\frac{3iabd(c+dx)^2\text{Li}_2\left(e^{2i(e+fx)}\right)}{f^2}+\frac{2ab(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{iab(c+dx)^4}{2d}+\frac{3iabd^3\text{Li}_4\left(e^{2i(e+fx)}\right)}{2f^4}-\frac{3ib^2d^2(c+dx)\text{Li}_2\left(e^{2i(e+fx)}\right)}{f^3}+\frac{3b^2d(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f^2}-\frac{b^2(c+dx)^3\cot(e+fx)}{f}-\frac{ib^2(c+dx)^3}{f}-\frac{b^2(c+dx)^4}{4d}+\frac{3b^2d^3\text{Li}_3\left(e^{2i(e+fx)}\right)}{2f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Cot[e + f*x])^2,x]
```

```
[Out] ((-I)*b^2*(c + d*x)^3)/f + (a^2*(c + d*x)^4)/(4*d) - ((I/2)*a*b*(c + d*x)^4
)/d - (b^2*(c + d*x)^4)/(4*d) - (b^2*(c + d*x)^3*Cot[e + f*x])/f + (3*b^2*d
*(c + d*x)^2*Log[1 - E^((2*I)*(e + f*x))])/f^2 + (2*a*b*(c + d*x)^3*Log[1 -
 E^((2*I)*(e + f*x))])/f - ((3*I)*b^2*d^2*(c + d*x)*PolyLog[2, E^((2*I)*(e
+ f*x))])/f^3 - ((3*I)*a*b*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(e + f*x))])/f
^2 + (3*b^2*d^3*PolyLog[3, E^((2*I)*(e + f*x))])/(2*f^4) + (3*a*b*d^2*(c +
d*x)*PolyLog[3, E^((2*I)*(e + f*x))])/f^3 + (((3*I)/2)*a*b*d^3*PolyLog[4, E
^((2*I)*(e + f*x))])/f^4
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3(a+b\cot(e+fx))^2\,dx = \int \left(a^2(c+dx)^3+2ab(c+dx)^3\cot(e+fx)+b^2(c+dx)^3\cot^2(e+fx\right.$$

$$= \frac{a^2(c+dx)^4}{4d}+(2ab)\int (c+dx)^3\cot(e+fx)\,dx+b^2\int (c+dx)^3\cot^2($$

$$= \frac{a^2(c+dx)^4}{4d}-\frac{iab(c+dx)^4}{2d}-\frac{b^2(c+dx)^3\cot(e+fx)}{f}-(4iab)\int \frac{e^{2i}}{}$$

$$= -\frac{ib^2(c+dx)^3}{f}+\frac{a^2(c+dx)^4}{4d}-\frac{iab(c+dx)^4}{2d}-\frac{b^2(c+dx)^4}{4d}-\frac{b^2(c+}{}$$

$$= -\frac{ib^2(c+dx)^3}{f}+\frac{a^2(c+dx)^4}{4d}-\frac{iab(c+dx)^4}{2d}-\frac{b^2(c+dx)^4}{4d}-\frac{b^2(c+}{}$$

$$= -\frac{ib^2(c+dx)^3}{f}+\frac{a^2(c+dx)^4}{4d}-\frac{iab(c+dx)^4}{2d}-\frac{b^2(c+dx)^4}{4d}-\frac{b^2(c+}{}$$

$$= -\frac{ib^2(c+dx)^3}{f}+\frac{a^2(c+dx)^4}{4d}-\frac{iab(c+dx)^4}{2d}-\frac{b^2(c+dx)^4}{4d}-\frac{b^2(c+}{}$$

$$= -\frac{ib^2(c+dx)^3}{f}+\frac{a^2(c+dx)^4}{4d}-\frac{iab(c+dx)^4}{2d}-\frac{b^2(c+dx)^4}{4d}-\frac{b^2(c+}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1313 vs. 2(295) = 590.
time = 7.01, size = 1313, normalized size = 4.45

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*(a + b*Cot[e + f*x])^2,x]
```

```
[Out] -1/4*(b^2*d^3*Csc[e]*(2*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*(-1 + E^((2*I)*e
))*Log[1 - E^((2*I)*(e + f*x))]) + 6*(-1 + E^((2*I)*e))*f*x*PolyLog[2, E^((
2*I)*(e + f*x))] + (3*I)*(-1 + E^((2*I)*e))*PolyLog[3, E^((2*I)*(e + f*x))]
))/(E^(I*e)*f^4) - (a*b*c*d^2*Csc[e]*(2*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*
(-1 + E^((2*I)*e))*Log[1 - E^((2*I)*(e + f*x))]) + 6*(-1 + E^((2*I)*e))*f*x
*PolyLog[2, E^((2*I)*(e + f*x))] + (3*I)*(-1 + E^((2*I)*e))*PolyLog[3, E^((
2*I)*(e + f*x))]))/(2*E^(I*e)*f^3) - (a*b*d^3*E^(I*e)*Csc[e]*(x^4 + (-1 + E
```

```
^((-2*I)*e))*x^4 + ((-1 + E^((2*I)*e))*(2*f^4*x^4 + (4*I)*f^3*x^3*Log[1 - E
^((2*I)*(e + f*x))] + 6*f^2*x^2*PolyLog[2, E^((2*I)*(e + f*x))] + (6*I)*f*x
*PolyLog[3, E^((2*I)*(e + f*x))] - 3*PolyLog[4, E^((2*I)*(e + f*x))]))/(2*E
^((2*I)*e)*f^4)))/2 + (3*b^2*c^2*d*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin
[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(f^2*(Cos[e]^2 + Sin[e]^2)) + (2*a*b*c^3*Cs
c[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(f*(C
os[e]^2 + Sin[e]^2)) + (Csc[e]*Csc[e + f*x]*(4*a^2*c^3*f*x*Cos[f*x] - 4*b^2
*c^3*f*x*Cos[f*x] + 6*a^2*c^2*d*f*x^2*Cos[f*x] - 6*b^2*c^2*d*f*x^2*Cos[f*x]
 + 4*a^2*c*d^2*f*x^3*Cos[f*x] - 4*b^2*c*d^2*f*x^3*Cos[f*x] + a^2*d^3*f*x^4*
Cos[f*x] - b^2*d^3*f*x^4*Cos[f*x] - 4*a^2*c^3*f*x*Cos[2*e + f*x] + 4*b^2*c^
3*f*x*Cos[2*e + f*x] - 6*a^2*c^2*d*f*x^2*Cos[2*e + f*x] + 6*b^2*c^2*d*f*x^2
*Cos[2*e + f*x] - 4*a^2*c*d^2*f*x^3*Cos[2*e + f*x] + 4*b^2*c*d^2*f*x^3*Cos[
2*e + f*x] - a^2*d^3*f*x^4*Cos[2*e + f*x] + b^2*d^3*f*x^4*Cos[2*e + f*x] +
8*b^2*c^3*Sin[f*x] + 24*b^2*c^2*d*x*Sin[f*x] + 8*a*b*c^3*f*x*Sin[f*x] + 24*
b^2*c*d^2*x^2*Sin[f*x] + 12*a*b*c^2*d*f*x^2*Sin[f*x] + 8*b^2*d^3*x^3*Sin[f*
x] + 8*a*b*c*d^2*f*x^3*Sin[f*x] + 2*a*b*d^3*f*x^4*Sin[f*x] + 8*a*b*c^3*f*x*
Sin[2*e + f*x] + 12*a*b*c^2*d*f*x^2*Sin[2*e + f*x] + 8*a*b*c*d^2*f*x^3*Sin[
2*e + f*x] + 2*a*b*d^3*f*x^4*Sin[2*e + f*x]))/(8*f) - (3*b^2*c*d^2*Csc[e]*S
ec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi
*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x +
 ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcT
an[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[e])/Sqrt
[1 + Tan[e]^2]))/(f^3*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)]) - (3*a*b*c^2*d*
Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e
]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I
)*(f*x + ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*
x + ArcTan[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[
e])/Sqrt[1 + Tan[e]^2]))/(f^2*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(266) = 532$.
time = 0.86, size = 1604, normalized size = 5.44

method	result	size
risch	Expression too large to display	1604

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*cot(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^3*b^2*x^4-b^2*c^3*x-1/4/d*b^2*c^4-2*I*b^2*(d^3*x^3+3*c*d^2*x^2+3*c^2
*d*x+c^3)/f/(exp(2*I*(f*x+e))-1)-6*I/f^3*b^2*c*d^2*e^2-6*I/f^3*b^2*d^3*poly
log(2,-exp(I*(f*x+e)))*x-6*I/f^3*b^2*d^3*polylog(2,exp(I*(f*x+e)))*x-3*I/f^
4*b*a*d^3*e^4-6*I/f^3*b^2*c*d^2*polylog(2,-exp(I*(f*x+e)))-6*I/f^3*b^2*c*d^
2*polylog(2,exp(I*(f*x+e)))-12*I/f*b*a*c^2*d*e*x+12*I/f^2*b*d^2*a*c*e^2*x-1
2*I/f^2*b*a*c*d^2*polylog(2,-exp(I*(f*x+e)))*x-12*I/f^2*b*a*c*d^2*polylog(2
```

```
,exp(I*(f*x+e)))*x-12/f^3*b*a*c*d^2*e^2*ln(exp(I*(f*x+e)))+6/f^3*b*a*c*d^2*
e^2*ln(exp(I*(f*x+e))-1)-6*I/f^2*b*a*d^3*polylog(2,exp(I*(f*x+e)))*x^2-6*I/
f^2*b*a*c^2*d*polylog(2,exp(I*(f*x+e)))-6*I/f^2*b*a*c^2*d*polylog(2,-exp(I*
(f*x+e)))-12*I/f^2*b^2*c*d^2*e*x+8*I/f^3*b*d^2*a*c*e^3-4*I/f^3*b*a*d^3*e^3*
x-6*I/f^2*b*a*d^3*polylog(2,-exp(I*(f*x+e)))*x^2-6*I/f^2*b*a*c^2*d*e^2-2*I*
d^2*a*b*c*x^3-3*I*d*a*b*c^2*x^2+2*I*a*b*c^3*x+1/2*I/d*a*b*c^4+1/4*d^3*a^2*x
^4+1/4/d*a^2*c^4-1/2*I*d^3*a*b*x^4+3/f^4*b^2*d^3*e^2*ln(exp(I*(f*x+e))-1)-6
/f^2*b^2*c^2*d*ln(exp(I*(f*x+e)))+3/f^2*b^2*c^2*d*ln(exp(I*(f*x+e))-1)+2/f*
b*a*c^3*ln(exp(I*(f*x+e))-1)+2/f*b*a*c^3*ln(exp(I*(f*x+e))+1)-2*I/f*b^2*d^3
*x^3+4*I/f^4*b^2*d^3*e^3-3/f^4*b^2*d^3*e^2*ln(1-exp(I*(f*x+e)))+3/f^2*b^2*d
^3*ln(exp(I*(f*x+e))+1)*x^2+3/f^2*b^2*d^3*ln(1-exp(I*(f*x+e)))*x^2+3/f^2*b^
2*c^2*d*ln(exp(I*(f*x+e))+1)-4/f*b*a*c^3*ln(exp(I*(f*x+e)))-6/f^4*b^2*d^3*e
^2*ln(exp(I*(f*x+e)))+12/f^2*b*a*c^2*d*e*ln(exp(I*(f*x+e)))-6/f^2*b*a*c^2*d
*e*ln(exp(I*(f*x+e))-1)+2/f*b*a*d^3*ln(exp(I*(f*x+e))+1)*x^3+2/f*b*a*d^3*ln
(1-exp(I*(f*x+e)))*x^3+2/f^4*b*a*d^3*ln(1-exp(I*(f*x+e)))*e^3+6/f*b*ln(exp(
I*(f*x+e))+1)*a*c^2*d*x+6/f*b*ln(1-exp(I*(f*x+e)))*a*c^2*d*x+6/f^2*b*ln(1-e
xp(I*(f*x+e)))*a*c^2*d*e-6/f^3*b*a*c*d^2*e^2*ln(1-exp(I*(f*x+e)))+6/f*b*a*c
*d^2*ln(exp(I*(f*x+e))+1)*x^2+6/f*b*a*c*d^2*ln(1-exp(I*(f*x+e)))*x^2+6/f^2*
b^2*c*d^2*ln(1-exp(I*(f*x+e)))*x+6/f^3*b^2*c*d^2*ln(1-exp(I*(f*x+e)))*e+12/
f^3*b*a*d^3*polylog(3,-exp(I*(f*x+e)))*x+3/2*d*a^2*c^2*x^2+a^2*c^3*x-d^2*b^
2*c*x^3-3/2*d*b^2*c^2*x^2+d^2*a^2*c*x^3+12*I/f^4*b*a*d^3*polylog(4,-exp(I*(
f*x+e)))+12*I/f^4*b*a*d^3*polylog(4,exp(I*(f*x+e)))+6*I/f^3*b^2*d^3*e^2*x-6
*I/f*b^2*c*d^2*x^2-6/f^3*b^2*c*d^2*e*ln(exp(I*(f*x+e))-1)+12/f^3*b^2*c*d^2*
e*ln(exp(I*(f*x+e)))+12/f^3*b*a*d^3*polylog(3,exp(I*(f*x+e)))*x+6/f^4*b^2*d
^3*polylog(3,exp(I*(f*x+e)))+6/f^4*b^2*d^3*polylog(3,-exp(I*(f*x+e)))+6/f^2
*b^2*c*d^2*ln(exp(I*(f*x+e))+1)*x+12/f^3*b*a*c*d^2*polylog(3,-exp(I*(f*x+e)
))+12/f^3*b*a*c*d^2*polylog(3,exp(I*(f*x+e)))-2/f^4*b*a*d^3*e^3*ln(exp(I*(f
*x+e))-1)+4/f^4*b*a*d^3*e^3*ln(exp(I*(f*x+e)))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4022 vs. $2(267) = 534$.
time = 1.50, size = 4022, normalized size = 13.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a^2*c^3 + (f*x + e)^4*a^2*d^3/f^3 + 4*(f*x + e)^3*a^2*c*d^
2/f^2 + 6*(f*x + e)^2*a^2*c^2*d/f - 4*(f*x + e)^3*a^2*d^3*e/f^3 - 12*(f*x +
 e)^2*a^2*c*d^2*e/f^2 - 12*(f*x + e)*a^2*c^2*d*e/f + 8*a*b*c^3*log(sin(f*x
+ e)) - 24*a*b*c^2*d*e*log(sin(f*x + e))/f + 6*(f*x + e)^2*a^2*d^3*e^2/f^3
+ 12*(f*x + e)*a^2*c*d^2*e^2/f^2 + 24*a*b*c*d^2*e^2*log(sin(f*x + e))/f^2 -
 4*(f*x + e)*a^2*d^3*e^3/f^3 - 8*a*b*d^3*e^3*log(sin(f*x + e))/f^3 + 4*((2*
a*b - I*b^2)*(f*x + e)^4*d^3 - 8*b^2*c^3*f^3 + 24*b^2*c^2*d*f^2*e - 24*b^2*
```

```
c*d^2*f*e^2 + 8*b^2*d^3*e^3 + 4*((2*a*b - I*b^2)*c*d^2*f - (2*a*b*e - I*b^2
*e)*d^3)*(f*x + e)^3 + 6*((2*a*b - I*b^2)*c^2*d*f^2 - 2*(2*a*b*e - I*b^2*e)
*c*d^2*f + (2*a*b*e^2 - I*b^2*e^2)*d^3)*(f*x + e)^2 - 4*(I*b^2*c^3*f^3 - 3*
I*b^2*c^2*d*f^2*e + 3*I*b^2*c*d^2*f*e^2 - I*b^2*d^3*e^3)*(f*x + e) - 4*(2*(
f*x + e)^3*a*b*d^3 + 3*b^2*c^2*d*f^2 - 6*b^2*c*d^2*f*e + 3*b^2*d^3*e^2 + 3*
(2*a*b*c*d^2*f - (2*a*b*e - b^2)*d^3)*(f*x + e)^2 + 6*(a*b*c^2*d*f^2 - (2*a
*b*e - b^2)*c*d^2*f + (a*b*e^2 - b^2*e)*d^3)*(f*x + e) - (2*(f*x + e)^3*a*b
*d^3 + 3*b^2*c^2*d*f^2 - 6*b^2*c*d^2*f*e + 3*b^2*d^3*e^2 + 3*(2*a*b*c*d^2*f
 - (2*a*b*e - b^2)*d^3)*(f*x + e)^2 + 6*(a*b*c^2*d*f^2 - (2*a*b*e - b^2)*c*
d^2*f + (a*b*e^2 - b^2*e)*d^3)*(f*x + e))*cos(2*f*x + 2*e) + (-2*I*(f*x + e
)^3*a*b*d^3 - 3*I*b^2*c^2*d*f^2 + 6*I*b^2*c*d^2*f*e - 3*I*b^2*d^3*e^2 + 3*(
-2*I*a*b*c*d^2*f + (2*I*a*b*e - I*b^2)*d^3)*(f*x + e)^2 + 6*(-I*a*b*c^2*d*f
^2 + (2*I*a*b*e - I*b^2)*c*d^2*f + (-I*a*b*e^2 + I*b^2*e)*d^3)*(f*x + e))*s
in(2*f*x + 2*e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) - 12*(b^2*c^2*d*f^
2 - 2*b^2*c*d^2*f*e + b^2*d^3*e^2 - (b^2*c^2*d*f^2 - 2*b^2*c*d^2*f*e + b^2*
d^3*e^2)*cos(2*f*x + 2*e) + (-I*b^2*c^2*d*f^2 + 2*I*b^2*c*d^2*f*e - I*b^2*d
^3*e^2)*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 4*(2*(f
*x + e)^3*a*b*d^3 + 3*(2*a*b*c*d^2*f - (2*a*b*e - b^2)*d^3)*(f*x + e)^2 + 6
*(a*b*c^2*d*f^2 - (2*a*b*e - b^2)*c*d^2*f + (a*b*e^2 - b^2*e)*d^3)*(f*x + e
) - (2*(f*x + e)^3*a*b*d^3 + 3*(2*a*b*c*d^2*f - (2*a*b*e - b^2)*d^3)*(f*x +
 e)^2 + 6*(a*b*c^2*d*f^2 - (2*a*b*e - b^2)*c*d^2*f + (a*b*e^2 - b^2*e)*d^3)
*(f*x + e))*cos(2*f*x + 2*e) - (2*I*(f*x + e)^3*a*b*d^3 + 3*(2*I*a*b*c*d^2*
f + (-2*I*a*b*e + I*b^2)*d^3)*(f*x + e)^2 + 6*(I*a*b*c^2*d*f^2 + (-2*I*a*b*
e + I*b^2)*c*d^2*f + (I*a*b*e^2 - I*b^2*e)*d^3)*(f*x + e))*sin(2*f*x + 2*e)
)*arctan2(sin(f*x + e), -cos(f*x + e) + 1) - ((2*a*b - I*b^2)*(f*x + e)^4*d
^3 + 4*((2*a*b - I*b^2)*c*d^2*f + (b^2*(I*e + 2) - 2*a*b*e)*d^3)*(f*x + e)^
3 + 6*((2*a*b - I*b^2)*c^2*d*f^2 + 2*(b^2*(I*e + 2) - 2*a*b*e)*c*d^2*f + (b
^2*(-I*e^2 - 4*e) + 2*a*b*e^2)*d^3)*(f*x + e)^2 + 4*(-I*b^2*c^3*f^3 + 3*b^2
*c^2*d*f^2*(I*e + 2) + 3*b^2*c*d^2*f*(-I*e^2 - 4*e) + b^2*d^3*(I*e^3 + 6*e^
2))*(f*x + e))*cos(2*f*x + 2*e) + 24*((f*x + e)^2*a*b*d^3 + a*b*c^2*d*f^2 -
 (2*a*b*e - b^2)*c*d^2*f + (a*b*e^2 - b^2*e)*d^3 + (2*a*b*c*d^2*f - (2*a*b*
e - b^2)*d^3)*(f*x + e) - ((f*x + e)^2*a*b*d^3 + a*b*c^2*d*f^2 - (2*a*b*e -
 b^2)*c*d^2*f + (a*b*e^2 - b^2*e)*d^3 + (2*a*b*c*d^2*f - (2*a*b*e - b^2)*d^
3)*(f*x + e))*cos(2*f*x + 2*e) - (I*(f*x + e)^2*a*b*d^3 + I*a*b*c^2*d*f^2 +
 (-2*I*a*b*e + I*b^2)*c*d^2*f + (I*a*b*e^2 - I*b^2*e)*d^3 + (2*I*a*b*c*d^2*
f + (-2*I*a*b*e + I*b^2)*d^3)*(f*x + e))*sin(2*f*x + 2*e))*dilog(-e^(I*f*x
+ I*e)) + 24*((f*x + e)^2*a*b*d^3 + a*b*c^2*d*f^2 - (2*a*b*e - b^2)*c*d^2*f
 + (a*b*e^2 - b^2*e)*d^3 + (2*a*b*c*d^2*f - (2*a*b*e - b^2)*d^3)*(f*x + e)
- ((f*x + e)^2*a*b*d^3 + a*b*c^2*d*f^2 - (2*a*b*e - b^2)*c*d^2*f + (a*b*e^2
 - b^2*e)*d^3 + (2*a*b*c*d^2*f - (2*a*b*e - b^2)*d^3)*(f*x + e))*cos(2*f*x
+ 2*e) - (I*(f*x + e)^2*a*b*d^3 + I*a*b*c^2*d*f^2 + (-2*I*a*b*e + I*b^2)*c*
d^2*f + (I*a*b*e^2 - I*b^2*e)*d^3 + (2*I*a*b*c*d^2*f + (-2*I*a*b*e + I*b^2)
*d^3)*(f*x + e))*sin(2*f*x + 2*e))*dilog(e^(I*f*x + I*e)) - 2*(-2*I*(f*x +
e)^3*a*b*d^3 - 3*I*b^2*c^2*d*f^2 + 6*I*b^2*c*d^2*f*e - 3*I*b^2*d^3*e^2 + 3*
(-2*I*a*b*c*d^2*f + (2*I*a*b*e - I*b^2)*d^3)*(f*x + e)^2 + 6*(-I*a*b*c^2*d*
```

```
f^2 + (2*I*a*b*e - I*b^2)*c*d^2*f + (-I*a*b*e^2 + I*b^2*e)*d^3)*(f*x + e) +
 (2*I*(f*x + e)^3*a*b*d^3 + 3*I*b^2*c^2*d*f^2 - 6*I*b^2*c*d^2*f*e + 3*I*b^2
*d^3*e^2 + 3*(2*I*a*b*c*d^2*f + (-2*I*a*b*e + I*b^2)*d^3)*(f*x + e)^2 + 6*(
I*a*b*c^2*d*f^2 + (-2*I*a*b*e + I*b^2)*c*d^2*f + (I*a*b*e^2 - I*b^2*e)*d^3)
*(f*x + e))*cos(2*f*x + 2*e) - (2*(f*x + e)^3*a*b*d^3 + 3*b^2*c^2*d*f^2 - 6
*b^2*c*d^2*f*e + 3*b^2*d^3*e^2 + 3*(2*a*b*c*d^2*f - (2*a*b*e - b^2)*d^3)*(f
*x + e)^2 + 6*(a*b*c^2*d*f^2 - (2*a*b*e - b^2)*c*d^2*f + (a*b*e^2 - b^2*e)*
d^3)*(f*x + e))*sin(2*f*x + 2*e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*c
os(f*x + e) + 1) - 2*(-2*I*(f*x + e)^3*a*b*d^3 - 3*I*b^2*c^2*d*f^2 + 6*I*b^
2*c*d^2*f*e - 3*I*b^2*d^3*e^2 + 3*(-2*I*a*b*c*d^2*f + (2*I*a*b*e - I*b^2)*d
^3)*(f*x + e)^2 + 6*(-I*a*b*c^2*d*f^2 + (2*I*a*b*e - I*b^2)*c*d^2*f + (-I*a
*b*e^2 + I*b^2*e)*d^3)*(f*x + e) + (2*I*(f*x + e)^3*a*b*d^3 + 3*I*b^2*c^2*d
*f^2 - 6*I*b^2*c*d^2*f*e + 3*I*b^2*d^3*e^2 + 3*(2*I*a*b*c*d^2*f + (-2*I*a*b
*e + I*b^2)*d^3)*(f*x + e)^2 + 6*(I*a*b*c^2*d*f^2 + (-2*I*a*b*e + I*b^2)*c*
d^2*f + (I*a*b*e^2 - I*b^2*e)*d^3)*(f*x + e))*c...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(267) = 534$.
time = 3.29, size = 1175, normalized size = 3.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x + 4*b^2
*c^3*f^3 - 3*I*a*b*d^3*polylog(4, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e))*si
n(2*f*x + 2*e) + 3*I*a*b*d^3*polylog(4, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*
e))*sin(2*f*x + 2*e) + 6*(I*a*b*d^3*f^2*x^2 + I*a*b*c^2*d*f^2 + I*b^2*c*d^2
*f + I*(2*a*b*c*d^2*f^2 + b^2*d^3*f)*x)*dilog(cos(2*f*x + 2*e) + I*sin(2*f*
x + 2*e))*sin(2*f*x + 2*e) + 6*(-I*a*b*d^3*f^2*x^2 - I*a*b*c^2*d*f^2 - I*b^
2*c*d^2*f - I*(2*a*b*c*d^2*f^2 + b^2*d^3*f)*x)*dilog(cos(2*f*x + 2*e) - I*s
in(2*f*x + 2*e))*sin(2*f*x + 2*e) - 2*(2*a*b*c^3*f^3 + 3*b^2*c^2*d*f^2 - 2*
a*b*d^3*e^3 + 3*(2*a*b*c*d^2*f + b^2*d^3)*e^2 - 6*(a*b*c^2*d*f^2 + b^2*c*d^
2*f)*e)*log(-1/2*cos(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2)*sin(2*f*x
 + 2*e) - 2*(2*a*b*c^3*f^3 + 3*b^2*c^2*d*f^2 - 2*a*b*d^3*e^3 + 3*(2*a*b*c*d
^2*f + b^2*d^3)*e^2 - 6*(a*b*c^2*d*f^2 + b^2*c*d^2*f)*e)*log(-1/2*cos(2*f*x
 + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2)*sin(2*f*x + 2*e) - 2*(2*a*b*d^3*f^3
*x^3 + 2*a*b*d^3*e^3 + 3*(2*a*b*c*d^2*f^3 + b^2*d^3*f^2)*x^2 + 6*(a*b*c^2*d
*f^3 + b^2*c*d^2*f^2)*x - 3*(2*a*b*c*d^2*f + b^2*d^3)*e^2 + 6*(a*b*c^2*d*f^
2 + b^2*c*d^2*f)*e)*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1)*sin(2*f
*x + 2*e) - 2*(2*a*b*d^3*f^3*x^3 + 2*a*b*d^3*e^3 + 3*(2*a*b*c*d^2*f^3 + b^2
*d^3*f^2)*x^2 + 6*(a*b*c^2*d*f^3 + b^2*c*d^2*f^2)*x - 3*(2*a*b*c*d^2*f + b^
2*d^3)*e^2 + 6*(a*b*c^2*d*f^2 + b^2*c*d^2*f)*e)*log(-cos(2*f*x + 2*e) - I*s
in(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e) - 3*(2*a*b*d^3*f*x + 2*a*b*c*d^2*f +
```

```
b^2*d^3)*polylog(3, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e))*sin(2*f*x + 2*e)
 - 3*(2*a*b*d^3*f*x + 2*a*b*c*d^2*f + b^2*d^3)*polylog(3, cos(2*f*x + 2*e)
- I*sin(2*f*x + 2*e))*sin(2*f*x + 2*e) + 4*(b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*f
^3*x^2 + 3*b^2*c^2*d*f^3*x + b^2*c^3*f^3)*cos(2*f*x + 2*e) - ((a^2 - b^2)*d
^3*f^4*x^4 + 4*(a^2 - b^2)*c*d^2*f^4*x^3 + 6*(a^2 - b^2)*c^2*d*f^4*x^2 + 4*
(a^2 - b^2)*c^3*f^4*x)*sin(2*f*x + 2*e))/(f^4*sin(2*f*x + 2*e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cot{(e + fx)})^2 (c + dx)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*cot(f*x+e))**2,x)
[Out] Integral((a + b*cot(e + f*x))**2*(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*x + c)^3*(b*cot(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\cot(e + f\,x))^2\,(c + d\,x)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^2*(c + d*x)^3,x)
[Out] int((a + b*cot(e + f*x))^2*(c + d*x)^3, x)
```

3.43 $\int (c+dx)^2(a+b\cot(e+fx))^2\,dx$

Optimal. Leaf size=227

$$-\frac{ib^2(c+dx)^2}{f}+\frac{a^2(c+dx)^3}{3d}-\frac{2iab(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{3d}-\frac{b^2(c+dx)^2\cot(e+fx)}{f}+\frac{2b^2d(c+dx)\log\left(1-\right.}{f^2}$$

```
[Out] -I*b^2*(d*x+c)^2/f+1/3*a^2*(d*x+c)^3/d-2/3*I*a*b*(d*x+c)^3/d-1/3*b^2*(d*x+c
)^3/d-b^2*(d*x+c)^2*cot(f*x+e)/f+2*b^2*d*(d*x+c)*ln(1-exp(2*I*(f*x+e)))/f^2
+2*a*b*(d*x+c)^2*ln(1-exp(2*I*(f*x+e)))/f-I*b^2*d^2*polylog(2,exp(2*I*(f*x+
e)))/f^3-2*I*a*b*d*(d*x+c)*polylog(2,exp(2*I*(f*x+e)))/f^2+a*b*d^2*polylog(
3,exp(2*I*(f*x+e)))/f^3
```

Rubi [A]

time = 0.26, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3803, 3798, 2221, 2611, 2320, 6724, 3801, 2317, 2438, 32}

$$\frac{a^2(c+dx)^3}{3d}-\frac{2iabd(c+dx)\text{Li}_2\left(e^{2i(e+fx)}\right)}{f^2}+\frac{2ab(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{2iab(c+dx)^3}{3d}+\frac{abd^2\text{Li}_3\left(e^{2i(e+fx)}\right)}{f^3}+\frac{2b^2d(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f^2}-\frac{b^2(c+dx)^2\cot(e+fx)}{f}-\frac{ib^2(c+dx)^2}{f}-\frac{b^2(c+dx)^3}{3d}-\frac{ib^2d^2\text{Li}_2\left(e^{2i(e+fx)}\right)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Cot[e + f*x])^2,x]
```

```
[Out] ((-I)*b^2*(c + d*x)^2)/f + (a^2*(c + d*x)^3)/(3*d) - (((2*I)/3)*a*b*(c + d*
x)^3)/d - (b^2*(c + d*x)^3)/(3*d) - (b^2*(c + d*x)^2*Cot[e + f*x])/f + (2*b
^2*d*(c + d*x)*Log[1 - E^((2*I)*(e + f*x))])/f^2 + (2*a*b*(c + d*x)^2*Log[1
 - E^((2*I)*(e + f*x))])/f - (I*b^2*d^2*PolyLog[2, E^((2*I)*(e + f*x))])/f^
3 - ((2*I)*a*b*d*(c + d*x)*PolyLog[2, E^((2*I)*(e + f*x))])/f^2 + (a*b*d^2*
PolyLog[3, E^((2*I)*(e + f*x))])/f^3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^2(a+b\cot(e+fx))^2\,dx &= \int \left(a^2(c+dx)^2+2ab(c+dx)^2\cot(e+fx)+b^2(c+dx)^2\cot^2(e+fx\right. \\
&= \frac{a^2(c+dx)^3}{3d}+(2ab)\int (c+dx)^2\cot(e+fx)\,dx+b^2\int (c+dx)^2\cot^2(\\
&= \frac{a^2(c+dx)^3}{3d}-\frac{2iab(c+dx)^3}{3d}-\frac{b^2(c+dx)^2\cot(e+fx)}{f}-(4iab)\int \frac{e}{} \\
&= -\frac{ib^2(c+dx)^2}{f}+\frac{a^2(c+dx)^3}{3d}-\frac{2iab(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{3d}-\frac{b^2(c+}{} \\
&= -\frac{ib^2(c+dx)^2}{f}+\frac{a^2(c+dx)^3}{3d}-\frac{2iab(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{3d}-\frac{b^2(c+}{} \\
&= -\frac{ib^2(c+dx)^2}{f}+\frac{a^2(c+dx)^3}{3d}-\frac{2iab(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{3d}-\frac{b^2(c+}{} \\
&= -\frac{ib^2(c+dx)^2}{f}+\frac{a^2(c+dx)^3}{3d}-\frac{2iab(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{3d}-\frac{b^2(c+}{}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 635 vs. 2(227) = 454.
time = 6.97, size = 635, normalized size = 2.80

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*(a + b*Cot[e + f*x])^2,x]
```

```
[Out] -1/6*(a*b*d^2*Csc[e]*(2*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*(-1 + E^((2*I)*e
))*Log[1 - E^((2*I)*(e + f*x))]) + 6*(-1 + E^((2*I)*e))*f*x*PolyLog[2, E^((
2*I)*(e + f*x))] + (3*I)*(-1 + E^((2*I)*e))*PolyLog[3, E^((2*I)*(e + f*x))]
))/(E^(I*e)*f^3) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Csc[e]*(2*a*b*Cos[e] + a^
2*Sin[e] - b^2*Sin[e]))/3 + (2*b^2*c*d*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]
*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(f^2*(Cos[e]^2 + Sin[e]^2)) + (2*a*b*c^
2*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(
f*(Cos[e]^2 + Sin[e]^2)) + (Csc[e]*Csc[e + f*x]*(b^2*c^2*Sin[f*x] + 2*b^2*c
*d*x*Sin[f*x] + b^2*d^2*x^2*Sin[f*x]))/f - (b^2*d^2*Csc[e]*Sec[e]*(E^(I*Arc
```

```
Tan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2
*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]]
))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]] +
I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[e])/Sqrt[1 + Tan[e]^2])
)/(f^3*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)]) - (2*a*b*c*d*Csc[e]*Sec[e]*(E^
(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 +
E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[T
an[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]
]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[e])/Sqrt[1 + Tan[
e]^2]))/(f^2*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(207) = 414$.
time = 0.68, size = 932, normalized size = 4.11

method	result
risch	$\frac{2b^2d^2\ln\left(e^{i(fx+e)}+1\right)x}{f^2}+\frac{2b^2d^2\ln\left(1-e^{i(fx+e)}\right)x}{f^2}+\frac{2b^2d^2\ln\left(1-e^{i(fx+e)}\right)e}{f^3}-\frac{2ib^2d^2x^2}{f}-\frac{2ib^2d^2e^2}{f^3}-\frac{2id^2ab\,x^3}{3}-2ba\,d^2e^2\,\mathrm{l}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*cot(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*d*a*b*c*x^2+4/f^3*b^2*d^2*e*ln(exp(I*(f*x+e)))-2/f^3*b^2*d^2*e*ln(exp(
I*(f*x+e))-1)+4/f^3*b*a*d^2*polylog(3,-exp(I*(f*x+e)))+d*a^2*c*x^2+a^2*c^2*
x-d*b^2*c*x^2+2/f^3*b*a*d^2*e^2*ln(exp(I*(f*x+e))-1)-4*I/f^2*b*a*d^2*polylo
g(2,exp(I*(f*x+e)))*x-4*I/f^2*b*a*d^2*polylog(2,-exp(I*(f*x+e)))*x-4*I/f^2*
b*a*c*d*e^2-4*I/f^2*b*a*c*d*polylog(2,-exp(I*(f*x+e)))-4*I/f^2*b*a*c*d*poly
log(2,exp(I*(f*x+e)))-8*I/f*b*a*c*d*e*x+2*I*a*b*c^2*x+4/f^2*b*ln(1-exp(I*(f
*x+e)))*a*c*d*e+4/f*b*ln(exp(I*(f*x+e))+1)*a*c*d*x+4/f*b*ln(1-exp(I*(f*x+e)
))*a*c*d*x-4/f^2*b*a*c*d*e*ln(exp(I*(f*x+e))-1)+8/f^2*b*a*c*d*e*ln(exp(I*(f
*x+e)))+4*I/f^2*b*a*d^2*e^2*x+2/3*I/d*a*b*c^3-2*I*b^2*(d^2*x^2+2*c*d*x+c^2)
/f/(exp(2*I*(f*x+e))-1)+2/f^2*b^2*c*d*ln(exp(I*(f*x+e))-1)+2/f^2*b^2*c*d*ln
(exp(I*(f*x+e))+1)+4/f^3*b*a*d^2*polylog(3,exp(I*(f*x+e)))-4/f^2*b^2*c*d*ln
(exp(I*(f*x+e)))-4/f*b*a*c^2*ln(exp(I*(f*x+e)))+2/f*b*a*c^2*ln(exp(I*(f*x+e
))+1)+2/f*b*a*c^2*ln(exp(I*(f*x+e))-1)+2/f^2*b^2*d^2*ln(exp(I*(f*x+e))+1)*x
+2/f^2*b^2*d^2*ln(1-exp(I*(f*x+e)))*x+2/f^3*b^2*d^2*ln(1-exp(I*(f*x+e)))*e-
2*I/f*b^2*d^2*x^2-2*I/f^3*b^2*d^2*e^2-2*I/f^3*b^2*d^2*polylog(2,exp(I*(f*x+
e)))-2*I/f^3*b^2*d^2*polylog(2,-exp(I*(f*x+e)))-2/3*I*d^2*a*b*x^3-1/3*d^2*b
^2*x^3-b^2*c^2*x-1/3/d*b^2*c^3+1/3*d^2*a^2*x^3+1/3/d*a^2*c^3-2/f^3*b*a*d^2*
e^2*ln(1-exp(I*(f*x+e)))+2/f*b*a*d^2*ln(1-exp(I*(f*x+e)))*x^2+8/3*I/f^3*b*a
*d^2*e^3-4*I/f^2*b^2*d^2*e*x-4/f^3*b*a*d^2*e^2*ln(exp(I*(f*x+e)))+2/f*b*a*d
^2*ln(exp(I*(f*x+e))+1)*x^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2031 vs. $2(207) = 414$.

time = 0.61, size = 2031, normalized size = 8.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a^2*c^2 + (f*x + e)^3*a^2*d^2/f^2 + 3*(f*x + e)^2*a^2*c*d/
f - 3*(f*x + e)^2*a^2*d^2*e/f^2 - 6*(f*x + e)*a^2*c*d*e/f + 6*a*b*c^2*log(s
in(f*x + e)) - 12*a*b*c*d*e*log(sin(f*x + e))/f + 3*(f*x + e)*a^2*d^2*e^2/f
^2 + 6*a*b*d^2*e^2*log(sin(f*x + e))/f^2 + 3*((2*a*b - I*b^2)*(f*x + e)^3*d
^2 - 6*b^2*c^2*f^2 + 12*b^2*c*d*f*e - 6*b^2*d^2*e^2 + 3*((2*a*b - I*b^2)*c*
d*f - (2*a*b*e - I*b^2*e)*d^2)*(f*x + e)^2 - 3*(I*b^2*c^2*f^2 - 2*I*b^2*c*d
*f*e + I*b^2*d^2*e^2)*(f*x + e) - 6*((f*x + e)^2*a*b*d^2 + b^2*c*d*f - b^2*
d^2*e + (2*a*b*c*d*f - (2*a*b*e - b^2)*d^2)*(f*x + e) - ((f*x + e)^2*a*b*d^
2 + b^2*c*d*f - b^2*d^2*e + (2*a*b*c*d*f - (2*a*b*e - b^2)*d^2)*(f*x + e))*
cos(2*f*x + 2*e) + (-I*(f*x + e)^2*a*b*d^2 - I*b^2*c*d*f + I*b^2*d^2*e + (-
2*I*a*b*c*d*f + (2*I*a*b*e - I*b^2)*d^2)*(f*x + e))*sin(2*f*x + 2*e))*arcta
n2(sin(f*x + e), cos(f*x + e) + 1) - 6*(b^2*c*d*f - b^2*d^2*e - (b^2*c*d*f
- b^2*d^2*e)*cos(2*f*x + 2*e) + (-I*b^2*c*d*f + I*b^2*d^2*e)*sin(2*f*x + 2*
e))*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 6*((f*x + e)^2*a*b*d^2 + (2*a
*b*c*d*f - (2*a*b*e - b^2)*d^2)*(f*x + e) - ((f*x + e)^2*a*b*d^2 + (2*a*b*c
*d*f - (2*a*b*e - b^2)*d^2)*(f*x + e))*cos(2*f*x + 2*e) - (I*(f*x + e)^2*a*
b*d^2 + (2*I*a*b*c*d*f + (-2*I*a*b*e + I*b^2)*d^2)*(f*x + e))*sin(2*f*x + 2
*e))*arctan2(sin(f*x + e), -cos(f*x + e) + 1) - ((2*a*b - I*b^2)*(f*x + e)^
3*d^2 + 3*((2*a*b - I*b^2)*c*d*f + (b^2*(I*e + 2) - 2*a*b*e)*d^2)*(f*x + e)
^2 + 3*(-I*b^2*c^2*f^2 + 2*b^2*c*d*f*(I*e + 2) + b^2*d^2*(-I*e^2 - 4*e))*(f
*x + e))*cos(2*f*x + 2*e) + 6*(2*(f*x + e)*a*b*d^2 + 2*a*b*c*d*f - (2*a*b*e
 - b^2)*d^2 - (2*(f*x + e)*a*b*d^2 + 2*a*b*c*d*f - (2*a*b*e - b^2)*d^2)*cos
(2*f*x + 2*e) - (2*I*(f*x + e)*a*b*d^2 + 2*I*a*b*c*d*f + (-2*I*a*b*e + I*b^
2)*d^2)*sin(2*f*x + 2*e))*dilog(-e^(I*f*x + I*e)) + 6*(2*(f*x + e)*a*b*d^2
+ 2*a*b*c*d*f - (2*a*b*e - b^2)*d^2 - (2*(f*x + e)*a*b*d^2 + 2*a*b*c*d*f -
(2*a*b*e - b^2)*d^2)*cos(2*f*x + 2*e) - (2*I*(f*x + e)*a*b*d^2 + 2*I*a*b*c*
d*f + (-2*I*a*b*e + I*b^2)*d^2)*sin(2*f*x + 2*e))*dilog(e^(I*f*x + I*e)) -
3*(-I*(f*x + e)^2*a*b*d^2 - I*b^2*c*d*f + I*b^2*d^2*e + (-2*I*a*b*c*d*f + (
2*I*a*b*e - I*b^2)*d^2)*(f*x + e) + (I*(f*x + e)^2*a*b*d^2 + I*b^2*c*d*f -
I*b^2*d^2*e + (2*I*a*b*c*d*f + (-2*I*a*b*e + I*b^2)*d^2)*(f*x + e))*cos(2*f
*x + 2*e) - ((f*x + e)^2*a*b*d^2 + b^2*c*d*f - b^2*d^2*e + (2*a*b*c*d*f - (
2*a*b*e - b^2)*d^2)*(f*x + e))*sin(2*f*x + 2*e))*log(cos(f*x + e)^2 + sin(f
*x + e)^2 + 2*cos(f*x + e) + 1) - 3*(-I*(f*x + e)^2*a*b*d^2 - I*b^2*c*d*f +
 I*b^2*d^2*e + (-2*I*a*b*c*d*f + (2*I*a*b*e - I*b^2)*d^2)*(f*x + e) + (I*(f
*x + e)^2*a*b*d^2 + I*b^2*c*d*f - I*b^2*d^2*e + (2*I*a*b*c*d*f + (-2*I*a*b*
e + I*b^2)*d^2)*(f*x + e))*cos(2*f*x + 2*e) - ((f*x + e)^2*a*b*d^2 + b^2*c*
d*f - b^2*d^2*e + (2*a*b*c*d*f - (2*a*b*e - b^2)*d^2)*(f*x + e))*sin(2*f*x
+ 2*e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - 12*(I*a
```

```
*b*d^2*cos(2*f*x + 2*e) - a*b*d^2*sin(2*f*x + 2*e) - I*a*b*d^2)*polylog(3,
-e^(I*f*x + I*e)) - 12*(I*a*b*d^2*cos(2*f*x + 2*e) - a*b*d^2*sin(2*f*x + 2*
e) - I*a*b*d^2)*polylog(3, e^(I*f*x + I*e)) + ((-2*I*a*b - b^2)*(f*x + e)^3
*d^2 - 3*((2*I*a*b + b^2)*c*d*f - (b^2*(e - 2*I) + 2*I*a*b*e)*d^2)*(f*x + e
)^2 - 3*(b^2*c^2*f^2 - 2*b^2*c*d*f*(e - 2*I) + b^2*d^2*(e^2 - 4*I*e))*(f*x
+ e))*sin(2*f*x + 2*e))/(-3*I*f^2*cos(2*f*x + 2*e) + 3*f^2*sin(2*f*x + 2*e)
 + 3*I*f^2))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(207) = 414$.
time = 2.79, size = 740, normalized size = 3.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^2*d^2*f^2*x^2 + 12*b^2*c*d*f^2*x + 6*b^2*c^2*f^2 - 3*a*b*d^2*poly
log(3, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e))*sin(2*f*x + 2*e) - 3*a*b*d^2*
polylog(3, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e))*sin(2*f*x + 2*e) + 3*(2*I
*a*b*d^2*f*x + 2*I*a*b*c*d*f + I*b^2*d^2)*dilog(cos(2*f*x + 2*e) + I*sin(2*
f*x + 2*e))*sin(2*f*x + 2*e) + 3*(-2*I*a*b*d^2*f*x - 2*I*a*b*c*d*f - I*b^2*
d^2)*dilog(cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e))*sin(2*f*x + 2*e) - 6*(a*b
*c^2*f^2 + b^2*c*d*f + a*b*d^2*e^2 - (2*a*b*c*d*f + b^2*d^2)*e)*log(-1/2*co
s(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2)*sin(2*f*x + 2*e) - 6*(a*b*c^
2*f^2 + b^2*c*d*f + a*b*d^2*e^2 - (2*a*b*c*d*f + b^2*d^2)*e)*log(-1/2*cos(2
*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2)*sin(2*f*x + 2*e) - 6*(a*b*d^2*f
^2*x^2 - a*b*d^2*e^2 + (2*a*b*c*d*f^2 + b^2*d^2*f)*x + (2*a*b*c*d*f + b^2*d
^2)*e)*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e) - 6
*(a*b*d^2*f^2*x^2 - a*b*d^2*e^2 + (2*a*b*c*d*f^2 + b^2*d^2*f)*x + (2*a*b*c*
d*f + b^2*d^2)*e)*log(-cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1)*sin(2*f*x
 + 2*e) + 6*(b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*cos(2*f*x + 2
*e) - 2*((a^2 - b^2)*d^2*f^3*x^3 + 3*(a^2 - b^2)*c*d*f^3*x^2 + 3*(a^2 - b^2
)*c^2*f^3*x)*sin(2*f*x + 2*e))/(f^3*sin(2*f*x + 2*e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cot(e + fx))^2 (c + dx)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*cot(f*x+e))**2,x)
```

```
[Out] Integral((a + b*cot(e + f*x))**2*(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*x + c)^2*(b*cot(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\cot(e + f\,x))^2\,(c + d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^2*(c + d*x)^2,x)
[Out] int((a + b*cot(e + f*x))^2*(c + d*x)^2, x)
```

3.44 $\int (c+dx)(a+b\cot(e+fx))^2\,dx$

Optimal. Leaf size=137

$$-b^2cx-\frac{1}{2}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{iab(c+dx)^2}{d}-\frac{b^2(c+dx)\cot(e+fx)}{f}+\frac{2ab(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f}+\frac{b^2d\log}{}$$

```
[Out] -b^2*c*x-1/2*b^2*d*x^2+1/2*a^2*(d*x+c)^2/d-I*a*b*(d*x+c)^2/d-b^2*(d*x+c)*co
t(f*x+e)/f+2*a*b*(d*x+c)*ln(1-exp(2*I*(f*x+e)))/f+b^2*d*ln(sin(f*x+e))/f^2-
I*a*b*d*polylog(2,exp(2*I*(f*x+e)))/f^2
```

Rubi [A]
time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3803, 3798, 2221, 2317, 2438, 3801, 3556}

$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{iab(c+dx)^2}{d}-\frac{iabd\text{Li}_2\left(e^{2i(e+fx)}\right)}{f^2}-\frac{b^2(c+dx)\cot(e+fx)}{f}-b^2cx+\frac{b^2d\log(\sin(e+fx))}{f^2}-\frac{1}{2}b^2dx^2$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Cot[e + f*x])^2,x]
```

```
[Out] -(b^2*c*x) - (b^2*d*x^2)/2 + (a^2*(c + d*x)^2)/(2*d) - (I*a*b*(c + d*x)^2)/
d - (b^2*(c + d*x)*Cot[e + f*x])/f + (2*a*b*(c + d*x)*Log[1 - E^((2*I)*(e +
 f*x))])/f + (b^2*d*Log[Sin[e + f*x]])/f^2 - (I*a*b*d*PolyLog[2, E^((2*I)*(
e + f*x))])/f^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)(a+b\cot(e+fx))^2\,dx &= \int \left(a^2(c+dx)+2ab(c+dx)\cot(e+fx)+b^2(c+dx)\cot^2(e+fx)\right)\,d \\
&= \frac{a^2(c+dx)^2}{2d}+(2ab)\int (c+dx)\cot(e+fx)\,dx+b^2\int (c+dx)\cot^2(e+ \\
&= \frac{a^2(c+dx)^2}{2d}-\frac{iab(c+dx)^2}{d}-\frac{b^2(c+dx)\cot(e+fx)}{f}-(4iab)\int \frac{e^{2i(e-}}{1-} \\
&= -b^2cx-\frac{1}{2}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{iab(c+dx)^2}{d}-\frac{b^2(c+dx)\cot(e+f}{f} \\
&= -b^2cx-\frac{1}{2}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{iab(c+dx)^2}{d}-\frac{b^2(c+dx)\cot(e+f}{f} \\
&= -b^2cx-\frac{1}{2}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{iab(c+dx)^2}{d}-\frac{b^2(c+dx)\cot(e+f}{f}
\end{aligned}$$

Mathematica [A]
time = 2.11, size = 200, normalized size = 1.46

$$\frac{(a+b\cot(e+fx))^2\sin(e+fx)\left(-2b^2f(c+dx)\cos(e+fx)+\left(-((e+fx)(2iabd(e+fx)+a^2(de-2cf-dfx)+b^2(-de+2cf+dfx)))+4abd(e+fx)\log\left(1-e^{2i(e+fx)}\right)+2b(bd-2ade+2acf)\log(\sin(e+fx))\right)\sin(e+fx)-2iabd\text{PolyLog}\left(2,e^{2i(e+fx)}\right)\sin(e+fx)\right)}{2f^2(b\cos(e+fx)+a\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Cot[e + f*x])^2,x]
```

```
[Out] ((a + b*Cot[e + f*x])^2*Sin[e + f*x]*(-2*b^2*f*(c + d*x)*Cos[e + f*x] + (-(
(e + f*x)*((2*I)*a*b*d*(e + f*x) + a^2*(d*e - 2*c*f - d*f*x) + b^2*(-(d*e)
+ 2*c*f + d*f*x))) + 4*a*b*d*(e + f*x)*Log[1 - E^((2*I)*(e + f*x))] + 2*b*(
b*d - 2*a*d*e + 2*a*c*f)*Log[Sin[e + f*x]])*Sin[e + f*x] - (2*I)*a*b*d*Poly
Log[2, E^((2*I)*(e + f*x))]*Sin[e + f*x]))/(2*f^2*(b*Cos[e + f*x] + a*Sin[e
 + f*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(127) = 254.
time = 0.58, size = 365, normalized size = 2.66

method	result
risch	$-\frac{b^2 d x^2}{2} - \frac{2iba d e^2}{f^2} + \frac{a^2 d x^2}{2} - b^2 cx - \frac{2ibad \operatorname{polylog}(2,e^{i(fx+e)})}{f^2} + a^2 cx - \frac{2ibad \operatorname{polylog}(2,-e^{i(fx+e)})}{f^2} + \frac{b^2 d \ln(e^{i(fx+}}{f^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*cot(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2*d*x^2-2*I/f^2*b*a*d*e^2+1/2*a^2*d*x^2-b^2*c*x-2*I/f^2*b*a*d*polylo
g(2,exp(I*(f*x+e)))+a^2*c*x-2*I/f^2*b*a*d*polylog(2,-exp(I*(f*x+e)))+1/f^2*
b^2*d*ln(exp(I*(f*x+e))-1)-2/f^2*b^2*d*ln(exp(I*(f*x+e)))+1/f^2*b^2*d*ln(ex
p(I*(f*x+e))+1)+2/f*b*a*c*ln(exp(I*(f*x+e))-1)-4/f*b*a*c*ln(exp(I*(f*x+e)))
+2/f*b*a*c*ln(exp(I*(f*x+e))+1)-2/f^2*b*a*d*e*ln(exp(I*(f*x+e))-1)+4/f^2*b*
a*d*e*ln(exp(I*(f*x+e)))+2/f*b*ln(1-exp(I*(f*x+e)))*a*d*x+2/f^2*b*ln(1-exp(
I*(f*x+e)))*a*d*e+2*I*a*b*c*x-I*a*b*d*x^2-4*I/f*b*a*d*e*x-2*I*b^2*(d*x+c)/f
/(exp(2*I*(f*x+e))-1)+2/f*b*ln(exp(I*(f*x+e))+1)*a*d*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 828 vs. 2(128) = 256.
time = 0.40, size = 828, normalized size = 6.04

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(f*x + e)*a^2*c + (f*x + e)^2*a^2*d/f - 2*(f*x + e)*a^2*d*e/f + 4*a*
b*c*log(sin(f*x + e)) - 4*a*b*d*e*log(sin(f*x + e))/f + 2*((2*a*b - I*b^2)*
(f*x + e)^2*d - 4*b^2*c*f + 4*b^2*d*e - 2*(I*b^2*c*f - I*b^2*d*e)*(f*x + e)
 - 2*(2*(f*x + e)*a*b*d + b^2*d - (2*(f*x + e)*a*b*d + b^2*d)*cos(2*f*x + 2
*e) + (-2*I*(f*x + e)*a*b*d - I*b^2*d)*sin(2*f*x + 2*e))*arctan2(sin(f*x +
e), cos(f*x + e) + 1) + 2*(b^2*d*cos(2*f*x + 2*e) + I*b^2*d*sin(2*f*x + 2*e
```

```
) - b^2*d)*arctan2(sin(f*x + e), cos(f*x + e) - 1) - 4*((f*x + e)*a*b*d*cos
(2*f*x + 2*e) + I*(f*x + e)*a*b*d*sin(2*f*x + 2*e) - (f*x + e)*a*b*d)*arcta
n2(sin(f*x + e), -cos(f*x + e) + 1) - ((2*a*b - I*b^2)*(f*x + e)^2*d + 2*(-
I*b^2*c*f + b^2*d*(I*e + 2))*(f*x + e))*cos(2*f*x + 2*e) - 4*(a*b*d*cos(2*f
*x + 2*e) + I*a*b*d*sin(2*f*x + 2*e) - a*b*d)*dilog(-e^(I*f*x + I*e)) - 4*(
a*b*d*cos(2*f*x + 2*e) + I*a*b*d*sin(2*f*x + 2*e) - a*b*d)*dilog(e^(I*f*x +
 I*e)) + (2*I*(f*x + e)*a*b*d + I*b^2*d + (-2*I*(f*x + e)*a*b*d - I*b^2*d)*
cos(2*f*x + 2*e) + (2*(f*x + e)*a*b*d + b^2*d)*sin(2*f*x + 2*e))*log(cos(f*
x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + (2*I*(f*x + e)*a*b*d + I*
b^2*d + (-2*I*(f*x + e)*a*b*d - I*b^2*d)*cos(2*f*x + 2*e) + (2*(f*x + e)*a*
b*d + b^2*d)*sin(2*f*x + 2*e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(
f*x + e) + 1) + ((-2*I*a*b - b^2)*(f*x + e)^2*d - 2*(b^2*c*f - b^2*d*(e - 2
*I))*(f*x + e))*sin(2*f*x + 2*e))/(-2*I*f*cos(2*f*x + 2*e) + 2*f*sin(2*f*x
+ 2*e) + 2*I*f))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(128) = 256$.
time = 3.96, size = 403, normalized size = 2.94

$$-\frac{2\,b^2dfx+i\,abd\mathrm{Li}_2(\cos(2\,fx+2\,e)+i\,\sin(2\,fx+2\,e))\sin(2\,fx+2\,e)-i\,abd\mathrm{Li}_2(\cos(2\,fx+2\,e)-i\,\sin(2\,fx+2\,e))\sin(2\,fx+2\,e)+2\,b^2cf-(2\,abcf-2\,abde+b^2d)\log\left(-\frac{1}{2}\,\cos(2\,fx+2\,e)+\frac{1}{2}i\,\sin(2\,fx+2\,e)+\frac{1}{2}\right)\sin(2\,fx+2\,e)-(2\,abcf-2\,abde+b^2d)\log\left(-\frac{1}{2}\,\cos(2\,fx+2\,e)-\frac{1}{2}i\,\sin(2\,fx+2\,e)+\frac{1}{2}\right)\sin(2\,fx+2\,e)-2\,(abdfx+abde)\log(-\cos(2\,fx+2\,e)+i\,\sin(2\,fx+2\,e)+1)\sin(2\,fx+2\,e)-2\,(abdfx+abde)\log(-\cos(2\,fx+2\,e)-i\,\sin(2\,fx+2\,e)+1)\sin(2\,fx+2\,e)+2\,(b^2dfx+b^2cf)\cos(2\,fx+2\,e)-((a^2-b^2)df^2x^2+2\,(a^2-b^2)cf^2x)\sin(2\,fx+2\,e)}{2\,f^2\sin(2\,fx+2\,e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*d*f*x + I*a*b*d*dilog(cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e))*si
n(2*f*x + 2*e) - I*a*b*d*dilog(cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e))*sin(2
*f*x + 2*e) + 2*b^2*c*f - (2*a*b*c*f - 2*a*b*d*e + b^2*d)*log(-1/2*cos(2*f*
x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2)*sin(2*f*x + 2*e) - (2*a*b*c*f - 2*
a*b*d*e + b^2*d)*log(-1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2)*
sin(2*f*x + 2*e) - 2*(a*b*d*f*x + a*b*d*e)*log(-cos(2*f*x + 2*e) + I*sin(2*
f*x + 2*e) + 1)*sin(2*f*x + 2*e) - 2*(a*b*d*f*x + a*b*d*e)*log(-cos(2*f*x +
 2*e) - I*sin(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e) + 2*(b^2*d*f*x + b^2*c*f)*
cos(2*f*x + 2*e) - ((a^2 - b^2)*d*f^2*x^2 + 2*(a^2 - b^2)*c*f^2*x)*sin(2*f*
x + 2*e))/(f^2*sin(2*f*x + 2*e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cot(e + fx))^2 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))**2,x)
```

```
[Out] Integral((a + b*cot(e + f*x))**2*(c + d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*cot(f*x + e) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\cot(e+f\,x))^2\,(c+d\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^2*(c + d*x),x)
```

```
[Out] int((a + b*cot(e + f*x))^2*(c + d*x), x)
```

3.45 $\int \frac{(a+b\cot(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{(a+b\cot(e+fx))^2}{c+dx},x\right)$$

```
[Out] Unintegrable((a+b*cot(f*x+e))^2/(d*x+c),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cot(e+fx))^2}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Cot[e + f*x])^2/(c + d*x),x]
```

```
[Out] Defer[Int][(a + b*Cot[e + f*x])^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{(a+b\cot(e+fx))^2}{c+dx}\,dx = \int \frac{(a+b\cot(e+fx))^2}{c+dx}\,dx$$

Mathematica [A]
time = 15.95, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cot(e+fx))^2}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Cot[e + f*x])^2/(c + d*x),x]
```

```
[Out] Integrate[(a + b*Cot[e + f*x])^2/(c + d*x), x]
```

Maple [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cot(fx+e))^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(f*x+e))^2/(d*x+c),x)
```

```
[Out] int((a+b*cot(f*x+e))^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] (((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*cos(2*f*x + 2*e)^2*log(d*x + c) - 2*
b^2*d*sin(2*f*x + 2*e) + ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(d*x + c)
*sin(2*f*x + 2*e)^2 - 2*((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*cos(2*f*x + 2
*e)*log(d*x + c) - (d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2
+ (d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 - 2*(d^2*f*x + c*d*f)*cos(2*f*x + 2*
e))*integrate((2*a*b*d*f*x + 2*a*b*c*f - b^2*d)*sin(f*x + e)/(d^2*f*x^2 + 2
*c*d*f*x + c^2*f + (d^2*f*x^2 + 2*c*d*f*x + c^2*f)*cos(f*x + e)^2 + (d^2*f*
x^2 + 2*c*d*f*x + c^2*f)*sin(f*x + e)^2 + 2*(d^2*f*x^2 + 2*c*d*f*x + c^2*f)
*cos(f*x + e)), x) + (d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^
2 + (d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 - 2*(d^2*f*x + c*d*f)*cos(2*f*x +
2*e))*integrate((2*a*b*d*f*x + 2*a*b*c*f - b^2*d)*sin(f*x + e)/(d^2*f*x^2 +
 2*c*d*f*x + c^2*f + (d^2*f*x^2 + 2*c*d*f*x + c^2*f)*cos(f*x + e)^2 + (d^2*
f*x^2 + 2*c*d*f*x + c^2*f)*sin(f*x + e)^2 - 2*(d^2*f*x^2 + 2*c*d*f*x + c^2*
f)*cos(f*x + e)), x) + ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(d*x + c))/
(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 + (d^2*f*x + c*d*f)
*sin(2*f*x + 2*e)^2 - 2*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b^2*cot(f*x + e)^2 + 2*a*b*cot(f*x + e) + a^2)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\cot\left(e + fx\right)\right)^{2}}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))**2/(d*x+c),x)
[Out] Integral((a + b*cot(e + f*x))**2/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^2/(d*x+c),x, algorithm="giac")
[Out] integrate((b*cot(f*x + e) + a)^2/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(a+b\,\mathrm{cot}\left(e+f\,x\right)\right)}^2}{c+d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^2/(c + d*x),x)
[Out] int((a + b*cot(e + f*x))^2/(c + d*x), x)
```

3.46 $\int \frac{(a+b\cot(e+fx))^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(a+b\cot(e+fx))^2}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable((a+b*cot(f*x+e))^2/(d*x+c)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cot(e+fx))^2}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Cot[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(a + b*Cot[e + f*x])^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{(a+b\cot(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(a+b\cot(e+fx))^2}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 12.97, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cot(e+fx))^2}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Cot[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] Integrate[(a + b*Cot[e + f*x])^2/(c + d*x)^2, x]
```

Maple [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cot\left(fx+e\right)\right)^2}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(f*x+e))^2/(d*x+c)^2,x)
```

```
[Out] int((a+b*cot(f*x+e))^2/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((a^2 - b^2)*d*f*x + 2*b^2*d*sin(2*f*x + 2*e) + (a^2 - b^2)*c*f + ((a^2 -
b^2)*d*f*x + (a^2 - b^2)*c*f)*cos(2*f*x + 2*e)^2 + ((a^2 - b^2)*d*f*x + (a^
2 - b^2)*c*f)*sin(2*f*x + 2*e)^2 - 2*((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*
cos(2*f*x + 2*e) + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2 + 2*c*d^
2*f*x + c^2*d*f)*cos(2*f*x + 2*e)^2 + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*s
in(2*f*x + 2*e)^2 - 2*(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*cos(2*f*x + 2*e))
*integrate(2*(a*b*d*f*x + a*b*c*f - b^2*d)*sin(f*x + e)/(d^3*f*x^3 + 3*c*d^
2*f*x^2 + 3*c^2*d*f*x + c^3*f + (d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x +
c^3*f)*cos(f*x + e)^2 + (d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)*s
in(f*x + e)^2 + 2*(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)*cos(f*x
 + e)), x) - (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2 + 2*c*d^2*f*x
+ c^2*d*f)*cos(2*f*x + 2*e)^2 + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*sin(2*f
*x + 2*e)^2 - 2*(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*cos(2*f*x + 2*e))*integ
rate(2*(a*b*d*f*x + a*b*c*f - b^2*d)*sin(f*x + e)/(d^3*f*x^3 + 3*c*d^2*f*x^
2 + 3*c^2*d*f*x + c^3*f + (d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)
*cos(f*x + e)^2 + (d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)*sin(f*x
 + e)^2 - 2*(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)*cos(f*x + e))
, x))/(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d
*f)*cos(2*f*x + 2*e)^2 + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*sin(2*f*x + 2*
e)^2 - 2*(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*cos(2*f*x + 2*e))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cot(f*x + e)^2 + 2*a*b*cot(f*x + e) + a^2)/(d^2*x^2 + 2*c*d*x
 + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\cot\left(e + fx\right)\right)^{2}}{\left(c + dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*cot(e + f*x))**2/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cot(f*x + e) + a)^2/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(a+b\,\mathrm{cot}\left(e+f\,x\right)\right)}^2}{{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^2/(c + d*x)^2,x)
```

```
[Out] int((a + b*cot(e + f*x))^2/(c + d*x)^2, x)
```

3.47 $\int (c+dx)^3(a+b\cot(e+fx))^3\,dx$

Optimal. Leaf size=603

$$-\frac{3ib^3d(c+dx)^2}{2f^2}-\frac{3iab^2(c+dx)^3}{f}-\frac{b^3(c+dx)^3}{2f}+\frac{a^3(c+dx)^4}{4d}-\frac{3ia^2b(c+dx)^4}{4d}-\frac{3ab^2(c+dx)^4}{4d}+\frac{ib^3(c+dx)^4}{4d}$$

[Out] -3*I*a*b^2*(d*x+c)^3/f+1/4*I*b^3*(d*x+c)^4/d-1/2*b^3*(d*x+c)^3/f+1/4*a^3*(d*x+c)^4/d-9/2*I*a^2*b*d*(d*x+c)^2*polylog(2,exp(2*I*(f*x+e)))/f^2-3/4*a*b^2*(d*x+c)^4/d+3/2*I*b^3*d*(d*x+c)^2*polylog(2,exp(2*I*(f*x+e)))/f^2-3/2*b^3*d*(d*x+c)^2*cot(f*x+e)/f^2-3*a*b^2*(d*x+c)^3*cot(f*x+e)/f-1/2*b^3*(d*x+c)^3*cot(f*x+e)^2/f+3*b^3*d^2*(d*x+c)*ln(1-exp(2*I*(f*x+e)))/f^3+9*a*b^2*d*(d*x+c)^2*ln(1-exp(2*I*(f*x+e)))/f^2+3*a^2*b*(d*x+c)^3*ln(1-exp(2*I*(f*x+e)))/f-b^3*(d*x+c)^3*ln(1-exp(2*I*(f*x+e)))/f+9/4*I*a^2*b*d^3*polylog(4,exp(2*I*(f*x+e)))/f^4-9*I*a*b^2*d^2*(d*x+c)*polylog(2,exp(2*I*(f*x+e)))/f^3-3/4*I*a^2*b*(d*x+c)^4/d-3/2*I*b^3*d*(d*x+c)^2/f^2+9/2*a*b^2*d^3*polylog(3,exp(2*I*(f*x+e)))/f^4+9/2*a^2*b*d^2*(d*x+c)*polylog(3,exp(2*I*(f*x+e)))/f^3-3/2*b^3*d^2*(d*x+c)*polylog(3,exp(2*I*(f*x+e)))/f^3-3/4*I*b^3*d^3*polylog(4,exp(2*I*(f*x+e)))/f^4-3/2*I*b^3*d^3*polylog(2,exp(2*I*(f*x+e)))/f^4

Rubi [A]
time = 0.66, antiderivative size = 603, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3803, 3798, 2221, 2611, 6744, 2320, 6724, 3801, 32, 2317, 2438}

$$\frac{a^3(c+dx)^4}{4d}+\frac{9a^2bd^2(c+dx)\text{Li}_3\left(e^{2i(e+fx)}\right)}{2f^3}-\frac{9ia^2bd(c+dx)^2\text{Li}_2\left(e^{2i(e+fx)}\right)}{2f^2}+\frac{3a^2b(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{3ia^2b(c+dx)^4}{4d}+\frac{9ia^2bd^3\text{Li}_4\left(e^{2i(e+fx)}\right)}{4f^4}-\frac{9iab^2d^2(c+dx)\text{Li}_2\left(e^{2i(e+fx)}\right)}{f^3}+\frac{9ab^2d(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f^2}-\frac{3ab^2(c+dx)^3\cot(e+fx)}{f}-\frac{3iab^2(c+dx)^3}{f}-\frac{3ab^2(c+dx)^4}{4d}+\frac{9ab^2d^3\text{Li}_3\left(e^{2i(e+fx)}\right)}{2f^4}-\frac{3b^3d^2(c+dx)\text{Li}_3\left(e^{2i(e+fx)}\right)}{2f^3}+\frac{3b^3d^2(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f^3}+\frac{3ib^3d(c+dx)^2\text{Li}_2\left(e^{2i(e+fx)}\right)}{2f^2}-\frac{3b^3d(c+dx)^2\cot(e+fx)}{2f^2}-\frac{b^3(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{b^3(c+dx)^3\cot^2(e+fx)}{2f}-\frac{3ib^3d(c+dx)^2}{2f^2}-\frac{b^3(c+dx)^3}{2f}+\frac{ib^3(c+dx)^4}{4d}-\frac{3ib^3d^3\text{Li}_2\left(e^{2i(e+fx)}\right)}{2f^4}-\frac{3ib^3d^3\text{Li}_4\left(e^{2i(e+fx)}\right)}{4f^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*(a + b*Cot[e + f*x])^3,x]

[Out] (((-3*I)/2)*b^3*d*(c + d*x)^2)/f^2 - ((3*I)*a*b^2*(c + d*x)^3)/f - (b^3*(c + d*x)^3)/(2*f) + (a^3*(c + d*x)^4)/(4*d) - (((3*I)/4)*a^2*b*(c + d*x)^4)/d - (3*a*b^2*(c + d*x)^4)/(4*d) + ((I/4)*b^3*(c + d*x)^4)/d - (3*b^3*d*(c + d*x)^2*Cot[e + f*x])/(2*f^2) - (3*a*b^2*(c + d*x)^3*Cot[e + f*x])/f - (b^3*(c + d*x)^3*Cot[e + f*x]^2)/(2*f) + (3*b^3*d^2*(c + d*x)*Log[1 - E^((2*I)*(e + f*x))])/f^3 + (9*a*b^2*d*(c + d*x)^2*Log[1 - E^((2*I)*(e + f*x))])/f^2 + (3*a^2*b*(c + d*x)^3*Log[1 - E^((2*I)*(e + f*x))])/f - (b^3*(c + d*x)^3*Log[1 - E^((2*I)*(e + f*x))])/f - (((3*I)/2)*b^3*d^3*PolyLog[2, E^((2*I)*(e + f*x))])/f^4 - ((9*I)*a*b^2*d^2*(c + d*x)*PolyLog[2, E^((2*I)*(e + f*x))])/f^3 - (((9*I)/2)*a^2*b*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(e + f*x))])/f^2 + (((3*I)/2)*b^3*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(e + f*x))])/f^2 + (9*a*b^2*d^3*PolyLog[3, E^((2*I)*(e + f*x))])/(2*f^4) + (9*a^2*b*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(e + f*x))])/(2*f^3) - (3*b^3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(e + f*x))])/(2*f^3) + (((9*I)/4)*a^2*b*d^3*PolyLog[4, E^((2*I)*(e + f*x))])/f^4 - (((3*I)/4)*b^3*d^3*PolyLog[4, E^((2*I)*(e + f*x))])/f^4

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3(a+b\cot(e+fx))^3\,dx = \int \left(a^3(c+dx)^3+3a^2b(c+dx)^3\cot(e+fx)+3ab^2(c+dx)^3\cot^2(e+f\right.$$
$$= \frac{a^3(c+dx)^4}{4d}+\left(3a^2b\right)\int (c+dx)^3\cot(e+fx)\,dx+\left(3ab^2\right)\int (c+dx)^3$$
$$= \frac{a^3(c+dx)^4}{4d}-\frac{3ia^2b(c+dx)^4}{4d}-\frac{3ab^2(c+dx)^3\cot(e+fx)}{f}-\frac{b^3(c+dx}{}$$
$$= -\frac{3iab^2(c+dx)^3}{f}+\frac{a^3(c+dx)^4}{4d}-\frac{3ia^2b(c+dx)^4}{4d}-\frac{3ab^2(c+dx)^4}{4d}+$$
$$= -\frac{3ib^3d(c+dx)^2}{2f^2}-\frac{3iab^2(c+dx)^3}{f}-\frac{b^3(c+dx)^3}{2f}+\frac{a^3(c+dx)^4}{4d}-$$
$$= -\frac{3ib^3d(c+dx)^2}{2f^2}-\frac{3iab^2(c+dx)^3}{f}-\frac{b^3(c+dx)^3}{2f}+\frac{a^3(c+dx)^4}{4d}-$$
$$= -\frac{3ib^3d(c+dx)^2}{2f^2}-\frac{3iab^2(c+dx)^3}{f}-\frac{b^3(c+dx)^3}{2f}+\frac{a^3(c+dx)^4}{4d}-$$
$$= -\frac{3ib^3d(c+dx)^2}{2f^2}-\frac{3iab^2(c+dx)^3}{f}-\frac{b^3(c+dx)^3}{2f}+\frac{a^3(c+dx)^4}{4d}-$$
$$= -\frac{3ib^3d(c+dx)^2}{2f^2}-\frac{3iab^2(c+dx)^3}{f}-\frac{b^3(c+dx)^3}{2f}+\frac{a^3(c+dx)^4}{4d}-$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2539 vs. $2(603) = 1206$.
time = 7.94, size = 2539, normalized size = 4.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*(a + b*Cot[e + f*x])^3,x]
```

```
[Out] ((-(b^3*c^3) - 3*b^3*c^2*d*x - 3*b^3*c*d^2*x^2 - b^3*d^3*x^3)*Csc[e + f*x]^
2)/(2*f) - (3*a*b^2*d^3*Csc[e]*(2*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*(-1 +
E^((2*I)*e))*Log[1 - E^((2*I)*(e + f*x))]) + 6*(-1 + E^((2*I)*e))*f*x*PolyL
og[2, E^((2*I)*(e + f*x))] + (3*I)*(-1 + E^((2*I)*e))*PolyLog[3, E^((2*I)*(
e + f*x))]))/(4*E^(I*e)*f^4) - (3*a^2*b*c*d^2*Csc[e]*(2*f^2*x^2*(2*E^((2*I)
*e)*f*x + (3*I)*(-1 + E^((2*I)*e))*Log[1 - E^((2*I)*(e + f*x))]) + 6*(-1 +
E^((2*I)*e))*f*x*PolyLog[2, E^((2*I)*(e + f*x))] + (3*I)*(-1 + E^((2*I)*e))
*PolyLog[3, E^((2*I)*(e + f*x))]))/(4*E^(I*e)*f^3) + (b^3*c*d^2*Csc[e]*(2*f
^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*(-1 + E^((2*I)*e))*Log[1 - E^((2*I)*(e +
f*x))]) + 6*(-1 + E^((2*I)*e))*f*x*PolyLog[2, E^((2*I)*(e + f*x))] + (3*I)*
```

```
(-1 + E^((2*I)*e))*PolyLog[3, E^((2*I)*(e + f*x))]))/(4*E^(I*e)*f^3) - (3*a
^2*b*d^3*E^(I*e)*Csc[e]*(x^4 + (-1 + E^((-2*I)*e))*x^4 + ((-1 + E^((2*I)*e)
)*(2*f^4*x^4 + (4*I)*f^3*x^3*Log[1 - E^((2*I)*(e + f*x))] + 6*f^2*x^2*PolyL
og[2, E^((2*I)*(e + f*x))] + (6*I)*f*x*PolyLog[3, E^((2*I)*(e + f*x))] - 3*
PolyLog[4, E^((2*I)*(e + f*x))]))/(2*E^((2*I)*e)*f^4)))/4 + (b^3*d^3*E^(I*e
)*Csc[e]*(x^4 + (-1 + E^((-2*I)*e))*x^4 + ((-1 + E^((2*I)*e))*(2*f^4*x^4 +
(4*I)*f^3*x^3*Log[1 - E^((2*I)*(e + f*x))] + 6*f^2*x^2*PolyLog[2, E^((2*I)*
(e + f*x))] + (6*I)*f*x*PolyLog[3, E^((2*I)*(e + f*x))] - 3*PolyLog[4, E^((
2*I)*(e + f*x))]))/(2*E^((2*I)*e)*f^4)))/4 + (3*b^3*c*d^2*Csc[e]*(-(f*x*Cos
[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(f^3*(Cos[e]^2 + Sin
[e]^2)) + (9*a*b^2*c^2*d*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[
e]*Sin[f*x]]*Sin[e]))/(f^2*(Cos[e]^2 + Sin[e]^2)) + (3*a^2*b*c^3*Csc[e]*(-(
f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(f*(Cos[e]^2
+ Sin[e]^2)) - (b^3*c^3*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e
]*Sin[f*x]]*Sin[e]))/(f*(Cos[e]^2 + Sin[e]^2)) + (3*x^2*(-(a^3*c^2*d) + (3*
I)*a^2*b*c^2*d + 3*a*b^2*c^2*d - I*b^3*c^2*d + a^3*c^2*d*Cos[2*e] + (3*I)*a
^2*b*c^2*d*Cos[2*e] - 3*a*b^2*c^2*d*Cos[2*e] - I*b^3*c^2*d*Cos[2*e] + I*a^3
*c^2*d*Sin[2*e] - 3*a^2*b*c^2*d*Sin[2*e] - (3*I)*a*b^2*c^2*d*Sin[2*e] + b^3
*c^2*d*Sin[2*e]))/(2*(-1 + Cos[2*e] + I*Sin[2*e])) + (x^3*(-(a^3*c*d^2) + (
3*I)*a^2*b*c*d^2 + 3*a*b^2*c*d^2 - I*b^3*c*d^2 + a^3*c*d^2*Cos[2*e] + (3*I)
*a^2*b*c*d^2*Cos[2*e] - 3*a*b^2*c*d^2*Cos[2*e] - I*b^3*c*d^2*Cos[2*e] + I*a
^3*c*d^2*Sin[2*e] - 3*a^2*b*c*d^2*Sin[2*e] - (3*I)*a*b^2*c*d^2*Sin[2*e] + b
^3*c*d^2*Sin[2*e]))/(-1 + Cos[2*e] + I*Sin[2*e]) + (x^4*(-(a^3*d^3) + (3*I)
*a^2*b*d^3 + 3*a*b^2*d^3 - I*b^3*d^3 + a^3*d^3*Cos[2*e] + (3*I)*a^2*b*d^3*C
os[2*e] - 3*a*b^2*d^3*Cos[2*e] - I*b^3*d^3*Cos[2*e] + I*a^3*d^3*Sin[2*e] -
3*a^2*b*d^3*Sin[2*e] - (3*I)*a*b^2*d^3*Sin[2*e] + b^3*d^3*Sin[2*e]))/(4*(-1
 + Cos[2*e] + I*Sin[2*e])) + x*(a^3*c^3 - 3*a*b^2*c^3 + ((3*I)*a^2*b*c^3)/(
-1 + Cos[2*e] + I*Sin[2*e]) + ((3*I)*a^2*b*c^3*Cos[2*e] - 3*a^2*b*c^3*Sin[2
*e])/(-1 + Cos[2*e] + I*Sin[2*e]) + ((-2*I)*b^3*c^3*Cos[2*e] + 2*b^3*c^3*Si
n[2*e])/((-1 + Cos[2*e] + I*Sin[2*e])*(1 + Cos[2*e] + Cos[4*e] + I*Sin[2*e]
 + I*Sin[4*e])) + ((-2*I)*b^3*c^3*Cos[4*e] + 2*b^3*c^3*Sin[4*e])/((-1 + Cos
[2*e] + I*Sin[2*e])*(1 + Cos[2*e] + Cos[4*e] + I*Sin[2*e] + I*Sin[4*e])) -
(I*b^3*c^3)/(-1 + Cos[6*e] + I*Sin[6*e]) + ((-I)*b^3*c^3*Cos[6*e] + b^3*c^3
*Sin[6*e])/(-1 + Cos[6*e] + I*Sin[6*e])) + (3*Csc[e]*Csc[e + f*x]*(b^3*c^2*
d*Sin[f*x] + 2*a*b^2*c^3*f*Sin[f*x] + 2*b^3*c*d^2*x*Sin[f*x] + 6*a*b^2*c^2*
d*f*x*Sin[f*x] + b^3*d^3*x^2*Sin[f*x] + 6*a*b^2*c*d^2*f*x^2*Sin[f*x] + 2*a*
b^2*d^3*f*x^3*Sin[f*x]))/(2*f^2) - (3*b^3*d^3*Csc[e]*Sec[e]*(E^(I*ArcTan[Ta
n[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*
x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]]))] +
Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]] + I*Poly
Log[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[e])/Sqrt[1 + Tan[e]^2]))/(2*f
^4*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)]) - (9*a*b^2*c*d^2*Csc[e]*Sec[e]*(E^
(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 +
E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[T
an[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]
```

```
]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[e])/Sqrt[1 + Tan[
e]^2]))/(f^3*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)]) - (9*a^2*b*c^2*d*Csc[e]*
Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - P
i*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x
+ ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + Arc
Tan[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[e])/Sqr
t[1 + Tan[e]^2]))/(2*f^2*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)]) + (3*b^3*c^2
*d*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Ta
n[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((
2*I)*(f*x + ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin
[f*x + ArcTan[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*T
an[e])/Sqrt[1 + Tan[e]^2]))/(2*f^2*Sqrt[Sec[e]^...
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3160 vs. $2(535) = 1070$.
time = 1.04, size = 3161, normalized size = 5.24

method	result	size
risch	Expression too large to display	3161

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*cot(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -9/f^2*b*a^2*c^2*d*e*ln(exp(I*(f*x+e))-1)+36/f^3*b^2*a*c*d^2*e*ln(exp(I*(f*
x+e)))+18/f^2*b*a^2*c^2*d*e*ln(exp(I*(f*x+e)))+3/f*b*a^2*d^3*ln(1-exp(I*(f*
x+e)))*x^3+3/f^4*b*a^2*d^3*ln(1-exp(I*(f*x+e)))*e^3+3/f*b*a^2*d^3*ln(exp(I*
(f*x+e))+1)*x^3-3*d^2*a*b^2*c*x^3-9/2*d*a*b^2*c^2*x^2+3*I*a^2*b*c^3*x+3/4*I
/d*a^2*b*c^4-18*I/f*b*a^2*c^2*d*e*x-36*I/f^2*b^2*a*c*d^2*e*x+18*I/f^2*b*a^2
*c*d^2*e^2*x-18*I/f^2*b*polylog(2,-exp(I*(f*x+e)))*a^2*c*d^2*x-18*I/f^2*b*p
olylog(2,exp(I*(f*x+e)))*a^2*c*d^2*x-9/2*I*d*a^2*b*c^2*x^2+6*I/f^2*b^3*poly
log(2,-exp(I*(f*x+e)))*c*d^2*x-9*I/f^2*b*a^2*d^3*polylog(2,exp(I*(f*x+e)))*
x^2-9*I/f^2*b*a^2*d^3*polylog(2,-exp(I*(f*x+e)))*x^2-18*I/f*b^2*a*c*d^2*x^2
-18*I/f^3*b^2*a*d^3*polylog(2,-exp(I*(f*x+e)))*x-18*I/f^3*b^2*a*d^3*polylog
(2,exp(I*(f*x+e)))*x-3*I/f^4*b^3*d^3*polylog(2,exp(I*(f*x+e)))-6*I/f^4*b^3*
d^3*polylog(4,-exp(I*(f*x+e)))-3*I/f^2*b^3*d^3*x^2+3/2*I/f^4*b^3*d^3*e^4-3*
I/f^4*b^3*d^3*e^2+3/f^3*b^3*c*d^2*ln(exp(I*(f*x+e))-1)-6/f^3*b^3*c*d^2*poly
log(3,-exp(I*(f*x+e)))-6/f^3*b^3*c*d^2*polylog(3,exp(I*(f*x+e)))-2/f^4*b^3*
d^3*e^3*ln(exp(I*(f*x+e)))+1/f^4*b^3*d^3*e^3*ln(exp(I*(f*x+e))-1)+18/f^4*b^
2*a*d^3*polylog(3,-exp(I*(f*x+e)))+18/f^4*b^2*a*d^3*polylog(3,exp(I*(f*x+e)
))-3/f^4*b^3*d^3*e*ln(exp(I*(f*x+e))-1)+6/f^4*b^3*d^3*e*ln(exp(I*(f*x+e)))+
3/f*b*a^2*c^3*ln(exp(I*(f*x+e))-1)-6/f^3*b^3*c*d^2*ln(exp(I*(f*x+e)))-6/f*b
*a^2*c^3*ln(exp(I*(f*x+e)))+3/f*b*a^2*c^3*ln(exp(I*(f*x+e))+1)+3/f^3*b^3*c*
d^2*ln(exp(I*(f*x+e))+1)-6/f^3*b^3*d^3*polylog(3,exp(I*(f*x+e)))*x-6/f^3*b^
3*d^3*polylog(3,-exp(I*(f*x+e)))*x+3/f^3*b^3*d^3*ln(1-exp(I*(f*x+e)))*x+3/f
^4*b^3*d^3*ln(1-exp(I*(f*x+e)))*e+3/f^3*b^3*d^3*ln(exp(I*(f*x+e))+1)*x+I*b^
```

```
3*c*d^2*x^3-6*I/f^4*b^3*d^3*polylog(4,exp(I*(f*x+e)))-3*I/f^4*b^3*d^3*polyl
og(2,-exp(I*(f*x+e)))+9/f^3*b*a^2*c*d^2*e^2*ln(exp(I*(f*x+e))-1)-18/f^3*b*a
^2*c*d^2*e^2*ln(exp(I*(f*x+e)))-18*I/f^3*b^2*a*c*d^2*e^2-9*I/f^2*b*a^2*c^2*
d*e^2+18*I/f^3*b^2*d^3*a*e^2*x-6*I/f^3*b*a^2*d^3*e^3*x+12*I/f^3*b*a^2*c*d^2
*e^3+6*I/f*b^3*c^2*d*e*x-6*I/f^2*b^3*c*d^2*e^2*x-18*I/f^3*b^2*a*c*d^2*polyl
og(2,-exp(I*(f*x+e)))-18*I/f^3*b^2*a*c*d^2*polylog(2,exp(I*(f*x+e)))-9*I/f^
2*b*a^2*c^2*d*polylog(2,-exp(I*(f*x+e)))-9*I/f^2*b*a^2*c^2*d*polylog(2,exp(
I*(f*x+e)))+6*I/f^2*b^3*polylog(2,exp(I*(f*x+e)))*c*d^2*x+3/2*I*b^3*c^2*d*x
^2-3/4*I*d^3*a^2*b*x^4-3*I*d^2*a^2*b*c*x^3+2/f*b^3*c^3*ln(exp(I*(f*x+e)))-1
/f*b^3*c^3*ln(exp(I*(f*x+e))+1)-1/f*b^3*c^3*ln(exp(I*(f*x+e))-1)+18/f^3*b*a
^2*c*d^2*polylog(3,-exp(I*(f*x+e)))-3/f^4*b*a^2*d^3*e^3*ln(exp(I*(f*x+e))-1
)+6/f^3*b^3*c*d^2*e^2*ln(exp(I*(f*x+e)))+6/f^4*b*a^2*d^3*e^3*ln(exp(I*(f*x+
e)))-3/f^3*b^3*c*d^2*e^2*ln(exp(I*(f*x+e))-1)+3*I/f^2*b^3*c^2*d*polylog(2,-
exp(I*(f*x+e)))+3*I/f^2*b^3*c^2*d*polylog(2,exp(I*(f*x+e)))+12*I/f^4*b^2*d^
3*a*e^3+3*I/f^2*b^3*d^3*polylog(2,exp(I*(f*x+e)))*x^2+3*I/f^2*b^3*d^3*polyl
og(2,-exp(I*(f*x+e)))*x^2+9/f^2*b*ln(1-exp(I*(f*x+e)))*a^2*c^2*d*e+18/f^3*b
^2*ln(1-exp(I*(f*x+e)))*a*c*d^2*e+9/f*b*ln(exp(I*(f*x+e))+1)*a^2*c^2*d*x+b^
2*(18*I*a*c^2*d*f*x+3*I*b*d^3*x^2+2*b*d^3*f*x^3*exp(2*I*(f*x+e))-6*I*b*c*d^
2*x*exp(2*I*(f*x+e))-18*I*a*c*d^2*f*x^2*exp(2*I*(f*x+e))+6*I*a*c^3*f+6*b*c*
d^2*f*x^2*exp(2*I*(f*x+e))+6*I*a*d^3*f*x^3+3*I*c^2*b*d-3*I*b*d^3*x^2*exp(2*
I*(f*x+e))+6*b*c^2*d*f*x*exp(2*I*(f*x+e))-6*I*a*c^3*f*exp(2*I*(f*x+e))-3*I*
c^2*b*d*exp(2*I*(f*x+e))-6*I*a*d^3*f*x^3*exp(2*I*(f*x+e))+2*b*c^3*f*exp(2*I
*(f*x+e))+6*I*c*b*d^2*x+18*I*a*c*d^2*f*x^2-18*I*a*c^2*d*f*x*exp(2*I*(f*x+e)
))/f^2/(exp(2*I*(f*x+e))-1)^2+d^2*a^3*c*x^3+3/2*d*a^3*c^2*x^2+a^3*c^3*x-3/4
*d^3*a*b^2*x^4-I*b^3*c^3*x-3*a*b^2*c^3*x-3/4/d*a*b^2*c^4+1/4*d^3*a^3*x^4+1/
4/d*a^3*c^4-3/f*b^3*ln(exp(I*(f*x+e))+1)*c*d^2*x^2-3/f*b^3*ln(1-exp(I*(f*x+
e)))*c*d^2*x^2-3/f*b^3*ln(1-exp(I*(f*x+e)))*c^2*d*x-3/f^2*b^3*ln(1-exp(I*(f
*x+e)))*c^2*d*e-3/f*b^3*ln(exp(I*(f*x+e))+1)*c^2*d*x+18/f^3*b*a^2*d^3*polyl
og(3,exp(I*(f*x+e)))*x+18/f^3*b*a^2*d^3*polylog(3,-exp(I*(f*x+e)))*x-9/f^4*
b^2*a*d^3*e^2*ln(1-exp(I*(f*x+e)))+9/f^2*b^2*a*d^3*ln(exp(I*(f*x+e))+1)*x^2
+3/f^3*b^3*ln(1-exp(I*(f*x+e)))*c*d^2*e^2+18/f^3*b*a^2*c*d^2*polylog(3,exp(
I*(f*x+e)))+9/f^2*b^2*a*d^3*ln(1-exp(I*(f*x+e)))*x^2-18/f^2*b^2*a*c^2*d*ln(
exp(I*(f*x+e)))+9/f^2*b^2*a*c^2*d*ln(exp(I*(f*x+e))+1)-9/f^3*b*ln(1-exp(I*(
f*x+e)))*a^2*c*d^2*e^2+18/f^2*b^2*ln(exp(I*(f*x+e))+1)*a*c*d^2*x+18/f^2*b^2
*ln(1-exp(I*(f*x+e)))*a*c*d^2*x+9/f*b*ln(exp(I*(f*x+e))+1)*a^2*c*d^2*x^2+9/
f*b*ln(1-exp(I*(f*x+e)))*a^2*c*d^2*x^2+9/f*b*ln(1-exp(I*(f*x+e)))*a^2*c^2*d
*x-18/f^3*b^2*a*c*d^2*e*ln(exp(I*(f*x+e))-1)-18/f^4*b^2*a*d^3*e^2*ln(exp(I*
(f*x+e)))+9/f^4*b^2*a*d^3*e^2*ln(exp(I*(f*x+e))-1)-6/f^2*b^3*c^2*d*e*ln(exp
(I*(f*x+e)))+9/f^2*b^2*a*c^2*d*ln(exp(I*(f*x+e))-1)+3/f^2*b^3*c^2*d*e*ln(ex
p(I*(f*x+e))-1)-1/f*b^3*d^3*ln(1-exp(I*(f*x+e)))*x^3-1/f^4*b^3*d^3*ln(1-exp
(I*(f*x+e)))*e^3-1/f*b^3*d^3*ln(exp(I*(f*x+e))+1)*x^3+1/4*I*b^3*d^3*x^4-6*I
/f*b^2*d^3*a*x^3+3*I/f^2*b^3*c^2*d*e^2+18*I/f^4*b*a^2*d^3*polylog(4,-exp(I*
(f*x+e)))+18*I/f^4*b*a^2*d^3*polylog(4,exp(I*(f*x+e)))-9/2*I/f^4*b*a^2*d^3*
e^4+2*I/f^3*b^3*d^3*e^3*x-6*I/f^3*b^3*d^3*e*x-4*I/f^3*b^3*c*d^2*e^3-1/4*I/d
*b^3*c^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10500 vs. 2(537) = 1074.
time = 15.69, size = 10500, normalized size = 17.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a^3*c^3 + (f*x + e)^4*a^3*d^3/f^3 + 4*(f*x + e)^3*a^3*c*d^
2/f^2 + 6*(f*x + e)^2*a^3*c^2*d/f - 4*(f*x + e)^3*a^3*d^3*e/f^3 - 12*(f*x +
 e)^2*a^3*c*d^2*e/f^2 - 12*(f*x + e)*a^3*c^2*d*e/f + 12*a^2*b*c^3*log(sin(f
*x + e)) - 36*a^2*b*c^2*d*e*log(sin(f*x + e))/f + 6*(f*x + e)^2*a^3*d^3*e^2
/f^3 + 12*(f*x + e)*a^3*c*d^2*e^2/f^2 + 36*a^2*b*c*d^2*e^2*log(sin(f*x + e)
)/f^2 - 4*(f*x + e)*a^3*d^3*e^3/f^3 - 12*a^2*b*d^3*e^3*log(sin(f*x + e))/f^
3 + 4*(24*a*b^2*c^3*f^3 - (3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^4*d^3 - 12*
(6*a*b^2*e - b^3)*c^2*d*f^2 + 24*(3*a*b^2*e^2 - b^3*e)*c*d^2*f - 4*((3*a^2*
b - 3*I*a*b^2 - b^3)*c*d^2*f - (3*a^2*b*e - 3*I*a*b^2*e - b^3*e)*d^3)*(f*x
+ e)^3 - 12*(2*a*b^2*e^3 - b^3*e^2)*d^3 - 6*((3*a^2*b - 3*I*a*b^2 - b^3)*c^
2*d*f^2 - 2*(3*a^2*b*e - 3*I*a*b^2*e - b^3*e)*c*d^2*f + (3*a^2*b*e^2 - 3*I*
a*b^2*e^2 - b^3*e^2)*d^3)*(f*x + e)^2 + 4*((3*I*a*b^2 + b^3)*c^3*f^3 + 3*(-
3*I*a*b^2*e - b^3*e)*c^2*d*f^2 + 3*(3*I*a*b^2*e^2 + b^3*e^2)*c*d^2*f + (-3*
I*a*b^2*e^3 - b^3*e^3)*d^3)*(f*x + e) - 4*(b^3*c^3*f^3 - (3*a^2*b - b^3)*(f
*x + e)^3*d^3 - 3*(b^3*e + 3*a*b^2)*c^2*d*f^2 + 3*(b^3*(e^2 - 1) + 6*a*b^2*
e)*c*d^2*f - (b^3*(e^3 - 3*e) + 9*a*b^2*e^2)*d^3 - 3*((3*a^2*b - b^3)*c*d^2
*f - (3*a^2*b*e - b^3*e - 3*a*b^2)*d^3)*(f*x + e)^2 - 3*((3*a^2*b - b^3)*c^
2*d*f^2 - 2*(3*a^2*b*e - b^3*e - 3*a*b^2)*c*d^2*f - (b^3*(e^2 - 1) - 3*a^2*
b*e^2 + 6*a*b^2*e)*d^3)*(f*x + e) + (b^3*c^3*f^3 - (3*a^2*b - b^3)*(f*x + e
)^3*d^3 - 3*(b^3*e + 3*a*b^2)*c^2*d*f^2 + 3*(b^3*(e^2 - 1) + 6*a*b^2*e)*c*d
^2*f - (b^3*(e^3 - 3*e) + 9*a*b^2*e^2)*d^3 - 3*((3*a^2*b - b^3)*c*d^2*f - (
3*a^2*b*e - b^3*e - 3*a*b^2)*d^3)*(f*x + e)^2 - 3*((3*a^2*b - b^3)*c^2*d*f^
2 - 2*(3*a^2*b*e - b^3*e - 3*a*b^2)*c*d^2*f - (b^3*(e^2 - 1) - 3*a^2*b*e^2
+ 6*a*b^2*e)*d^3)*(f*x + e))*cos(4*f*x + 4*e) - 2*(b^3*c^3*f^3 - (3*a^2*b -
 b^3)*(f*x + e)^3*d^3 - 3*(b^3*e + 3*a*b^2)*c^2*d*f^2 + 3*(b^3*(e^2 - 1) +
6*a*b^2*e)*c*d^2*f - (b^3*(e^3 - 3*e) + 9*a*b^2*e^2)*d^3 - 3*((3*a^2*b - b^
3)*c*d^2*f - (3*a^2*b*e - b^3*e - 3*a*b^2)*d^3)*(f*x + e)^2 - 3*((3*a^2*b -
 b^3)*c^2*d*f^2 - 2*(3*a^2*b*e - b^3*e - 3*a*b^2)*c*d^2*f - (b^3*(e^2 - 1)
- 3*a^2*b*e^2 + 6*a*b^2*e)*d^3)*(f*x + e))*cos(2*f*x + 2*e) - (-I*b^3*c^3*f
^3 + (3*I*a^2*b - I*b^3)*(f*x + e)^3*d^3 + 3*(I*b^3*e + 3*I*a*b^2)*c^2*d*f^
2 + 3*(b^3*(-I*e^2 + I) - 6*I*a*b^2*e)*c*d^2*f + (b^3*(I*e^3 - 3*I*e) + 9*I
*a*b^2*e^2)*d^3 + 3*((3*I*a^2*b - I*b^3)*c*d^2*f + (-3*I*a^2*b*e + I*b^3*e
+ 3*I*a*b^2)*d^3)*(f*x + e)^2 + 3*((3*I*a^2*b - I*b^3)*c^2*d*f^2 + 2*(-3*I*
a^2*b*e + I*b^3*e + 3*I*a*b^2)*c*d^2*f + (b^3*(-I*e^2 + I) + 3*I*a^2*b*e^2
- 6*I*a*b^2*e)*d^3)*(f*x + e))*sin(4*f*x + 4*e) - 2*(I*b^3*c^3*f^3 + (-3*I*
a^2*b + I*b^3)*(f*x + e)^3*d^3 + 3*(-I*b^3*e - 3*I*a*b^2)*c^2*d*f^2 + 3*(b^
```

```
3*(I*e^2 - I) + 6*I*a*b^2*e)*c*d^2*f + (b^3*(-I*e^3 + 3*I*e) - 9*I*a*b^2*e^
2)*d^3 + 3*((-3*I*a^2*b + I*b^3)*c*d^2*f + (3*I*a^2*b*e - I*b^3*e - 3*I*a*b
^2)*d^3)*(f*x + e)^2 + 3*((-3*I*a^2*b + I*b^3)*c^2*d*f^2 + 2*(3*I*a^2*b*e -
 I*b^3*e - 3*I*a*b^2)*c*d^2*f + (b^3*(I*e^2 - I) - 3*I*a^2*b*e^2 + 6*I*a*b^
2*e)*d^3)*(f*x + e))*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), cos(f*x + e) +
 1) - 4*(b^3*c^3*f^3 - 3*(b^3*e + 3*a*b^2)*c^2*d*f^2 + 3*(b^3*(e^2 - 1) + 6
*a*b^2*e)*c*d^2*f - (b^3*(e^3 - 3*e) + 9*a*b^2*e^2)*d^3 + (b^3*c^3*f^3 - 3*
(b^3*e + 3*a*b^2)*c^2*d*f^2 + 3*(b^3*(e^2 - 1) + 6*a*b^2*e)*c*d^2*f - (b^3*
(e^3 - 3*e) + 9*a*b^2*e^2)*d^3)*cos(4*f*x + 4*e) - 2*(b^3*c^3*f^3 - 3*(b^3*
e + 3*a*b^2)*c^2*d*f^2 + 3*(b^3*(e^2 - 1) + 6*a*b^2*e)*c*d^2*f - (b^3*(e^3
- 3*e) + 9*a*b^2*e^2)*d^3)*cos(2*f*x + 2*e) - (-I*b^3*c^3*f^3 + 3*(I*b^3*e
+ 3*I*a*b^2)*c^2*d*f^2 + 3*(b^3*(-I*e^2 + I) - 6*I*a*b^2*e)*c*d^2*f + (b^3*
(I*e^3 - 3*I*e) + 9*I*a*b^2*e^2)*d^3)*sin(4*f*x + 4*e) - 2*(I*b^3*c^3*f^3 +
 3*(-I*b^3*e - 3*I*a*b^2)*c^2*d*f^2 + 3*(b^3*(I*e^2 - I) + 6*I*a*b^2*e)*c*d
^2*f + (b^3*(-I*e^3 + 3*I*e) - 9*I*a*b^2*e^2)*d^3)*sin(2*f*x + 2*e))*arctan
2(sin(f*x + e), cos(f*x + e) - 1) - 4*((3*a^2*b - b^3)*(f*x + e)^3*d^3 + 3*
((3*a^2*b - b^3)*c*d^2*f - (3*a^2*b*e - b^3*e - 3*a*b^2)*d^3)*(f*x + e)^2 +
 3*((3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a^2*b*e - b^3*e - 3*a*b^2)*c*d^2*f - (
b^3*(e^2 - 1) - 3*a^2*b*e^2 + 6*a*b^2*e)*d^3)*(f*x + e) + ((3*a^2*b - b^3)*
(f*x + e)^3*d^3 + 3*((3*a^2*b - b^3)*c*d^2*f - (3*a^2*b*e - b^3*e - 3*a*b^2
)*d^3)*(f*x + e)^2 + 3*((3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a^2*b*e - b^3*e -
3*a*b^2)*c*d^2*f - (b^3*(e^2 - 1) - 3*a^2*b*e^2 + 6*a*b^2*e)*d^3)*(f*x + e)
)*cos(4*f*x + 4*e) - 2*((3*a^2*b - b^3)*(f*x + e)^3*d^3 + 3*((3*a^2*b - b^3
)*c*d^2*f - (3*a^2*b*e - b^3*e - 3*a*b^2)*d^3)*(f*x + e)^2 + 3*((3*a^2*b -
b^3)*c^2*d*f^2 - 2*(3*a^2*b*e - b^3*e - 3*a*b^2)*c*d^2*f - (b^3*(e^2 - 1) -
 3*a^2*b*e^2 + 6*a*b^2*e)*d^3)*(f*x + e))*cos(2*f*x + 2*e) - ((-3*I*a^2*b +
 I*b^3)*(f*x + e)^3*d^3 + 3*((-3*I*a^2*b + I*b^3)*c*d^2*f + (3*I*a^2*b*e -
I*b^3*e - 3*I*a*b^2)*d^3)*(f*x + e)^2 + 3*((-3*I*a^2*b + I*b^3)*c^2*d*f^2 +
 2*(3*I*a^2*b*e - I*b^3*e - 3*I*a*b^2)*c*d^2*f + (b^3*(I*e^2 - I) - 3*I*a^2
*b*e^2 + 6*I*a*b^2*e)*d^3)*(f*x + e))*sin(4*f*x + 4*e) - 2*((3*I*a^2*b - I*
b^3)*(f*x + e)^3*d^3 + 3*((3*I*a^2*b - I*b^3)*c...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2746 vs. $2(537) = 1074$.
time = 2.80, size = 2746, normalized size = 4.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(a^3 - 3*a*b^2)*d^3*f^4*x^4 - 8*b^3*c^3*f^3 - 8*(b^3*d^3*f^3 - (a^3
 - 3*a*b^2)*c*d^2*f^4)*x^3 - 12*(2*b^3*c*d^2*f^3 - (a^3 - 3*a*b^2)*c^2*d*f^
4)*x^2 - 8*(3*b^3*c^2*d*f^3 - (a^3 - 3*a*b^2)*c^3*f^4)*x - 2*((a^3 - 3*a*b^
2)*d^3*f^4*x^4 + 4*(a^3 - 3*a*b^2)*c*d^2*f^4*x^3 + 6*(a^3 - 3*a*b^2)*c^2*d*
```

f^4*x^2 + 4*(a^3 - 3*a*b^2)*c^3*f^4*x)*cos(2*f*x + 2*e) + 6*(-I*(3*a^2*b - b^3)*d^3*f^2*x^2 - 6*I*a*b^2*c*d^2*f - I*b^3*d^3 - I*(3*a^2*b - b^3)*c^2*d*f^2 - 2*I*(3*a*b^2*d^3*f + (3*a^2*b - b^3)*c*d^2*f^2)*x + (I*(3*a^2*b - b^3)*d^3*f^2*x^2 + 6*I*a*b^2*c*d^2*f + I*b^3*d^3 + I*(3*a^2*b - b^3)*c^2*d*f^2 + 2*I*(3*a*b^2*d^3*f + (3*a^2*b - b^3)*c*d^2*f^2)*x)*cos(2*f*x + 2*e))*dilog(cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) + 6*(I*(3*a^2*b - b^3)*d^3*f^2*x^2 + 6*I*a*b^2*c*d^2*f + I*b^3*d^3 + I*(3*a^2*b - b^3)*c^2*d*f^2 + 2*I*(3*a*b^2*d^3*f + (3*a^2*b - b^3)*c*d^2*f^2)*x + (-I*(3*a^2*b - b^3)*d^3*f^2*x^2 - 6*I*a*b^2*c*d^2*f - I*b^3*d^3 - I*(3*a^2*b - b^3)*c^2*d*f^2 - 2*I*(3*a*b^2*d^3*f + (3*a^2*b - b^3)*c*d^2*f^2)*x)*cos(2*f*x + 2*e))*dilog(cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) + 4*(9*a*b^2*c^2*d*f^2 + 3*b^3*c*d^2*f + (3*a^2*b - b^3)*c^3*f^3 - (3*a^2*b - b^3)*d^3*e^3 - (9*a*b^2*c^2*d*f^2 + 3*b^3*c*d^2*f + (3*a^2*b - b^3)*c^3*f^3 - (3*a^2*b - b^3)*d^3*e^3 + 3*(3*a*b^2*d^3 + (3*a^2*b - b^3)*c*d^2*f)*e^2 - 3*(6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b - b^3)*c^2*d*f^2)*e)*cos(2*f*x + 2*e) + 3*(3*a*b^2*d^3 + (3*a^2*b - b^3)*c*d^2*f)*e^2 - 3*(6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b - b^3)*c^2*d*f^2)*e)*log(-1/2*cos(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2) + 4*(9*a*b^2*c^2*d*f^2 + 3*b^3*c*d^2*f + (3*a^2*b - b^3)*c^3*f^3 - (3*a^2*b - b^3)*d^3*e^3 - (9*a*b^2*c^2*d*f^2 + 3*b^3*c*d^2*f + (3*a^2*b - b^3)*c^3*f^3 - (3*a^2*b - b^3)*d^3*e^3 + 3*(3*a*b^2*d^3 + (3*a^2*b - b^3)*c*d^2*f)*e^2 - 3*(6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b - b^3)*c^2*d*f^2)*e)*cos(2*f*x + 2*e) + 3*(3*a*b^2*d^3 + (3*a^2*b - b^3)*c*d^2*f)*e^2 - 3*(6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b - b^3)*c^2*d*f^2)*e)*log(-1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2) + 4*((3*a^2*b - b^3)*d^3*f^3*x^3 + (3*a^2*b - b^3)*d^3*e^3 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b - b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b - b^3)*c^2*d*f^3)*x - ((3*a^2*b - b^3)*d^3*f^3*x^3 + (3*a^2*b - b^3)*d^3*e^3 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b - b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b - b^3)*c^2*d*f^3)*x - 3*(3*a*b^2*d^3 + (3*a^2*b - b^3)*c*d^2*f)*e^2 + 3*(6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b - b^3)*c^2*d*f^2)*e)*cos(2*f*x + 2*e) - 3*(3*a*b^2*d^3 + (3*a^2*b - b^3)*c*d^2*f)*e^2 + 3*(6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b - b^3)*c^2*d*f^2)*e)*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1) + 4*((3*a^2*b - b^3)*d^3*f^3*x^3 + (3*a^2*b - b^3)*d^3*e^3 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b - b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b - b^3)*c^2*d*f^3)*x - ((3*a^2*b - b^3)*d^3*f^3*x^3 + (3*a^2*b - b^3)*d^3*e^3 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b - b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b - b^3)*c^2*d*f^3)*x - 3*(3*a*b^2*d^3 + (3*a^2*b - b^3)*c*d^2*f)*e^2 + 3*(6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b - b^3)*c^2*d*f^2)*e)*cos(2*f*x + 2*e) - 3*(3*a*b^2*d^3 + (3*a^2*b - b^3)*c*d^2*f)*e^2 + 3*(6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b - b^3)*c^2*d*f^2)*e)*log(-cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1) + 3*(-I*(3*a^2*b - b^3)*d^3*cos(2*f*x + 2*e) + I*(3*a^2*b - b^3)*d^3)*polylog(4, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) + 3*(I*(3*a^2*b - b^3)*d^3*cos(2*f*x + 2*e) - I*(3*a^2*b - b^3)*d^3)*polylog(4, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) + 6*(3*a*b^2*d^3 + (3*a^2*b - b^3)*d^3*f*x + (3*a^2*b - b^3)*c*d^2*f - (3*a*b^2*d^3 + (3*a^2*b - b^3)*d^3*f*x + (3*a^2

```
*b - b^3)*c*d^2*f)*cos(2*f*x + 2*e))*polylog(3, cos(2*f*x + 2*e) + I*sin(2*
f*x + 2*e)) + 6*(3*a*b^2*d^3 + (3*a^2*b - b^3)*d^3*f*x + (3*a^2*b - b^3)*c*
d^2*f - (3*a*b^2*d^3 + (3*a^2*b - b^3)*d^3*f*x + (3*a^2*b - b^3)*c*d^2*f)*c
os(2*f*x + 2*e))*polylog(3, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) - 12*(2*
a*b^2*d^3*f^3*x^3 + 2*a*b^2*c^3*f^3 + b^3*c^2*d*f^2 + (6*a*b^2*c*d^2*f^3 +
b^3*d^3*f^2)*x^2 + 2*(3*a*b^2*c^2*d*f^3 + b^3*c*d^2*f^2)*x)*sin(2*f*x + 2*e
))/(f^4*cos(2*f*x + 2*e) - f^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cot(e + fx))^3 (c + dx)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*cot(f*x+e))**3,x)
```

```
[Out] Integral((a + b*cot(e + f*x))**3*(c + d*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*(b*cot(f*x + e) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\cot(e + f\,x))^3 \, (c + d\,x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^3*(c + d*x)^3,x)
```

```
[Out] int((a + b*cot(e + f*x))^3*(c + d*x)^3, x)
```

3.48 $\int (c+dx)^2(a+b\cot(e+fx))^3\,dx$

Optimal. Leaf size=433

$$-\frac{b^3cdx}{f}-\frac{b^3d^2x^2}{2f}-\frac{3iab^2(c+dx)^2}{f}+\frac{a^3(c+dx)^3}{3d}-\frac{ia^2b(c+dx)^3}{d}-\frac{ab^2(c+dx)^3}{d}+\frac{ib^3(c+dx)^3}{3d}-\frac{b^3d(c+dx)\cot}{f^2}$$

```
[Out] -b^3*c*d*x/f-1/2*b^3*d^2*x^2/f-I*a^2*b*(d*x+c)^3/d+1/3*a^3*(d*x+c)^3/d-3*I*
a^2*b*d*(d*x+c)*polylog(2,exp(2*I*(f*x+e)))/f^2-a*b^2*(d*x+c)^3/d+1/3*I*b^3
*(d*x+c)^3/d-b^3*d*(d*x+c)*cot(f*x+e)/f^2-3*a*b^2*(d*x+c)^2*cot(f*x+e)/f-1/
2*b^3*(d*x+c)^2*cot(f*x+e)^2/f+6*a*b^2*d*(d*x+c)*ln(1-exp(2*I*(f*x+e)))/f^2
+3*a^2*b*(d*x+c)^2*ln(1-exp(2*I*(f*x+e)))/f-b^3*(d*x+c)^2*ln(1-exp(2*I*(f*x
+e)))/f+b^3*d^2*ln(sin(f*x+e))/f^3-3*I*a*b^2*d^2*polylog(2,exp(2*I*(f*x+e))
)/f^3+I*b^3*d*(d*x+c)*polylog(2,exp(2*I*(f*x+e)))/f^2-3*I*a*b^2*(d*x+c)^2/f
+3/2*a^2*b*d^2*polylog(3,exp(2*I*(f*x+e)))/f^3-1/2*b^3*d^2*polylog(3,exp(2*
I*(f*x+e)))/f^3
```

Rubi [A]
time = 0.45, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3803, 3798, 2221, 2611, 2320, 6724, 3801, 2317, 2438, 32, 3556}

$$\frac{a^3(c+dx)^3}{3d}-\frac{3ia^2bd(c+dx)\mathrm{Li}_2\left(e^{2i(e+fx)}\right)}{f^2}+\frac{3a^2b(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ia^2b(c+dx)^3}{d}+\frac{3a^2bd^2\mathrm{Li}_3\left(e^{2i(e+fx)}\right)}{2f^3}+\frac{6ab^2d(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f^2}-\frac{3ab^2(c+dx)^2\cot(e+fx)}{f}-\frac{3iab^2(c+dx)^2}{f}-\frac{ab^2(c+dx)^3}{d}-\frac{3iab^2d^2\mathrm{Li}_2\left(e^{2i(e+fx)}\right)}{f^3}+\frac{ib^3d(c+dx)\mathrm{Li}_2\left(e^{2i(e+fx)}\right)}{f^2}-\frac{b^3d(c+dx)\cot(e+fx)}{f^2}-\frac{b^3(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{b^3(c+dx)^2\cot^2(e+fx)}{2f}-\frac{b^3cdx}{f}+\frac{ib^3(c+dx)^3}{3d}-\frac{b^3d^2\mathrm{Li}_3\left(e^{2i(e+fx)}\right)}{2f^3}+\frac{b^3d^2\log(\sin(e+fx))}{f^3}-\frac{b^3d^2x^2}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Cot[e + f*x])^3,x]
```

```
[Out] -((b^3*c*d*x)/f) - (b^3*d^2*x^2)/(2*f) - ((3*I)*a*b^2*(c + d*x)^2)/f + (a^3
*(c + d*x)^3)/(3*d) - (I*a^2*b*(c + d*x)^3)/d - (a*b^2*(c + d*x)^3)/d + ((I
/3)*b^3*(c + d*x)^3)/d - (b^3*d*(c + d*x)*Cot[e + f*x])/f^2 - (3*a*b^2*(c +
 d*x)^2*Cot[e + f*x])/f - (b^3*(c + d*x)^2*Cot[e + f*x]^2)/(2*f) + (6*a*b^2
*d*(c + d*x)*Log[1 - E^((2*I)*(e + f*x))])/f^2 + (3*a^2*b*(c + d*x)^2*Log[1
 - E^((2*I)*(e + f*x))])/f - (b^3*(c + d*x)^2*Log[1 - E^((2*I)*(e + f*x))])
/f + (b^3*d^2*Log[Sin[e + f*x]])/f^3 - ((3*I)*a*b^2*d^2*PolyLog[2, E^((2*I)
*(e + f*x))])/f^3 - ((3*I)*a^2*b*d*(c + d*x)*PolyLog[2, E^((2*I)*(e + f*x))
])/f^2 + (I*b^3*d*(c + d*x)*PolyLog[2, E^((2*I)*(e + f*x))])/f^2 + (3*a^2*b
*d^2*PolyLog[3, E^((2*I)*(e + f*x))])/(2*f^3) - (b^3*d^2*PolyLog[3, E^((2*I
)*(e + f*x))])/(2*f^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2(a+b\cot(e+fx))^3\,dx = \int \left(a^3(c+dx)^2+3a^2b(c+dx)^2\cot(e+fx)+3ab^2(c+dx)^2\cot^2(e+f\right.$$

$$= \frac{a^3(c+dx)^3}{3d}+\left(3a^2b\right)\int (c+dx)^2\cot(e+fx)\,dx+\left(3ab^2\right)\int (c+dx)^2$$

$$= \frac{a^3(c+dx)^3}{3d}-\frac{ia^2b(c+dx)^3}{d}-\frac{3ab^2(c+dx)^2\cot(e+fx)}{f}-\frac{b^3(c+dx}{}$$

$$= -\frac{3iab^2(c+dx)^2}{f}+\frac{a^3(c+dx)^3}{3d}-\frac{ia^2b(c+dx)^3}{d}-\frac{ab^2(c+dx)^3}{d}+\frac{ib^3(}{}$$

$$= -\frac{b^3cdx}{f}-\frac{b^3d^2x^2}{2f}-\frac{3iab^2(c+dx)^2}{f}+\frac{a^3(c+dx)^3}{3d}-\frac{ia^2b(c+dx)^3}{d}-$$

$$= -\frac{b^3cdx}{f}-\frac{b^3d^2x^2}{2f}-\frac{3iab^2(c+dx)^2}{f}+\frac{a^3(c+dx)^3}{3d}-\frac{ia^2b(c+dx)^3}{d}-$$

$$= -\frac{b^3cdx}{f}-\frac{b^3d^2x^2}{2f}-\frac{3iab^2(c+dx)^2}{f}+\frac{a^3(c+dx)^3}{3d}-\frac{ia^2b(c+dx)^3}{d}-$$

$$= -\frac{b^3cdx}{f}-\frac{b^3d^2x^2}{2f}-\frac{3iab^2(c+dx)^2}{f}+\frac{a^3(c+dx)^3}{3d}-\frac{ia^2b(c+dx)^3}{d}-$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1825 vs. 2(433) = 866.

time = 7.44, size = 1825, normalized size = 4.21

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*(a + b*Cot[e + f*x])^3,x]
```

```
[Out] -1/4*(a^2*b*d^2*Csc[e]*(2*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*(-1 + E^((2*I)
*e))*Log[1 - E^((2*I)*(e + f*x))]) + 6*(-1 + E^((2*I)*e))*f*x*PolyLog[2, E^
((2*I)*(e + f*x))] + (3*I)*(-1 + E^((2*I)*e))*PolyLog[3, E^((2*I)*(e + f*x)
)]))/(E^(I*e)*f^3) + (b^3*d^2*Csc[e]*(2*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*
(-1 + E^((2*I)*e))*Log[1 - E^((2*I)*(e + f*x))]) + 6*(-1 + E^((2*I)*e))*f*x
*PolyLog[2, E^((2*I)*(e + f*x))] + (3*I)*(-1 + E^((2*I)*e))*PolyLog[3, E^((
2*I)*(e + f*x))]))/(12*E^(I*e)*f^3) + (b^3*d^2*Csc[e]*(-(f*x*Cos[e]) + Log[
Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(f^3*(Cos[e]^2 + Sin[e]^2)) + (
6*a*b^2*c*d*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*
Sin[e]))/(f^2*(Cos[e]^2 + Sin[e]^2)) + (3*a^2*b*c^2*Csc[e]*(-(f*x*Cos[e]) +
 Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(f*(Cos[e]^2 + Sin[e]^2))
- (b^3*c^2*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*S
in[e]))/(f*(Cos[e]^2 + Sin[e]^2)) + (Csc[e]*Csc[e + f*x]^2*(6*b^3*c*d*Cos[e
] + 18*a*b^2*c^2*f*Cos[e] + 6*b^3*d^2*x*Cos[e] + 36*a*b^2*c*d*f*x*Cos[e] +
18*a^2*b*c^2*f^2*x*Cos[e] - 6*b^3*c^2*f^2*x*Cos[e] + 18*a*b^2*d^2*f*x^2*Cos
[e] + 18*a^2*b*c*d*f^2*x^2*Cos[e] - 6*b^3*c*d*f^2*x^2*Cos[e] + 6*a^2*b*d^2*
f^2*x^3*Cos[e] - 2*b^3*d^2*f^2*x^3*Cos[e] - 6*b^3*c*d*Cos[e + 2*f*x] - 18*a
*b^2*c^2*f*Cos[e + 2*f*x] - 6*b^3*d^2*x*Cos[e + 2*f*x] - 36*a*b^2*c*d*f*x*C
os[e + 2*f*x] - 9*a^2*b*c^2*f^2*x*Cos[e + 2*f*x] + 3*b^3*c^2*f^2*x*Cos[e +
2*f*x] - 18*a*b^2*d^2*f*x^2*Cos[e + 2*f*x] - 9*a^2*b*c*d*f^2*x^2*Cos[e + 2*
f*x] + 3*b^3*c*d*f^2*x^2*Cos[e + 2*f*x] - 3*a^2*b*d^2*f^2*x^3*Cos[e + 2*f*x
] + b^3*d^2*f^2*x^3*Cos[e + 2*f*x] - 9*a^2*b*c^2*f^2*x*Cos[3*e + 2*f*x] + 3
*b^3*c^2*f^2*x*Cos[3*e + 2*f*x] - 9*a^2*b*c*d*f^2*x^2*Cos[3*e + 2*f*x] + 3*
b^3*c*d*f^2*x^2*Cos[3*e + 2*f*x] - 3*a^2*b*d^2*f^2*x^3*Cos[3*e + 2*f*x] + b
^3*d^2*f^2*x^3*Cos[3*e + 2*f*x] - 6*b^3*c^2*f*Sin[e] - 12*b^3*c*d*f*x*Sin[e
] + 6*a^3*c^2*f^2*x*Sin[e] - 18*a*b^2*c^2*f^2*x*Sin[e] - 6*b^3*d^2*f*x^2*Si
n[e] + 6*a^3*c*d*f^2*x^2*Sin[e] - 18*a*b^2*c*d*f^2*x^2*Sin[e] + 2*a^3*d^2*f
^2*x^3*Sin[e] - 6*a*b^2*d^2*f^2*x^3*Sin[e] + 3*a^3*c^2*f^2*x*Sin[e + 2*f*x]
 - 9*a*b^2*c^2*f^2*x*Sin[e + 2*f*x] + 3*a^3*c*d*f^2*x^2*Sin[e + 2*f*x] - 9*
a*b^2*c*d*f^2*x^2*Sin[e + 2*f*x] + a^3*d^2*f^2*x^3*Sin[e + 2*f*x] - 3*a*b^2
*d^2*f^2*x^3*Sin[e + 2*f*x] - 3*a^3*c^2*f^2*x*Sin[3*e + 2*f*x] + 9*a*b^2*c^
2*f^2*x*Sin[3*e + 2*f*x] - 3*a^3*c*d*f^2*x^2*Sin[3*e + 2*f*x] + 9*a*b^2*c*d
*f^2*x^2*Sin[3*e + 2*f*x] - a^3*d^2*f^2*x^3*Sin[3*e + 2*f*x] + 3*a*b^2*d^2*
f^2*x^3*Sin[3*e + 2*f*x]))/(12*f^2) - (3*a*b^2*d^2*Csc[e]*Sec[e]*(E^(I*ArcT
an[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2*
I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]])
)] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]] + I
*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[e])/Sqrt[1 + Tan[e]^2]))
```

```
/(f^3*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)]) - (3*a^2*b*c*d*Csc[e]*Sec[e]*(E
^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 +
 E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[
Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e
]]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[e])/Sqrt[1 + Tan
[e]^2]))/(f^2*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)]) + (b^3*c*d*Csc[e]*Sec[e
]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log
[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x + Arc
Tan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[T
an[e]]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[e])/Sqrt[1 +
 Tan[e]^2]))/(f^2*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1787 vs. $2(399) = 798$.
time = 0.87, size = 1788, normalized size = 4.13

method	result	size
risch	Expression too large to display	1788

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*cot(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] I/d*a^2*b*c^3+3*I*a^2*b*c^2*x-3*I*d*a^2*b*c*x^2-I*d^2*a^2*b*x^3-12*I/f*b*a^
2*c*d*e*x+12/f^2*b*a^2*c*d*e*ln(exp(I*(f*x+e)))-6*I/f^2*b*polylog(2,-exp(I*
(f*x+e)))*a^2*d^2*x-6*I/f^2*b*polylog(2,exp(I*(f*x+e)))*a^2*d^2*x+6*I/f^2*b
*a^2*d^2*e^2*x-6*I/f^2*b*a^2*c*d*e^2+4*I/f*b^3*c*d*e*x-12*I/f^2*b^2*a*d^2*e
*x-6*I/f^2*b*a^2*c*d*polylog(2,exp(I*(f*x+e)))-6*I/f^2*b*a^2*c*d*polylog(2,
-exp(I*(f*x+e)))+3/f^3*b*a^2*d^2*e^2*ln(exp(I*(f*x+e))-1)-6/f^3*b*a^2*d^2*e
^2*ln(exp(I*(f*x+e)))+2*I/f^2*b^3*polylog(2,-exp(I*(f*x+e)))*d^2*x+2*I/f^2*
b^3*polylog(2,exp(I*(f*x+e)))*d^2*x+2*I/f^2*b^3*c*d*polylog(2,exp(I*(f*x+e)
))-1/f*b^3*ln(1-exp(I*(f*x+e)))*d^2*x^2-1/f*b^3*ln(exp(I*(f*x+e))+1)*d^2*x^
2-1/f^3*b^3*d^2*e^2*ln(exp(I*(f*x+e))-1)+2/f^3*b^3*d^2*e^2*ln(exp(I*(f*x+e)
))+3/f*b*a^2*c^2*ln(exp(I*(f*x+e))-1)+1/f^3*b^3*ln(1-exp(I*(f*x+e)))*d^2*e^
2+I*b^3*c*d*x^2-6/f*b*a^2*c^2*ln(exp(I*(f*x+e)))+3/f*b*a^2*c^2*ln(exp(I*(f*
x+e))+1)+6/f*b*ln(exp(I*(f*x+e))+1)*a^2*c*d*x+6/f*b*ln(1-exp(I*(f*x+e)))*a^
2*c*d*x+6/f^2*b*ln(1-exp(I*(f*x+e)))*a^2*c*d*e-6/f^2*b*a^2*c*d*e*ln(exp(I*(
f*x+e))-1)+6/f^3*b^2*ln(1-exp(I*(f*x+e)))*a*d^2*e+3/f*b*ln(1-exp(I*(f*x+e))
)*a^2*d^2*x^2+3/f*b*ln(exp(I*(f*x+e))+1)*a^2*d^2*x^2+6/f^2*b^2*ln(exp(I*(f*
x+e))+1)*a*d^2*x-2/f*b^3*ln(exp(I*(f*x+e))+1)*c*d*x-3/f^3*b*ln(1-exp(I*(f*x
+e)))*a^2*d^2*e^2-3*d*a*b^2*c*x^2+6/f^3*b*a^2*d^2*polylog(3,-exp(I*(f*x+e))
)+6/f^3*b*a^2*d^2*polylog(3,exp(I*(f*x+e)))-4/3*I/f^3*b^3*d^2*e^3-6*I/f^3*b
^2*a*d^2*e^2-6*I/f^3*b^2*a*d^2*polylog(2,-exp(I*(f*x+e)))-6*I/f^3*b^2*a*d^2
*polylog(2,exp(I*(f*x+e)))+2*I/f^2*b^3*c*d*e^2-2*I/f^2*b^3*d^2*e^2*x-6*I/f*
b^2*a*d^2*x^2+2*I/f^2*b^3*c*d*polylog(2,-exp(I*(f*x+e)))+4*I/f^3*b*a^2*d^2*
e^3-2/f*b^3*ln(1-exp(I*(f*x+e)))*c*d*x-2/f^2*b^3*ln(1-exp(I*(f*x+e)))*c*d*e
```

```
+6/f^2*b^2*ln(1-exp(I*(f*x+e)))*a*d^2*x+6/f^2*b^2*a*c*d*ln(exp(I*(f*x+e))+1
)+6/f^2*b^2*a*c*d*ln(exp(I*(f*x+e))-1)-12/f^2*b^2*a*c*d*ln(exp(I*(f*x+e)))+
2/f^2*b^3*c*d*e*ln(exp(I*(f*x+e))-1)-4/f^2*b^3*c*d*e*ln(exp(I*(f*x+e)))+12/
f^3*b^2*a*d^2*e*ln(exp(I*(f*x+e)))-6/f^3*b^2*a*d^2*e*ln(exp(I*(f*x+e))-1)+1
/3*d^2*a^3*x^3+1/3/d*a^3*c^3+2*b^2*(-3*I*a*d^2*f*x^2*exp(2*I*(f*x+e))-6*I*a
*c*d*f*x*exp(2*I*(f*x+e))+b*d^2*f*x^2*exp(2*I*(f*x+e))-3*I*a*c^2*f*exp(2*I*
(f*x+e))+3*I*a*d^2*f*x^2-I*b*d^2*x*exp(2*I*(f*x+e))+2*b*c*d*f*x*exp(2*I*(f*
x+e))+6*I*a*c*d*f*x-I*c*b*d*exp(2*I*(f*x+e))+b*c^2*f*exp(2*I*(f*x+e))+3*I*a
*c^2*f+I*b*d^2*x+I*c*b*d)/f^2/(exp(2*I*(f*x+e))-1)^2-3*a*b^2*c^2*x-1/d*a*b^
2*c^3-1/3*I/d*b^3*c^3-1/f*b^3*c^2*ln(exp(I*(f*x+e))-1)-2/f^3*b^3*d^2*polylo
g(3,exp(I*(f*x+e)))+1/f^3*b^3*d^2*ln(exp(I*(f*x+e))-1)-2/f^3*b^3*d^2*ln(exp
(I*(f*x+e)))+1/f^3*b^3*d^2*ln(exp(I*(f*x+e))+1)-2/f^3*b^3*d^2*polylog(3,-ex
p(I*(f*x+e)))-1/f*b^3*c^2*ln(exp(I*(f*x+e))+1)+2/f*b^3*c^2*ln(exp(I*(f*x+e)
))+1/3*I*b^3*d^2*x^3+d*a^3*c*x^2+a^3*c^2*x-d^2*a*b^2*x^3-I*b^3*c^2*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5334 vs. $2(402) = 804$.
time = 3.14, size = 5334, normalized size = 12.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a^3*c^2 + (f*x + e)^3*a^3*d^2/f^2 + 3*(f*x + e)^2*a^3*c*d/
f - 3*(f*x + e)^2*a^3*d^2*e/f^2 - 6*(f*x + e)*a^3*c*d*e/f + 9*a^2*b*c^2*log
(sin(f*x + e)) - 18*a^2*b*c*d*e*log(sin(f*x + e))/f + 3*(f*x + e)*a^3*d^2*e
^2/f^2 + 9*a^2*b*d^2*e^2*log(sin(f*x + e))/f^2 + 3*(36*a*b^2*c^2*f^2 - 2*(3
*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^3*d^2 - 12*(6*a*b^2*e - b^3)*c*d*f - 6*
((3*a^2*b - 3*I*a*b^2 - b^3)*c*d*f - (3*a^2*b*e - 3*I*a*b^2*e - b^3*e)*d^2)
*(f*x + e)^2 + 12*(3*a*b^2*e^2 - b^3*e)*d^2 + 6*((3*I*a*b^2 + b^3)*c^2*f^2
+ 2*(-3*I*a*b^2*e - b^3*e)*c*d*f + (3*I*a*b^2*e^2 + b^3*e^2)*d^2)*(f*x + e)
 - 6*(b^3*c^2*f^2 - (3*a^2*b - b^3)*(f*x + e)^2*d^2 - 2*(b^3*e + 3*a*b^2)*c
*d*f + (b^3*(e^2 - 1) + 6*a*b^2*e)*d^2 - 2*((3*a^2*b - b^3)*c*d*f - (3*a^2*
b*e - b^3*e - 3*a*b^2)*d^2)*(f*x + e) + (b^3*c^2*f^2 - (3*a^2*b - b^3)*(f*x
 + e)^2*d^2 - 2*(b^3*e + 3*a*b^2)*c*d*f + (b^3*(e^2 - 1) + 6*a*b^2*e)*d^2 -
 2*((3*a^2*b - b^3)*c*d*f - (3*a^2*b*e - b^3*e - 3*a*b^2)*d^2)*(f*x + e))*c
os(4*f*x + 4*e) - 2*(b^3*c^2*f^2 - (3*a^2*b - b^3)*(f*x + e)^2*d^2 - 2*(b^3
*e + 3*a*b^2)*c*d*f + (b^3*(e^2 - 1) + 6*a*b^2*e)*d^2 - 2*((3*a^2*b - b^3)*
c*d*f - (3*a^2*b*e - b^3*e - 3*a*b^2)*d^2)*(f*x + e))*cos(2*f*x + 2*e) - (-
I*b^3*c^2*f^2 + (3*I*a^2*b - I*b^3)*(f*x + e)^2*d^2 + 2*(I*b^3*e + 3*I*a*b^
2)*c*d*f + (b^3*(-I*e^2 + I) - 6*I*a*b^2*e)*d^2 + 2*((3*I*a^2*b - I*b^3)*c*
d*f + (-3*I*a^2*b*e + I*b^3*e + 3*I*a*b^2)*d^2)*(f*x + e))*sin(4*f*x + 4*e)
 - 2*(I*b^3*c^2*f^2 + (-3*I*a^2*b + I*b^3)*(f*x + e)^2*d^2 + 2*(-I*b^3*e -
3*I*a*b^2)*c*d*f + (b^3*(I*e^2 - I) + 6*I*a*b^2*e)*d^2 + 2*((-3*I*a^2*b + I
```

```
*b^3)*c*d*f + (3*I*a^2*b*e - I*b^3*e - 3*I*a*b^2)*d^2)*(f*x + e))*sin(2*f*x
 + 2*e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) - 6*(b^3*c^2*f^2 - 2*(b^3*
e + 3*a*b^2)*c*d*f + (b^3*(e^2 - 1) + 6*a*b^2*e)*d^2 + (b^3*c^2*f^2 - 2*(b^
3*e + 3*a*b^2)*c*d*f + (b^3*(e^2 - 1) + 6*a*b^2*e)*d^2)*cos(4*f*x + 4*e) -
2*(b^3*c^2*f^2 - 2*(b^3*e + 3*a*b^2)*c*d*f + (b^3*(e^2 - 1) + 6*a*b^2*e)*d^
2)*cos(2*f*x + 2*e) - (-I*b^3*c^2*f^2 + 2*(I*b^3*e + 3*I*a*b^2)*c*d*f + (b^
3*(-I*e^2 + I) - 6*I*a*b^2*e)*d^2)*sin(4*f*x + 4*e) - 2*(I*b^3*c^2*f^2 + 2*
(-I*b^3*e - 3*I*a*b^2)*c*d*f + (b^3*(I*e^2 - I) + 6*I*a*b^2*e)*d^2)*sin(2*f
*x + 2*e))*arctan2(sin(f*x + e), cos(f*x + e) - 1) - 6*((3*a^2*b - b^3)*(f*
x + e)^2*d^2 + 2*((3*a^2*b - b^3)*c*d*f - (3*a^2*b*e - b^3*e - 3*a*b^2)*d^2
)*(f*x + e) + ((3*a^2*b - b^3)*(f*x + e)^2*d^2 + 2*((3*a^2*b - b^3)*c*d*f -
 (3*a^2*b*e - b^3*e - 3*a*b^2)*d^2)*(f*x + e))*cos(4*f*x + 4*e) - 2*((3*a^2
*b - b^3)*(f*x + e)^2*d^2 + 2*((3*a^2*b - b^3)*c*d*f - (3*a^2*b*e - b^3*e -
 3*a*b^2)*d^2)*(f*x + e))*cos(2*f*x + 2*e) - ((-3*I*a^2*b + I*b^3)*(f*x + e
)^2*d^2 + 2*((-3*I*a^2*b + I*b^3)*c*d*f + (3*I*a^2*b*e - I*b^3*e - 3*I*a*b^
2)*d^2)*(f*x + e))*sin(4*f*x + 4*e) - 2*((3*I*a^2*b - I*b^3)*(f*x + e)^2*d^
2 + 2*((3*I*a^2*b - I*b^3)*c*d*f + (-3*I*a^2*b*e + I*b^3*e + 3*I*a*b^2)*d^2
)*(f*x + e))*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), -cos(f*x + e) + 1) - 2
*((3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^3*d^2 + 3*((3*a^2*b - 3*I*a*b^2 - b
^3)*c*d*f - (3*a*b^2*(-I*e - 2) + 3*a^2*b*e - b^3*e)*d^2)*(f*x + e)^2 - 3*(
(3*I*a*b^2 + b^3)*c^2*f^2 + 2*(3*a*b^2*(-I*e - 2) - b^3*e)*c*d*f + (b^3*(e^
2 - 2) + 3*a*b^2*(I*e^2 + 4*e))*d^2)*(f*x + e))*cos(4*f*x + 4*e) + 4*((3*a^
2*b - 3*I*a*b^2 - b^3)*(f*x + e)^3*d^2 - 3*(3*a*b^2 + I*b^3)*c^2*f^2 + 3*(b
^3*(2*I*e - 1) + 6*a*b^2*e)*c*d*f + 3*((3*a^2*b - 3*I*a*b^2 - b^3)*c*d*f +
(b^3*(e - I) + 3*a*b^2*(I*e + 1) - 3*a^2*b*e)*d^2)*(f*x + e)^2 + 3*(b^3*(-I
*e^2 + e) - 3*a*b^2*e^2)*d^2 + 3*((-3*I*a*b^2 - b^3)*c^2*f^2 + 2*(b^3*(e -
I) + 3*a*b^2*(I*e + 1))*c*d*f - (b^3*(e^2 - 2*I*e - 1) - 3*a*b^2*(-I*e^2 -
2*e))*d^2)*(f*x + e))*cos(2*f*x + 2*e) - 12*((3*a^2*b - b^3)*(f*x + e)*d^2
+ (3*a^2*b - b^3)*c*d*f - (3*a^2*b*e - b^3*e - 3*a*b^2)*d^2 + ((3*a^2*b - b
^3)*(f*x + e)*d^2 + (3*a^2*b - b^3)*c*d*f - (3*a^2*b*e - b^3*e - 3*a*b^2)*d
^2)*cos(4*f*x + 4*e) - 2*((3*a^2*b - b^3)*(f*x + e)*d^2 + (3*a^2*b - b^3)*c
*d*f - (3*a^2*b*e - b^3*e - 3*a*b^2)*d^2)*cos(2*f*x + 2*e) - ((-3*I*a^2*b +
 I*b^3)*(f*x + e)*d^2 + (-3*I*a^2*b + I*b^3)*c*d*f + (3*I*a^2*b*e - I*b^3*e
 - 3*I*a*b^2)*d^2)*sin(4*f*x + 4*e) - 2*((3*I*a^2*b - I*b^3)*(f*x + e)*d^2
+ (3*I*a^2*b - I*b^3)*c*d*f + (-3*I*a^2*b*e + I*b^3*e + 3*I*a*b^2)*d^2)*sin
(2*f*x + 2*e))*dilog(-e^(I*f*x + I*e)) - 12*((3*a^2*b - b^3)*(f*x + e)*d^2
+ (3*a^2*b - b^3)*c*d*f - (3*a^2*b*e - b^3*e - 3*a*b^2)*d^2 + ((3*a^2*b - b
^3)*(f*x + e)*d^2 + (3*a^2*b - b^3)*c*d*f - (3*a^2*b*e - b^3*e - 3*a*b^2)*d
^2)*cos(4*f*x + 4*e) - 2*((3*a^2*b - b^3)*(f*x + e)*d^2 + (3*a^2*b - b^3)*c
*d*f - (3*a^2*b*e - b^3*e - 3*a*b^2)*d^2)*cos(2*f*x + 2*e) - ((-3*I*a^2*b +
 I*b^3)*(f*x + e)*d^2 + (-3*I*a^2*b + I*b^3)*c*d*f + (3*I*a^2*b*e - I*b^3*e
 - 3*I*a*b^2)*d^2)*sin(4*f*x + 4*e) - 2*((3*I*a^2*b - I*b^3)*(f*x + e)*d^2
+ (3*I*a^2*b - I*b^3)*c*d*f + (-3*I*a^2*b*e + I*b^3*e + 3*I*a*b^2)*d^2)*sin
(2*f*x + 2*e))*dilog(e^(I*f*x + I*e)) + 3*(I*b^3*c^2*f^2 + (-3*I*a^2*b + I*
b^3)*(f*x + e)^2*d^2 + 2*(-I*b^3*e - 3*I*a*b^2)*c*d*f + (b^3*(I*e^2 - I) +
```

```
6*I*a*b^2*e)*d^2 + 2*((-3*I*a^2*b + I*b^3)*c*d*...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1579 vs. 2(402) = 804.
time = 5.48, size = 1579, normalized size = 3.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(4*(a^3 - 3*a*b^2)*d^2*f^3*x^3 - 12*b^3*c^2*f^2 - 12*(b^3*d^2*f^2 - (
a^3 - 3*a*b^2)*c*d*f^3)*x^2 - 12*(2*b^3*c*d*f^2 - (a^3 - 3*a*b^2)*c^2*f^3)*
x - 4*((a^3 - 3*a*b^2)*d^2*f^3*x^3 + 3*(a^3 - 3*a*b^2)*c*d*f^3*x^2 + 3*(a^3
 - 3*a*b^2)*c^2*f^3*x)*cos(2*f*x + 2*e) + 6*(-3*I*a*b^2*d^2 - I*(3*a^2*b -
b^3)*d^2*f*x - I*(3*a^2*b - b^3)*c*d*f + (3*I*a*b^2*d^2 + I*(3*a^2*b - b^3)
*d^2*f*x + I*(3*a^2*b - b^3)*c*d*f)*cos(2*f*x + 2*e))*dilog(cos(2*f*x + 2*e
) + I*sin(2*f*x + 2*e)) + 6*(3*I*a*b^2*d^2 + I*(3*a^2*b - b^3)*d^2*f*x + I*
(3*a^2*b - b^3)*c*d*f + (-3*I*a*b^2*d^2 - I*(3*a^2*b - b^3)*d^2*f*x - I*(3*
a^2*b - b^3)*c*d*f)*cos(2*f*x + 2*e))*dilog(cos(2*f*x + 2*e) - I*sin(2*f*x
+ 2*e)) + 6*(6*a*b^2*c*d*f + b^3*d^2 + (3*a^2*b - b^3)*c^2*f^2 + (3*a^2*b -
 b^3)*d^2*e^2 - (6*a*b^2*c*d*f + b^3*d^2 + (3*a^2*b - b^3)*c^2*f^2 + (3*a^2
*b - b^3)*d^2*e^2 - 2*(3*a*b^2*d^2 + (3*a^2*b - b^3)*c*d*f)*e)*cos(2*f*x +
2*e) - 2*(3*a*b^2*d^2 + (3*a^2*b - b^3)*c*d*f)*e)*log(-1/2*cos(2*f*x + 2*e)
 + 1/2*I*sin(2*f*x + 2*e) + 1/2) + 6*(6*a*b^2*c*d*f + b^3*d^2 + (3*a^2*b -
b^3)*c^2*f^2 + (3*a^2*b - b^3)*d^2*e^2 - (6*a*b^2*c*d*f + b^3*d^2 + (3*a^2*
b - b^3)*c^2*f^2 + (3*a^2*b - b^3)*d^2*e^2 - 2*(3*a*b^2*d^2 + (3*a^2*b - b^
3)*c*d*f)*e)*cos(2*f*x + 2*e) - 2*(3*a*b^2*d^2 + (3*a^2*b - b^3)*c*d*f)*e)*
log(-1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2) + 6*((3*a^2*b - b
^3)*d^2*f^2*x^2 - (3*a^2*b - b^3)*d^2*e^2 + 2*(3*a*b^2*d^2*f + (3*a^2*b - b
^3)*c*d*f^2)*x - ((3*a^2*b - b^3)*d^2*f^2*x^2 - (3*a^2*b - b^3)*d^2*e^2 + 2
*(3*a*b^2*d^2*f + (3*a^2*b - b^3)*c*d*f^2)*x + 2*(3*a*b^2*d^2 + (3*a^2*b -
b^3)*c*d*f)*e)*cos(2*f*x + 2*e) + 2*(3*a*b^2*d^2 + (3*a^2*b - b^3)*c*d*f)*e
)*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1) + 6*((3*a^2*b - b^3)*d^2*
f^2*x^2 - (3*a^2*b - b^3)*d^2*e^2 + 2*(3*a*b^2*d^2*f + (3*a^2*b - b^3)*c*d*
f^2)*x - ((3*a^2*b - b^3)*d^2*f^2*x^2 - (3*a^2*b - b^3)*d^2*e^2 + 2*(3*a*b^
2*d^2*f + (3*a^2*b - b^3)*c*d*f^2)*x + 2*(3*a*b^2*d^2 + (3*a^2*b - b^3)*c*d
*f)*e)*cos(2*f*x + 2*e) + 2*(3*a*b^2*d^2 + (3*a^2*b - b^3)*c*d*f)*e)*log(-c
os(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1) - 3*((3*a^2*b - b^3)*d^2*cos(2*f*
x + 2*e) - (3*a^2*b - b^3)*d^2)*polylog(3, cos(2*f*x + 2*e) + I*sin(2*f*x +
 2*e)) - 3*((3*a^2*b - b^3)*d^2*cos(2*f*x + 2*e) - (3*a^2*b - b^3)*d^2)*pol
ylog(3, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) - 12*(3*a*b^2*d^2*f^2*x^2 +
3*a*b^2*c^2*f^2 + b^3*c*d*f + (6*a*b^2*c*d*f^2 + b^3*d^2*f)*x)*sin(2*f*x +
2*e))/(f^3*cos(2*f*x + 2*e) - f^3)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cot(e+fx))^3 (c+dx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*cot(f*x+e))**3,x)
[Out] Integral((a + b*cot(e + f*x))**3*(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*x + c)^2*(b*cot(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\cot(e+fx))^3 (c+dx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^3*(c + d*x)^2,x)
[Out] int((a + b*cot(e + f*x))^3*(c + d*x)^2, x)
```

3.49 $\int (c+dx)(a+b\cot(e+fx))^3\,dx$

Optimal. Leaf size=278

$$-3ab^2cx-\frac{b^3dx}{2f}-\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}-\frac{3ia^2b(c+dx)^2}{2d}+\frac{ib^3(c+dx)^2}{2d}-\frac{b^3d\cot(e+fx)}{2f^2}-\frac{3ab^2(c+dx)\cot(}{f}$$

```
[Out] -3*a*b^2*c*x-1/2*b^3*d*x/f-3/2*a*b^2*d*x^2+1/2*a^3*(d*x+c)^2/d-3/2*I*a^2*b*
(d*x+c)^2/d+1/2*I*b^3*(d*x+c)^2/d-1/2*b^3*d*cot(f*x+e)/f^2-3*a*b^2*(d*x+c)*
cot(f*x+e)/f-1/2*b^3*(d*x+c)*cot(f*x+e)^2/f+3*a^2*b*(d*x+c)*ln(1-exp(2*I*(f
*x+e)))/f-b^3*(d*x+c)*ln(1-exp(2*I*(f*x+e)))/f+3*a*b^2*d*ln(sin(f*x+e))/f^2
-3/2*I*a^2*b*d*polylog(2,exp(2*I*(f*x+e)))/f^2+1/2*I*b^3*d*polylog(2,exp(2*
I*(f*x+e)))/f^2
```

Rubi [A]
time = 0.23, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3803, 3798, 2221, 2317, 2438, 3801, 3556, 3554, 8}

$$\frac{a^3(c+dx)^2}{2d}+\frac{3a^2b(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{3ia^2b(c+dx)^2}{2d}-\frac{3ia^2bd\text{Li}_2\left(e^{2i(e+fx)}\right)}{2f^2}-\frac{3ab^2(c+dx)\cot(e+fx)}{f}-3ab^2cx+\frac{3ab^2d\log(\sin(e+fx))}{f^2}-\frac{3}{2}ab^2dx^2-\frac{b^3(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{b^3(c+dx)\cot^2(e+fx)}{2f}+\frac{ib^3(c+dx)^2}{2d}+\frac{ib^3d\text{Li}_2\left(e^{2i(e+fx)}\right)}{2f^2}-\frac{b^3d\cot(e+fx)}{2f^2}-\frac{b^3dx}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Cot[e + f*x])^3,x]
```

```
[Out] -3*a*b^2*c*x - (b^3*d*x)/(2*f) - (3*a*b^2*d*x^2)/2 + (a^3*(c + d*x)^2)/(2*d
) - (((3*I)/2)*a^2*b*(c + d*x)^2)/d + ((I/2)*b^3*(c + d*x)^2)/d - (b^3*d*Co
t[e + f*x])/(2*f^2) - (3*a*b^2*(c + d*x)*Cot[e + f*x])/f - (b^3*(c + d*x)*C
ot[e + f*x]^2)/(2*f) + (3*a^2*b*(c + d*x)*Log[1 - E^((2*I)*(e + f*x))])/f -
 (b^3*(c + d*x)*Log[1 - E^((2*I)*(e + f*x))])/f + (3*a*b^2*d*Log[Sin[e + f*
x]])/f^2 - (((3*I)/2)*a^2*b*d*PolyLog[2, E^((2*I)*(e + f*x))])/f^2 + ((I/2)
*b^3*d*PolyLog[2, E^((2*I)*(e + f*x))])/f^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)(a+b\cot(e+fx))^3\,dx &= \int \left(a^3(c+dx)+3a^2b(c+dx)\cot(e+fx)+3ab^2(c+dx)\cot^2(e+fx\right.\\
&= \frac{a^3(c+dx)^2}{2d}+\left(3a^2b\right)\int (c+dx)\cot(e+fx)\,dx+\left(3ab^2\right)\int (c+dx)\,\mathrm{co}\\
&= \frac{a^3(c+dx)^2}{2d}-\frac{3ia^2b(c+dx)^2}{2d}-\frac{3ab^2(c+dx)\cot(e+fx)}{f}-\frac{b^3(c+dx}{}\\
&= -3ab^2cx-\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}-\frac{3ia^2b(c+dx)^2}{2d}+\frac{ib^3(c+dx)^2}{2d}-\\
&= -3ab^2cx-\frac{b^3dx}{2f}-\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}-\frac{3ia^2b(c+dx)^2}{2d}+\frac{ib^3(c+}{2d}\\
&= -3ab^2cx-\frac{b^3dx}{2f}-\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}-\frac{3ia^2b(c+dx)^2}{2d}+\frac{ib^3(c+}{2d}\\
&= -3ab^2cx-\frac{b^3dx}{2f}-\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}-\frac{3ia^2b(c+dx)^2}{2d}+\frac{ib^3(c+}{2d}
\end{aligned}
$$

Mathematica [A]
time = 3.38, size = 276, normalized size = 0.99

$$
\frac{(a+b\cot(e+fx))^3\sin(e+fx)\left(\left(-\left((e+fx)\left(3ia^2bd(e+fx)-ib^3d(e+fx)+3ab^2(-de+2cf+dfx)+a^3(-2cf+d(e-fx))\right)\right)-2b(-3a^2+b^2)\,d(e+fx)\log\left(1-e^{2i(e+fx)}\right)+2b(3abd+b^2(de-cf)+a^2(-3de+3cf))\log(\sin(e+fx))\right)\sin^2(e+fx)+ib(-3a^2+b^2)\,d\mathrm{PolyLog}\left(2,e^{2i(e+fx)}\right)\sin^2(e+fx)-\frac{1}{2}b^2(2bf(c+dx)+(bd+6af(c+dx))\sin(2(e+fx)))\right)}{2f^2(b\cos(e+fx)+a\sin(e+fx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Cot[e + f*x])^3,x]
```

```
[Out] ((a + b*Cot[e + f*x])^3*Sin[e + f*x]*((-((e + f*x)*((3*I)*a^2*b*d*(e + f*x)
 - I*b^3*d*(e + f*x) + 3*a*b^2*(-(d*e) + 2*c*f + d*f*x) + a^3*(-2*c*f + d*(
e - f*x)))) - 2*b*(-3*a^2 + b^2)*d*(e + f*x)*Log[1 - E^((2*I)*(e + f*x))] +
 2*b*(3*a*b*d + b^2*(d*e - c*f) + a^2*(-3*d*e + 3*c*f))*Log[Sin[e + f*x]])*
Sin[e + f*x]^2 + I*b*(-3*a^2 + b^2)*d*PolyLog[2, E^((2*I)*(e + f*x))]*Sin[e
 + f*x]^2 - (b^2*(2*b*f*(c + d*x) + (b*d + 6*a*f*(c + d*x))*Sin[2*(e + f*x)
]))/2))/(2*f^2*(b*Cos[e + f*x] + a*Sin[e + f*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 744 vs. 2(248) = 496.
time = 0.77, size = 745, normalized size = 2.68

method	result
risch	$-\frac{3b\,a^2de\ln\left(\mathrm{e}^{i(fx+e)}-1\right)}{f^2}+\frac{6b\,a^2de\ln\left(\mathrm{e}^{i(fx+e)}\right)}{f^2}+\frac{ib^3d\,\mathrm{polylog}\left(2,-\mathrm{e}^{i(fx+e)}\right)}{f^2}+\frac{ib^3d\,\mathrm{polylog}\left(2,\mathrm{e}^{i(fx+e)}\right)}{f^2}+3ia^2bcx-\frac{3i}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*cot(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/f^2*b*a^2*d*e*ln(exp(I*(f*x+e))-1)+6/f^2*b*a^2*d*e*ln(exp(I*(f*x+e)))+3/
f^2*b*ln(1-exp(I*(f*x+e)))*a^2*d*e+3/f*b*ln(exp(I*(f*x+e))+1)*a^2*d*x+3/f*b
*ln(1-exp(I*(f*x+e)))*a^2*d*x+2*I/f*b^3*d*e*x-3*I/f^2*b*a^2*d*e^2-3*I/f^2*b
*a^2*d*polylog(2,-exp(I*(f*x+e)))-3*I/f^2*b*a^2*d*polylog(2,exp(I*(f*x+e)))
+3*I*a^2*b*c*x-3/2*I*a^2*b*d*x^2-1/f*b^3*ln(1-exp(I*(f*x+e)))*d*x-1/f*b^3*l
n(exp(I*(f*x+e))+1)*d*x-1/f^2*b^3*ln(1-exp(I*(f*x+e)))*d*e+3/f^2*b^2*a*d*ln
(exp(I*(f*x+e))-1)-6/f^2*b^2*a*d*ln(exp(I*(f*x+e)))+I/f^2*b^3*d*polylog(2,-
exp(I*(f*x+e)))+1/f^2*b^3*d*e*ln(exp(I*(f*x+e))-1)-2/f^2*b^3*d*e*ln(exp(I*(
f*x+e)))+3/f*b*a^2*c*ln(exp(I*(f*x+e))+1)+I/f^2*b^3*d*e^2+I/f^2*b^3*d*polyl
og(2,exp(I*(f*x+e)))-6/f*b*a^2*c*ln(exp(I*(f*x+e)))+3/f*b*a^2*c*ln(exp(I*(f
*x+e))-1)+3/f^2*b^2*a*d*ln(exp(I*(f*x+e))+1)-6*I/f*b*a^2*d*e*x-I*b^3*c*x+1/
2*a^3*d*x^2+a^3*c*x-1/f*b^3*c*ln(exp(I*(f*x+e))+1)+b^2*(-6*I*a*d*f*x*exp(2*
I*(f*x+e))-6*I*a*c*f*exp(2*I*(f*x+e))+2*b*d*f*x*exp(2*I*(f*x+e))+6*I*a*d*f*
x-I*b*d*exp(2*I*(f*x+e))+2*b*c*f*exp(2*I*(f*x+e))+6*I*a*c*f+I*b*d)/f^2/(exp
(2*I*(f*x+e))-1)^2-3/2*a*b^2*d*x^2-3*a*b^2*c*x+1/2*I*b^3*d*x^2-1/f*b^3*c*ln
(exp(I*(f*x+e))-1)+2/f*b^3*c*ln(exp(I*(f*x+e)))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2149 vs. $2(250) = 500$.
time = 0.78, size = 2149, normalized size = 7.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(f*x + e)*a^3*c + (f*x + e)^2*a^3*d/f - 2*(f*x + e)*a^3*d*e/f + 6*a^
2*b*c*log(sin(f*x + e)) - 6*a^2*b*d*e*log(sin(f*x + e))/f + 2*(12*a*b^2*c*f
 - (3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^2*d + 2*((3*I*a*b^2 + b^3)*c*f + (
-3*I*a*b^2*e - b^3*e)*d)*(f*x + e) - 2*(6*a*b^2*e - b^3)*d - 2*(b^3*c*f - (
3*a^2*b - b^3)*(f*x + e)*d - (b^3*e + 3*a*b^2)*d + (b^3*c*f - (3*a^2*b - b^
3)*(f*x + e)*d - (b^3*e + 3*a*b^2)*d)*cos(4*f*x + 4*e) - 2*(b^3*c*f - (3*a^
2*b - b^3)*(f*x + e)*d - (b^3*e + 3*a*b^2)*d)*cos(2*f*x + 2*e) - (-I*b^3*c*
f + (3*I*a^2*b - I*b^3)*(f*x + e)*d + (I*b^3*e + 3*I*a*b^2)*d)*sin(4*f*x +
4*e) - 2*(I*b^3*c*f + (-3*I*a^2*b + I*b^3)*(f*x + e)*d + (-I*b^3*e - 3*I*a*
b^2)*d)*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) - 2*(b^3*
c*f - (b^3*e + 3*a*b^2)*d + (b^3*c*f - (b^3*e + 3*a*b^2)*d)*cos(4*f*x + 4*e
) - 2*(b^3*c*f - (b^3*e + 3*a*b^2)*d)*cos(2*f*x + 2*e) - (-I*b^3*c*f + (I*b
^3*e + 3*I*a*b^2)*d)*sin(4*f*x + 4*e) - 2*(I*b^3*c*f + (-I*b^3*e - 3*I*a*b^
2)*d)*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), cos(f*x + e) - 1) - 2*((3*a^2
```

```
*b - b^3)*(f*x + e)*d*cos(4*f*x + 4*e) - 2*(3*a^2*b - b^3)*(f*x + e)*d*cos(
2*f*x + 2*e) - (-3*I*a^2*b + I*b^3)*(f*x + e)*d*sin(4*f*x + 4*e) - 2*(3*I*a
^2*b - I*b^3)*(f*x + e)*d*sin(2*f*x + 2*e) + (3*a^2*b - b^3)*(f*x + e)*d)*a
rctan2(sin(f*x + e), -cos(f*x + e) + 1) - ((3*a^2*b - 3*I*a*b^2 - b^3)*(f*x
 + e)^2*d - 2*((3*I*a*b^2 + b^3)*c*f + (3*a*b^2*(-I*e - 2) - b^3*e)*d)*(f*x
 + e))*cos(4*f*x + 4*e) + 2*((3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^2*d - 2*
(3*a*b^2 + I*b^3)*c*f + 2*((-3*I*a*b^2 - b^3)*c*f + (b^3*(e - I) + 3*a*b^2*
(I*e + 1))*d)*(f*x + e) + (b^3*(2*I*e - 1) + 6*a*b^2*e)*d)*cos(2*f*x + 2*e)
 - 2*((3*a^2*b - b^3)*d*cos(4*f*x + 4*e) - 2*(3*a^2*b - b^3)*d*cos(2*f*x +
2*e) - (-3*I*a^2*b + I*b^3)*d*sin(4*f*x + 4*e) - 2*(3*I*a^2*b - I*b^3)*d*si
n(2*f*x + 2*e) + (3*a^2*b - b^3)*d)*dilog(-e^(I*f*x + I*e)) - 2*((3*a^2*b -
 b^3)*d*cos(4*f*x + 4*e) - 2*(3*a^2*b - b^3)*d*cos(2*f*x + 2*e) - (-3*I*a^2
*b + I*b^3)*d*sin(4*f*x + 4*e) - 2*(3*I*a^2*b - I*b^3)*d*sin(2*f*x + 2*e) +
 (3*a^2*b - b^3)*d)*dilog(e^(I*f*x + I*e)) - (-I*b^3*c*f + (3*I*a^2*b - I*b
^3)*(f*x + e)*d + (I*b^3*e + 3*I*a*b^2)*d + (-I*b^3*c*f + (3*I*a^2*b - I*b^
3)*(f*x + e)*d + (I*b^3*e + 3*I*a*b^2)*d)*cos(4*f*x + 4*e) - 2*(-I*b^3*c*f
+ (3*I*a^2*b - I*b^3)*(f*x + e)*d + (I*b^3*e + 3*I*a*b^2)*d)*cos(2*f*x + 2*
e) + (b^3*c*f - (3*a^2*b - b^3)*(f*x + e)*d - (b^3*e + 3*a*b^2)*d)*sin(4*f*
x + 4*e) - 2*(b^3*c*f - (3*a^2*b - b^3)*(f*x + e)*d - (b^3*e + 3*a*b^2)*d)*
sin(2*f*x + 2*e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1)
 - (-I*b^3*c*f + (3*I*a^2*b - I*b^3)*(f*x + e)*d + (I*b^3*e + 3*I*a*b^2)*d
+ (-I*b^3*c*f + (3*I*a^2*b - I*b^3)*(f*x + e)*d + (I*b^3*e + 3*I*a*b^2)*d)*
cos(4*f*x + 4*e) - 2*(-I*b^3*c*f + (3*I*a^2*b - I*b^3)*(f*x + e)*d + (I*b^3
*e + 3*I*a*b^2)*d)*cos(2*f*x + 2*e) + (b^3*c*f - (3*a^2*b - b^3)*(f*x + e)*
d - (b^3*e + 3*a*b^2)*d)*sin(4*f*x + 4*e) - 2*(b^3*c*f - (3*a^2*b - b^3)*(f
*x + e)*d - (b^3*e + 3*a*b^2)*d)*sin(2*f*x + 2*e))*log(cos(f*x + e)^2 + sin
(f*x + e)^2 - 2*cos(f*x + e) + 1) - ((3*I*a^2*b + 3*a*b^2 - I*b^3)*(f*x + e
)^2*d + 2*((3*a*b^2 - I*b^3)*c*f - (3*a*b^2*(e - 2*I) - I*b^3*e)*d)*(f*x +
e))*sin(4*f*x + 4*e) + 2*((3*I*a^2*b + 3*a*b^2 - I*b^3)*(f*x + e)^2*d + 2*(
-3*I*a*b^2 + b^3)*c*f + 2*((3*a*b^2 - I*b^3)*c*f - (3*a*b^2*(e - I) - b^3*(
I*e + 1))*d)*(f*x + e) - (b^3*(2*e + I) - 6*I*a*b^2*e)*d)*sin(2*f*x + 2*e))
/(-2*I*f*cos(4*f*x + 4*e) + 4*I*f*cos(2*f*x + 2*e) + 2*f*sin(4*f*x + 4*e) -
 4*f*sin(2*f*x + 2*e) - 2*I*f))/f
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(250) = 500$.
time = 2.61, size = 750, normalized size = 2.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^3 - 3*a*b^2)*d*f^2*x^2 - 4*b^3*c*f - 4*(b^3*d*f - (a^3 - 3*a*b^2
)*c*f^2)*x - 2*((a^3 - 3*a*b^2)*d*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*f^2*x)*cos(
```

```
2*f*x + 2*e) - (-I*(3*a^2*b - b^3)*d*cos(2*f*x + 2*e) + I*(3*a^2*b - b^3)*d
)*dilog(cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) - (I*(3*a^2*b - b^3)*d*cos(2
*f*x + 2*e) - I*(3*a^2*b - b^3)*d)*dilog(cos(2*f*x + 2*e) - I*sin(2*f*x + 2
*e)) + 2*(3*a*b^2*d + (3*a^2*b - b^3)*c*f - (3*a^2*b - b^3)*d*e - (3*a*b^2*
d + (3*a^2*b - b^3)*c*f - (3*a^2*b - b^3)*d*e)*cos(2*f*x + 2*e))*log(-1/2*c
os(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2) + 2*(3*a*b^2*d + (3*a^2*b -
 b^3)*c*f - (3*a^2*b - b^3)*d*e - (3*a*b^2*d + (3*a^2*b - b^3)*c*f - (3*a^2
*b - b^3)*d*e)*cos(2*f*x + 2*e))*log(-1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*
x + 2*e) + 1/2) + 2*((3*a^2*b - b^3)*d*f*x + (3*a^2*b - b^3)*d*e - ((3*a^2*
b - b^3)*d*f*x + (3*a^2*b - b^3)*d*e)*cos(2*f*x + 2*e))*log(-cos(2*f*x + 2*
e) + I*sin(2*f*x + 2*e) + 1) + 2*((3*a^2*b - b^3)*d*f*x + (3*a^2*b - b^3)*d
*e - ((3*a^2*b - b^3)*d*f*x + (3*a^2*b - b^3)*d*e)*cos(2*f*x + 2*e))*log(-c
os(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1) - 2*(6*a*b^2*d*f*x + 6*a*b^2*c*f
+ b^3*d)*sin(2*f*x + 2*e))/(f^2*cos(2*f*x + 2*e) - f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cot(e+fx))^3 (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))**3,x)
```

```
[Out] Integral((a + b*cot(e + f*x))**3*(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*cot(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\cot(e+f\,x))^3\,(c+d\,x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^3*(c + d*x),x)
```

```
[Out] int((a + b*cot(e + f*x))^3*(c + d*x), x)
```

3.50 $\int \frac{(a+b\cot(e+fx))^3}{c+dx} \, dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{(a+b\cot(e+fx))^3}{c+dx},x\right)$$

```
[Out] Unintegrable((a+b*cot(f*x+e))^3/(d*x+c),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cot(e+fx))^3}{c+dx} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Cot[e + f*x])^3/(c + d*x),x]
[Out] Defer[Int][(a + b*Cot[e + f*x])^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{(a+b\cot(e+fx))^3}{c+dx} \, dx = \int \frac{(a+b\cot(e+fx))^3}{c+dx} \, dx$$

Mathematica [A]
time = 9.25, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cot(e+fx))^3}{c+dx} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Cot[e + f*x])^3/(c + d*x),x]
[Out] Integrate[(a + b*Cot[e + f*x])^3/(c + d*x), x]
```

Maple [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cot(fx+e))^3}{dx+c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(f*x+e))^3/(d*x+c),x)
```

```
[Out] int((a+b*cot(f*x+e))^3/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] (((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^
2)*c^2*f^2)*cos(4*f*x + 4*e)^2*log(d*x + c) + ((a^3 - 3*a*b^2)*d^2*f^2*x^2
+ 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c)*sin(4
*f*x + 4*e)^2 - 4*(b^3*d^2*f*x + b^3*c*d*f - ((a^3 - 3*a*b^2)*d^2*f^2*x^2 +
 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c))*cos(2
*f*x + 2*e)^2 - 4*(b^3*d^2*f*x + b^3*c*d*f - ((a^3 - 3*a*b^2)*d^2*f^2*x^2 +
 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c))*sin(2
*f*x + 2*e)^2 + (2*(b^3*d^2*f*x + b^3*c*d*f - 2*((a^3 - 3*a*b^2)*d^2*f^2*x^
2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c))*co
s(2*f*x + 2*e) + 2*((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2
*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c) + (6*a*b^2*d^2*f*x + 6*a*b^2*c*d
*f - b^3*d^2)*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 2*(b^3*d^2*f*x + b^3*c*d
*f - 2*((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 -
3*a*b^2)*c^2*f^2)*log(d*x + c))*cos(2*f*x + 2*e) + (d^3*f^2*x^2 + 2*c*d^2*f
^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(4*f*x + 4*
e)^2 + 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e)^2 + (d^
3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sin(4*f*x + 4*e)^2 - 4*(d^3*f^2*x^2
+ 2*c*d^2*f^2*x + c^2*d*f^2)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(d^3*f^2
*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sin(2*f*x + 2*e)^2 + 2*(d^3*f^2*x^2 + 2*c
*d^2*f^2*x + c^2*d*f^2 - 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*
f*x + 2*e))*cos(4*f*x + 4*e) - 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*
cos(2*f*x + 2*e))*integrate(-((3*a^2*b - b^3)*d^2*f^2*x^2 - 3*a*b^2*c*d*f +
 b^3*d^2 + (3*a^2*b - b^3)*c^2*f^2 - (3*a*b^2*d^2*f - 2*(3*a^2*b - b^3)*c*d
*f^2)*x)*sin(f*x + e)/(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*
f^2 + (d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*f^2)*cos(f*x + e
)^2 + (d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*f^2)*sin(f*x + e
)^2 + 2*(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*f^2)*cos(f*x +
 e)), x) - (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^
2*f^2*x + c^2*d*f^2)*cos(4*f*x + 4*e)^2 + 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x +
c^2*d*f^2)*cos(2*f*x + 2*e)^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*s
in(4*f*x + 4*e)^2 - 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sin(4*f*x +
 4*e)*sin(2*f*x + 2*e) + 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sin(2*
f*x + 2*e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*(d^3*f^2*x^2
+ 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) - 4*(d^3*f^
```

```
2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e))*integrate(-((3*a^2*b -
 b^3)*d^2*f^2*x^2 - 3*a*b^2*c*d*f + b^3*d^2 + (3*a^2*b - b^3)*c^2*f^2 - (3*
a*b^2*d^2*f - 2*(3*a^2*b - b^3)*c*d*f^2)*x)*sin(f*x + e)/(d^3*f^2*x^3 + 3*c
*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*f^2 + (d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3
*c^2*d*f^2*x + c^3*f^2)*cos(f*x + e)^2 + (d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3
*c^2*d*f^2*x + c^3*f^2)*sin(f*x + e)^2 - 2*(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 +
 3*c^2*d*f^2*x + c^3*f^2)*cos(f*x + e)), x) + ((a^3 - 3*a*b^2)*d^2*f^2*x^2
+ 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c) + (6*
a*b^2*d^2*f*x + 6*a*b^2*c*d*f - b^3*d^2 - (6*a*b^2*d^2*f*x + 6*a*b^2*c*d*f
- b^3*d^2)*cos(2*f*x + 2*e) + 2*(b^3*d^2*f*x + b^3*c*d*f - 2*((a^3 - 3*a*b^
2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log
(d*x + c))*sin(2*f*x + 2*e))*sin(4*f*x + 4*e) - (6*a*b^2*d^2*f*x + 6*a*b^2*
c*d*f - b^3*d^2)*sin(2*f*x + 2*e))/(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2
 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(4*f*x + 4*e)^2 + 4*(d^3*f^
2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e)^2 + (d^3*f^2*x^2 + 2*c*
d^2*f^2*x + c^2*d*f^2)*sin(4*f*x + 4*e)^2 - 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x
+ c^2*d*f^2)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(d^3*f^2*x^2 + 2*c*d^2*f
^2*x + c^2*d*f^2)*sin(2*f*x + 2*e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2
*d*f^2 - 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) - 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e)
)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b^3*cot(f*x + e)^3 + 3*a*b^2*cot(f*x + e)^2 + 3*a^2*b*cot(f*x + e
) + a^3)/(d*x + c), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \cot \left(e + f x\right)\right)^{3}}{c + d x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))**3/(d*x+c),x)
```

```
[Out] Integral((a + b*cot(e + f*x))**3/(c + d*x), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cot(f*x + e) + a)^3/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\cot(e+f\,x))^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^3/(c + d*x),x)
```

```
[Out] int((a + b*cot(e + f*x))^3/(c + d*x), x)
```

3.51 $\int \frac{(a+b\cot(e+fx))^3}{(c+dx)^2} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(a+b\cot(e+fx))^3}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable((a+b*cot(f*x+e))^3/(d*x+c)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cot(e+fx))^3}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Cot[e + f*x])^3/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(a + b*Cot[e + f*x])^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{(a+b\cot(e+fx))^3}{(c+dx)^2} dx = \int \frac{(a+b\cot(e+fx))^3}{(c+dx)^2} dx$$

Mathematica [A]
time = 12.19, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cot(e+fx))^3}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Cot[e + f*x])^3/(c + d*x)^2,x]
```

```
[Out] Integrate[(a + b*Cot[e + f*x])^3/(c + d*x)^2, x]
```

Maple [A]
time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cot(fx+e))^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(f*x+e))^3/(d*x+c)^2,x)
```

```
[Out] int((a+b*cot(f*x+e))^3/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^
2)*c^2*f^2 + ((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (
a^3 - 3*a*b^2)*c^2*f^2)*cos(4*f*x + 4*e)^2 + 4*((a^3 - 3*a*b^2)*d^2*f^2*x^2
 + b^3*c*d*f + (a^3 - 3*a*b^2)*c^2*f^2 + (b^3*d^2*f + 2*(a^3 - 3*a*b^2)*c*d
*f^2)*x)*cos(2*f*x + 2*e)^2 + ((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b
^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*sin(4*f*x + 4*e)^2 + 4*((a^3 - 3*a
*b^2)*d^2*f^2*x^2 + b^3*c*d*f + (a^3 - 3*a*b^2)*c^2*f^2 + (b^3*d^2*f + 2*(a
^3 - 3*a*b^2)*c*d*f^2)*x)*sin(2*f*x + 2*e)^2 + 2*((a^3 - 3*a*b^2)*d^2*f^2*x
^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2 - (2*(a^3 - 3*a*
b^2)*d^2*f^2*x^2 + b^3*c*d*f + 2*(a^3 - 3*a*b^2)*c^2*f^2 + (b^3*d^2*f + 4*(
a^3 - 3*a*b^2)*c*d*f^2)*x)*cos(2*f*x + 2*e) - (3*a*b^2*d^2*f*x + 3*a*b^2*c*
d*f - b^3*d^2)*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) - 2*(2*(a^3 - 3*a*b^2)*d^
2*f^2*x^2 + b^3*c*d*f + 2*(a^3 - 3*a*b^2)*c^2*f^2 + (b^3*d^2*f + 4*(a^3 - 3
*a*b^2)*c*d*f^2)*x)*cos(2*f*x + 2*e) - (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c
^2*d^2*f^2*x + c^3*d*f^2 + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x
 + c^3*d*f^2)*cos(4*f*x + 4*e)^2 + 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2
*d^2*f^2*x + c^3*d*f^2)*cos(2*f*x + 2*e)^2 + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2
 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x + 4*e)^2 - 4*(d^4*f^2*x^3 + 3*c*d
^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 + 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(2*f*
x + 2*e)^2 + 2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2
 - 2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(2*f*
x + 2*e))*cos(4*f*x + 4*e) - 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f
^2*x + c^3*d*f^2)*cos(2*f*x + 2*e))*integrate(-((3*a^2*b - b^3)*d^2*f^2*x^2
 - 6*a*b^2*c*d*f + 3*b^3*d^2 + (3*a^2*b - b^3)*c^2*f^2 - 2*(3*a*b^2*d^2*f -
 (3*a^2*b - b^3)*c*d*f^2)*x)*sin(f*x + e)/(d^4*f^2*x^4 + 4*c*d^3*f^2*x^3 +
6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2 + (d^4*f^2*x^4 + 4*c*d^3*f^2*x^
3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2)*cos(f*x + e)^2 + (d^4*f^2*
x^4 + 4*c*d^3*f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2)*sin(f*
x + e)^2 + 2*(d^4*f^2*x^4 + 4*c*d^3*f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f
^2*x + c^4*f^2)*cos(f*x + e)), x) + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*
d^2*f^2*x + c^3*d*f^2 + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x +
c^3*d*f^2)*cos(4*f*x + 4*e)^2 + 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^
2*f^2*x + c^3*d*f^2)*cos(2*f*x + 2*e)^2 + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 +
```

```
3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x + 4*e)^2 - 4*(d^4*f^2*x^3 + 3*c*d^3*
f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(2*f*x +
 2*e)^2 + 2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2 -
2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(2*f*x +
 2*e))*cos(4*f*x + 4*e) - 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*
x + c^3*d*f^2)*cos(2*f*x + 2*e))*integrate(-((3*a^2*b - b^3)*d^2*f^2*x^2 -
6*a*b^2*c*d*f + 3*b^3*d^2 + (3*a^2*b - b^3)*c^2*f^2 - 2*(3*a*b^2*d^2*f - (3
*a^2*b - b^3)*c*d*f^2)*x)*sin(f*x + e)/(d^4*f^2*x^4 + 4*c*d^3*f^2*x^3 + 6*c
^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2 + (d^4*f^2*x^4 + 4*c*d^3*f^2*x^3 +
 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2)*cos(f*x + e)^2 + (d^4*f^2*x^4
 + 4*c*d^3*f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2)*sin(f*x +
 e)^2 - 2*(d^4*f^2*x^4 + 4*c*d^3*f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*
x + c^4*f^2)*cos(f*x + e)), x) - 2*(3*a*b^2*d^2*f*x + 3*a*b^2*c*d*f - b^3*d
^2 - (3*a*b^2*d^2*f*x + 3*a*b^2*c*d*f - b^3*d^2)*cos(2*f*x + 2*e) + (2*(a^3
 - 3*a*b^2)*d^2*f^2*x^2 + b^3*c*d*f + 2*(a^3 - 3*a*b^2)*c^2*f^2 + (b^3*d^2*
f + 4*(a^3 - 3*a*b^2)*c*d*f^2)*x)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e) + 2*(3
*a*b^2*d^2*f*x + 3*a*b^2*c*d*f - b^3*d^2)*sin(2*f*x + 2*e))/(d^4*f^2*x^3 +
3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2 + (d^4*f^2*x^3 + 3*c*d^3*f^2*
x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(4*f*x + 4*e)^2 + 4*(d^4*f^2*x^3 + 3*
c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(2*f*x + 2*e)^2 + (d^4*f^2*
x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x + 4*e)^2 - 4
*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x +
4*e)*sin(2*f*x + 2*e) + 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x
+ c^3*d*f^2)*sin(2*f*x + 2*e)^2 + 2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*
d^2*f^2*x + c^3*d*f^2 - 2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x
+ c^3*d*f^2)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) - 4*(d^4*f^2*x^3 + 3*c*d^3*
f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(2*f*x + 2*e))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*cot(f*x + e)^3 + 3*a*b^2*cot(f*x + e)^2 + 3*a^2*b*cot(f*x + e
) + a^3)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\cot(e + fx))^3}{(c + dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))**3/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*cot(e + f*x))**3/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cot(f*x + e) + a)^3/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\cot(e+f\,x))^3}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^3/(c + d*x)^2,x)
```

```
[Out] int((a + b*cot(e + f*x))^3/(c + d*x)^2, x)
```

3.52 $\int \frac{(c+dx)^3}{a+b\cot(e+fx)}\,dx$

Optimal. Leaf size=242

$$\frac{(c+dx)^4}{4(a-ib)d}-\frac{b(c+dx)^3\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)\,f}+\frac{3ibd(c+dx)^2\text{PolyLog}\left(2,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2\,(a^2+b^2)\,f^2}-\frac{3bd^2(c+dx)\text{Poly}}{2\,(a^2}$$

```
[Out] 1/4*(d*x+c)^4/(a-I*b)/d-b*(d*x+c)^3*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/
(a^2+b^2)/f+3/2*I*b*d*(d*x+c)^2*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))
/(a^2+b^2)/f^2-3/2*b*d^2*(d*x+c)*polylog(3,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b)
)/(a^2+b^2)/f^3-3/4*I*b*d^3*polylog(4,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^
2+b^2)/f^4
```

Rubi [A]
time = 0.25, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3812, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3bd^2(c+dx)\text{Li}_3\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^3\,(a^2+b^2)}+\frac{3ibd(c+dx)^2\text{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^2\,(a^2+b^2)}-\frac{b(c+dx)^3\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f\,(a^2+b^2)}-\frac{3ibd^3\text{Li}_4\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{4f^4\,(a^2+b^2)}+\frac{(c+dx)^4}{4d(a-ib)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Cot[e + f*x]),x]
```

```
[Out] (c + d*x)^4/(4*(a - I*b)*d) - (b*(c + d*x)^3*Log[1 - ((a + I*b)*E^((2*I)*(e
 + f*x)))/(a - I*b)])/((a^2 + b^2)*f) + (((3*I)/2)*b*d*(c + d*x)^2*PolyLog[
2, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/((a^2 + b^2)*f^2) - (3*b*d^2
*(c + d*x)*PolyLog[3, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/(2*(a^2 +
 b^2)*f^3) - (((3*I)/4)*b*d^3*PolyLog[4, ((a + I*b)*E^((2*I)*(e + f*x)))/(a
 - I*b)])/((a^2 + b^2)*f^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3812

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist
[2*I*b, Int[(c + d*x)^m*E^(2*I*k*Pi)*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2
 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])), x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+b\cot(e+fx)}\,dx = \frac{(c+dx)^4}{4(a-ib)d} + (2ib)\int \frac{e^{2i(e+fx)}(c+dx)^3}{(a-ib)^2+(-a^2-b^2)\,e^{2i(e+fx)}}\,dx$$

$$= \frac{(c+dx)^4}{4(a-ib)d} - \frac{b(c+dx)^3\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)\,f} + \frac{(3bd)\int(c+dx)^2\log\left(1+\frac{(-a^2}{}\right.}{(a^2+b^2)\,f}$$

$$= \frac{(c+dx)^4}{4(a-ib)d} - \frac{b(c+dx)^3\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)\,f} + \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right.}{2\,(a^2+b^2)\,f^2}$$

$$= \frac{(c+dx)^4}{4(a-ib)d} - \frac{b(c+dx)^3\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)\,f} + \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right.}{2\,(a^2+b^2)\,f^2}$$

$$= \frac{(c+dx)^4}{4(a-ib)d} - \frac{b(c+dx)^3\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)\,f} + \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right.}{2\,(a^2+b^2)\,f^2}$$

$$= \frac{(c+dx)^4}{4(a-ib)d} - \frac{b(c+dx)^3\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)\,f} + \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right.}{2\,(a^2+b^2)\,f^2}$$

Mathematica [A]
time = 2.18, size = 419, normalized size = 1.73

$$\frac{4ac^3f^4x+4ibc^3f^4x+6ac^2df^4x^2+6ibc^2df^4x^2+4acd^2f^4x^3+4ibcd^2f^4x^3+ad^3f^4x^4+ibd^3f^4x^4-4bc^3f^3\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)-12bc^2df^3x\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)-12bcd^2f^3x^2\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)-4bd^3f^3x^3\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)+6ibdf^2(c+dx)^2\mathrm{PolyLog}\left(2,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)-6bd^2f(c+dx)\mathrm{PolyLog}\left(3,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)-3ibd^3\mathrm{PolyLog}\left(4,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{4\,(a^2+b^2)\,f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*Cot[e + f*x]),x]
```

```
[Out] (4*a*c^3*f^4*x + (4*I)*b*c^3*f^4*x + 6*a*c^2*d*f^4*x^2 + (6*I)*b*c^2*d*f^4*
x^2 + 4*a*c*d^2*f^4*x^3 + (4*I)*b*c*d^2*f^4*x^3 + a*d^3*f^4*x^4 + I*b*d^3*f
^4*x^4 - 4*b*c^3*f^3*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] - 1
2*b*c^2*d*f^3*x*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] - 12*b*c
*d^2*f^3*x^2*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] - 4*b*d^3*f
^3*x^3*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] + (6*I)*b*d*f^2*(
c + d*x)^2*PolyLog[2, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] - 6*b*d^2*
f*(c + d*x)*PolyLog[3, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] - (3*I)*b
*d^3*PolyLog[4, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/(4*(a^2 + b^2)*
f^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1401 vs. 2(215) = 430.
time = 1.05, size = 1402, normalized size = 5.79

method	result	size

risch	Expression too large to display	1402

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*cot(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] d^2/(I*b+a)*c*x^3+3/2*d/(I*b+a)*c^2*x^2+3/f^2/(b-I*a)*b*c^2*d/(a-I*b)*e^2-4
/f^3/(b-I*a)*b*c*d^2/(a-I*b)*e^3+2/f^3/(b-I*a)*b*d^3/(a-I*b)*e^3*x+3/2/f^2/
(b-I*a)*b*c^2*d/(a-I*b)*polylog(2,(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))+3/2/f^2
/(b-I*a)*b*d^3/(a-I*b)*polylog(2,(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*x^2-I/f/
(b-I*a)*b*c^3/(I*b-a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)+2*I
/f/(b-I*a)*b*c^3/(I*b-a)*ln(exp(I*(f*x+e)))+3/2/f^4/(b-I*a)*b*d^3/(a-I*b)*e
^4-3/4/f^4/(b-I*a)*b*d^3/(a-I*b)*polylog(4,(I*b+a)*exp(2*I*(f*x+e))/(a-I*b)
)+2/(b-I*a)*b*c*d^2/(a-I*b)*x^3+3/(b-I*a)*b*c^2*d/(a-I*b)*x^2+3/2*I/f^3/(b-
I*a)*b*d^3/(a-I*b)*polylog(3,(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*x+I/f^4/(b-I
*a)*b*d^3*e^3/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))+6/f/(b-I*a)*b*
c^2*d/(a-I*b)*e*x+3/f^2/(b-I*a)*b*c*d^2/(a-I*b)*polylog(2,(I*b+a)*exp(2*I*(
f*x+e))/(a-I*b))*x+3/2*I/f^3/(b-I*a)*b*c*d^2/(a-I*b)*polylog(3,(I*b+a)*exp(
2*I*(f*x+e))/(a-I*b))-2*I/f^4/(b-I*a)*b*d^3*e^3/(I*b-a)*ln(exp(I*(f*x+e)))+
I/f/(b-I*a)*b*d^3/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*x^3+I/f^4/
(b-I*a)*b*d^3*e^3/(I*b-a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)
-6/f^2/(b-I*a)*b*c*d^2/(a-I*b)*e^2*x+1/2/(b-I*a)*b*d^3/(a-I*b)*x^4+1/4*d^3/
(I*b+a)*x^4+1/(I*b+a)*c^3*x+1/4/d/(I*b+a)*c^4-6*I/f^2/(b-I*a)*b*c^2*d*e/(I*
b-a)*ln(exp(I*(f*x+e)))+3*I/f/(b-I*a)*b*c*d^2/(a-I*b)*ln(1-(I*b+a)*exp(2*I*
(f*x+e))/(a-I*b))*x^2+6*I/f^3/(b-I*a)*b*c*d^2*e^2/(I*b-a)*ln(exp(I*(f*x+e))
)+3*I/f/(b-I*a)*b*c^2*d/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*x+3*
I/f^2/(b-I*a)*b*c^2*d*e/(I*b-a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-
a+I*b)+3*I/f^2/(b-I*a)*b*c^2*d/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b
))*e-3*I/f^3/(b-I*a)*b*c*d^2*e^2/(I*b-a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*
(f*x+e))-a+I*b)-3*I/f^3/(b-I*a)*b*c*d^2*e^2/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f
*x+e))/(a-I*b))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1045 vs. $2(207) = 414$.
time = 0.73, size = 1045, normalized size = 4.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/12*(18*c^2*d*(2*(f*x + e)*a/((a^2 + b^2)*f) - 2*b*log(a*tan(f*x + e) + b
)/((a^2 + b^2)*f) + b*log(tan(f*x + e)^2 + 1)/((a^2 + b^2)*f))*e - 6*(2*(f*
x + e)*a/(a^2 + b^2) - 2*b*log(a*tan(f*x + e) + b)/(a^2 + b^2) + b*log(tan(
f*x + e)^2 + 1)/(a^2 + b^2))*c^3 - (3*(f*x + e)^4*(a + I*b)*d^3 - 12*I*b*d^
```

```
3*polylog(4, (I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) + 12*((a + I*b)*c*d^2
*f - (a*e + I*b*e)*d^3)*(f*x + e)^3 + 18*((a + I*b)*c^2*d*f^2 - 2*(a*e + I*
b*e)*c*d^2*f + (a*e^2 + I*b*e^2)*d^3)*(f*x + e)^2 + 12*(3*(a*e^2 + I*b*e^2)
*c*d^2*f - (a*e^3 + I*b*e^3)*d^3)*(f*x + e) + 12*(-3*I*b*c*d^2*f*e^2 + I*b*
d^3*e^3)*arctan2(b*cos(2*f*x + 2*e) + a*sin(2*f*x + 2*e) + b, a*cos(2*f*x +
 2*e) - b*sin(2*f*x + 2*e) - a) + 4*(-4*I*(f*x + e)^3*b*d^3 + 9*(-I*b*c*d^2
*f + I*b*d^3*e)*(f*x + e)^2 + 9*(-I*b*c^2*d*f^2 + 2*I*b*c*d^2*f*e - I*b*d^3
*e^2)*(f*x + e))*arctan2(-(2*a*b*cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f*x +
 2*e))/(a^2 + b^2), (2*a*b*sin(2*f*x + 2*e) + a^2 + b^2 - (a^2 - b^2)*cos(2
*f*x + 2*e))/(a^2 + b^2)) + 6*(4*I*(f*x + e)^2*b*d^3 + 3*I*b*c^2*d*f^2 - 6*
I*b*c*d^2*f*e + 3*I*b*d^3*e^2 + 6*(I*b*c*d^2*f - I*b*d^3*e)*(f*x + e))*dilo
g((I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) - 6*(3*b*c*d^2*f*e^2 - b*d^3*e^3
)*log((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)
*sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e)) - 2*(4*(f
*x + e)^3*b*d^3 + 9*(b*c*d^2*f - b*d^3*e)*(f*x + e)^2 + 9*(b*c^2*d*f^2 - 2*
b*c*d^2*f*e + b*d^3*e^2)*(f*x + e))*log(((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4
*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2
 - b^2)*cos(2*f*x + 2*e))/(a^2 + b^2)) - 6*(4*(f*x + e)*b*d^3 + 3*b*c*d^2*f
 - 3*b*d^3*e)*polylog(3, (I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)))/((a^2 +
b^2)*f^3))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1047 vs. $2(207) = 414$.
time = 3.38, size = 1047, normalized size = 4.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*d^3*f^4*x^4 + 8*a*c*d^2*f^4*x^3 + 12*a*c^2*d*f^4*x^2 + 8*a*c^3*f^4
*x - 3*I*b*d^3*polylog(4, ((a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2
- 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) + 3*I*b*d^3*polylog(4, ((a^
2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 - 2*a*b + I*b^2)*sin(2*f*x +
2*e))/(a^2 + b^2)) - 6*(-I*b*d^3*f^2*x^2 - 2*I*b*c*d^2*f^2*x - I*b*c^2*d*f^
2)*dilog(-(a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2
*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) - 6*(I*b*d^3*f^2*x^2 + 2*I
*b*c*d^2*f^2*x + I*b*c^2*d*f^2)*dilog(-(a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*c
os(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1
) - 4*(b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^2 - b*d^3*e^3)*log(1/2*a
^2 + I*a*b - 1/2*b^2 - 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^
2)*sin(2*f*x + 2*e)) - 4*(b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^2 - b
*d^3*e^3)*log(-1/2*a^2 + I*a*b + 1/2*b^2 + 1/2*(a^2 + b^2)*cos(2*f*x + 2*e)
 + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3
*x^2 + 3*b*c^2*d*f^3*x + 3*b*c^2*d*f^2*e - 3*b*c*d^2*f*e^2 + b*d^3*e^3)*log
```

```
((a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*
b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2
+ 3*b*c^2*d*f^3*x + 3*b*c^2*d*f^2*e - 3*b*c*d^2*f*e^2 + b*d^3*e^3)*log((a^2
 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*s
in(2*f*x + 2*e))/(a^2 + b^2)) - 6*(b*d^3*f*x + b*c*d^2*f)*polylog(3, ((a^2
+ 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 - 2*a*b - I*b^2)*sin(2*f*x + 2*e
))/(a^2 + b^2)) - 6*(b*d^3*f*x + b*c*d^2*f)*polylog(3, ((a^2 - 2*I*a*b - b^
2)*cos(2*f*x + 2*e) + (-I*a^2 - 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2
)))/((a^2 + b^2)*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{a+b\cot(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*cot(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**3/(a + b*cot(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*cot(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{a+b\cot(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*cot(e + f*x)),x)
```

```
[Out] int((c + d*x)^3/(a + b*cot(e + f*x)), x)
```

3.53 $\int \frac{(c+dx)^2}{a+b\cot(e+fx)}\,dx$

Optimal. Leaf size=181

$$\frac{(c+dx)^3}{3(a-ib)d}-\frac{b(c+dx)^2\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)\,f}+\frac{ibd(c+dx)\mathrm{PolyLog}\left(2,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)\,f^2}-\frac{bd^2\mathrm{PolyLog}\left(3,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2\,(a^2+b^2)\,f^3}$$

```
[Out] 1/3*(d*x+c)^3/(a-I*b)/d-b*(d*x+c)^2*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/
(a^2+b^2)/f+I*b*d*(d*x+c)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+
b^2)/f^2-1/2*b*d^2*polylog(3,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)/f^
3
```

Rubi [A]
time = 0.21, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3812, 2221, 2611, 2320, 6724}

$$\frac{ibd(c+dx)\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f^2\,(a^2+b^2)}-\frac{b(c+dx)^2\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f\,(a^2+b^2)}-\frac{bd^2\mathrm{Li}_3\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^3\,(a^2+b^2)}+\frac{(c+dx)^3}{3d(a-ib)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Cot[e + f*x]),x]
```

```
[Out] (c + d*x)^3/(3*(a - I*b)*d) - (b*(c + d*x)^2*Log[1 - ((a + I*b)*E^((2*I)*(e
 + f*x)))/(a - I*b)])/((a^2 + b^2)*f) + (I*b*d*(c + d*x)*PolyLog[2, ((a + I
*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/((a^2 + b^2)*f^2) - (b*d^2*PolyLog[3,
((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/(2*(a^2 + b^2)*f^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3812

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist
[2*I*b, Int[(c + d*x)^m*E^(2*I*k*Pi)*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2
 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])), x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+b\cot(e+fx)}\,dx = \frac{(c+dx)^3}{3(a-ib)d} + (2ib)\int \frac{e^{2i(e+fx)}(c+dx)^2}{(a-ib)^2+\left(-a^2-b^2\right)e^{2i(e+fx)}}\,dx$$

$$= \frac{(c+dx)^3}{3(a-ib)d} - \frac{b(c+dx)^2\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} + \frac{(2bd)\int(c+dx)\log\left(1+\frac{(-a^2-b^2\cdots}{(a\cdots}\right.}{\left(a^2+b^2\right)f}\cdots$$

$$= \frac{(c+dx)^3}{3(a-ib)d} - \frac{b(c+dx)^2\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} + \frac{ibd(c+dx)\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f^2} - \cdots$$

$$= \frac{(c+dx)^3}{3(a-ib)d} - \frac{b(c+dx)^2\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} + \frac{ibd(c+dx)\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f^2} - \cdots$$

$$= \frac{(c+dx)^3}{3(a-ib)d} - \frac{b(c+dx)^2\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} + \frac{ibd(c+dx)\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f^2} - \cdots$$

Mathematica [A]
time = 1.63, size = 243, normalized size = 1.34

$$\frac{b\left(\frac{4i(a+ib)e^{2ie}f^3x(3c^2+3cdx+d^2x^2)}{a(-1+e^{2ie})+ib(1+e^{2ie})} - 6f^2(c+dx)^2\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right) + 6idf(c+dx)\mathrm{PolyLog}\left(2,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right) - 3d^2\mathrm{PolyLog}\left(3,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)\right)}{6\left(a^2+b^2\right)f^3} + \frac{x(3c^2+3cdx+d^2x^2)\sin(e)}{3(b\cos(e)+a\sin(e))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Cot[e + f*x]),x]
```

```
[Out] (b*(((4*I)*(a + I*b)*E^((2*I)*e)*f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))/(a*(-1
+ E^((2*I)*e)) + I*b*(1 + E^((2*I)*e))) - 6*f^2*(c + d*x)^2*Log[1 - ((a + I
*b)*E^((2*I)*(e + f*x)))/(a - I*b)] + (6*I)*d*f*(c + d*x)*PolyLog[2, ((a +
I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] - 3*d^2*PolyLog[3, ((a + I*b)*E^((2*I)
*(e + f*x)))/(a - I*b)]))/(6*(a^2 + b^2)*f^3) + (x*(3*c^2 + 3*c*d*x + d^2*x
^2)*Sin[e])/(3*(b*Cos[e] + a*Sin[e]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(163) = 326$.
time = 0.92, size = 897, normalized size = 4.96

method	result
risch	$\frac{d^2x^3}{3ib+3a} + \frac{dc\,x^2}{ib+a} + \frac{c^2x}{ib+a} + \frac{c^3}{3d(ib+a)} - \frac{ib\,d^2\ln\left(1-\frac{(ib+a)e^{2i(fx+e)}}{-ib+a}\right)e^2}{f^3(-ia+b)(-ib+a)} - \frac{ib\,d^2e^2\ln\left(a\,e^{2i(fx+e)}+ib\,e^{2i(fx+e)}-a+ib\right)}{f^3(-ia+b)(ib-a)} + \frac{}{3(-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*cot(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*d^2/(I*b+a)*x^3+d/(I*b+a)*c*x^2+1/(I*b+a)*c^2*x+1/3/d/(I*b+a)*c^3-I/f^3
/(b-I*a)*b*d^2/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*e^2-I/f^3/(b-
I*a)*b*d^2*e^2/(I*b-a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)+2/
3/(b-I*a)*b*d^2/(a-I*b)*x^3-2/f^2/(b-I*a)*b*d^2/(a-I*b)*e^2*x-4/3/f^3/(b-I*
a)*b*d^2/(a-I*b)*e^3-I/f/(b-I*a)*b*c^2/(I*b-a)*ln(a*exp(2*I*(f*x+e))+I*b*ex
p(2*I*(f*x+e))-a+I*b)+2*I/f/(b-I*a)*b*c^2/(I*b-a)*ln(exp(I*(f*x+e)))+2*I/f^
3/(b-I*a)*b*d^2*e^2/(I*b-a)*ln(exp(I*(f*x+e)))-4*I/f^2/(b-I*a)*b*c*d*e/(I*b
-a)*ln(exp(I*(f*x+e)))+1/f^2/(b-I*a)*b*d^2/(a-I*b)*polylog(2,(I*b+a)*exp(2*
I*(f*x+e))/(a-I*b))*x+2*I/f/(b-I*a)*b*c*d/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x
+e))/(a-I*b))*x+2*I/f^2/(b-I*a)*b*c*d*e/(I*b-a)*ln(a*exp(2*I*(f*x+e))+I*b*e
xp(2*I*(f*x+e))-a+I*b)+I/f/(b-I*a)*b*d^2/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+
e))/(a-I*b))*x^2+1/2*I/f^3/(b-I*a)*b*d^2/(a-I*b)*polylog(3,(I*b+a)*exp(2*I*
(f*x+e))/(a-I*b))+2*I/f^2/(b-I*a)*b*c*d/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e
))/(a-I*b))*e+2/(b-I*a)*b*c*d/(a-I*b)*x^2+4/f/(b-I*a)*b*c*d/(a-I*b)*e*x+2/f
^2/(b-I*a)*b*c*d/(a-I*b)*e^2+1/f^2/(b-I*a)*b*c*d/(a-I*b)*polylog(2,(I*b+a)*
exp(2*I*(f*x+e))/(a-I*b))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(157) = 314$.
time = 0.65, size = 760, normalized size = 4.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/6*(6*c*d*(2*(f*x + e)*a/((a^2 + b^2)*f) - 2*b*log(a*tan(f*x + e) + b)/((
a^2 + b^2)*f) + b*log(tan(f*x + e)^2 + 1)/((a^2 + b^2)*f))*e - 3*(2*(f*x +
e)*a/(a^2 + b^2) - 2*b*log(a*tan(f*x + e) + b)/(a^2 + b^2) + b*log(tan(f*x
+ e)^2 + 1)/(a^2 + b^2))*c^2 - (2*(f*x + e)^3*(a + I*b)*d^2 - 6*I*b*d^2*arc
tan2(b*cos(2*f*x + 2*e) + a*sin(2*f*x + 2*e) + b, a*cos(2*f*x + 2*e) - b*si
n(2*f*x + 2*e) - a)*e^2 - 3*b*d^2*e^2*log((a^2 + b^2)*cos(2*f*x + 2*e)^2 +
4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^
2 - b^2)*cos(2*f*x + 2*e)) + 6*(f*x + e)*(a*e^2 + I*b*e^2)*d^2 - 3*b*d^2*po
lylog(3, (I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) + 6*((a + I*b)*c*d*f - (a
*e + I*b*e)*d^2)*(f*x + e)^2 + 6*(-I*(f*x + e)^2*b*d^2 + 2*(-I*b*c*d*f + I*
b*d^2*e)*(f*x + e))*arctan2(-(2*a*b*cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f*
x + 2*e))/(a^2 + b^2), (2*a*b*sin(2*f*x + 2*e) + a^2 + b^2 - (a^2 - b^2)*co
s(2*f*x + 2*e))/(a^2 + b^2)) + 6*(I*(f*x + e)*b*d^2 + I*b*c*d*f - I*b*d^2*e
)*dilog((I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) - 3*((f*x + e)^2*b*d^2 + 2
*(b*c*d*f - b*d^2*e)*(f*x + e))*log(((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b
*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b
^2)*cos(2*f*x + 2*e))/(a^2 + b^2)))/((a^2 + b^2)*f^2))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 748 vs. $2(157) = 314$.
time = 2.95, size = 748, normalized size = 4.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*d^2*f^3*x^3 + 12*a*c*d*f^3*x^2 + 12*a*c^2*f^3*x - 3*b*d^2*polylog
(3, ((a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 - 2*a*b - I*b^2)*sin(2
*f*x + 2*e))/(a^2 + b^2)) - 3*b*d^2*polylog(3, ((a^2 - 2*I*a*b - b^2)*cos(2
*f*x + 2*e) + (-I*a^2 - 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 6*(
-I*b*d^2*f*x - I*b*c*d*f)*dilog(-(a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f
*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) - 6
*(I*b*d^2*f*x + I*b*c*d*f)*dilog(-(a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*
f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) - 6
*(b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)*log(1/2*a^2 + I*a*b - 1/2*b^2 - 1/2*
(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 6*(b
*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)*log(-1/2*a^2 + I*a*b + 1/2*b^2 + 1/2*(a
^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 6*(b*d
^2*f^2*x^2 + 2*b*c*d*f^2*x + 2*b*c*d*f*e - b*d^2*e^2)*log((a^2 + b^2 - (a^2
 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2
*e))/(a^2 + b^2)) - 6*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + 2*b*c*d*f*e - b*d^2*
e^2)*log((a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a
*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)))/((a^2 + b^2)*f^3)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{a+b\cot(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*cot(f*x+e)),x)
[Out] Integral((c + d*x)**2/(a + b*cot(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="giac")
[Out] integrate((d*x + c)^2/(b*cot(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{a+b\cot(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*cot(e + f*x)),x)
[Out] int((c + d*x)^2/(a + b*cot(e + f*x)), x)
```

3.54 $\int \frac{c+dx}{a+b\cot(e+fx)}\,dx$

Optimal. Leaf size=126

$$\frac{(c+dx)^2}{2(a-ib)d}-\frac{b(c+dx)\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)\,f}+\frac{ibd\text{PolyLog}\left(2,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2\,(a^2+b^2)\,f^2}$$

```
[Out] 1/2*(d*x+c)^2/(a-I*b)/d-b*(d*x+c)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a
^2+b^2)/f+1/2*I*b*d*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)/f
^2
```

Rubi [A]
time = 0.12, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3812, 2221, 2317, 2438}

$$-\frac{b(c+dx)\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f\,(a^2+b^2)}+\frac{ibd\text{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^2\,(a^2+b^2)}+\frac{(c+dx)^2}{2d(a-ib)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Cot[e + f*x]),x]
```

```
[Out] (c + d*x)^2/(2*(a - I*b)*d) - (b*(c + d*x)*Log[1 - ((a + I*b)*E^((2*I)*(e +
 f*x)))/(a - I*b)])/((a^2 + b^2)*f) + ((I/2)*b*d*PolyLog[2, ((a + I*b)*E^((
2*I)*(e + f*x)))/(a - I*b)])/((a^2 + b^2)*f^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3812

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist
[2*I*b, Int[(c + d*x)^m*E^(2*I*k*Pi)*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2
 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])), x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{a+b\cot(e+fx)}\,dx &= \frac{(c+dx)^2}{2(a-ib)d}+(2ib)\int \frac{e^{2i(e+fx)}(c+dx)}{(a-ib)^2+\left(-a^2-b^2\right)e^{2i(e+fx)}}\,dx\\
&= \frac{(c+dx)^2}{2(a-ib)d}-\frac{b(c+dx)\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f}+\frac{(bd)\int \log\left(1+\frac{\left(-a^2-b^2\right)e^{2i(e+fx)}}{(a-ib)^2}\right.}{\left(a^2+b^2\right)f}\\
&= \frac{(c+dx)^2}{2(a-ib)d}-\frac{b(c+dx)\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f}-\frac{(ibd)\text{Subst}\left(\int \frac{\log\left(1+\frac{\left(-a^2-b^2\right)x}{(a-ib)^2}\right)}{x}\right.}{2\left(a^2+b^2\right)f^2}\\
&= \frac{(c+dx)^2}{2(a-ib)d}-\frac{b(c+dx)\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f}+\frac{ibd\text{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2\left(a^2+b^2\right)f^2}
\end{aligned}$$

Mathematica [A]
time = 1.01, size = 113, normalized size = 0.90

$$\frac{f\left((a+ib)fx(2c+dx)-2b(c+dx)\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)\right)+ibd\text{PolyLog}\left(2,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2\left(a^2+b^2\right)f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*Cot[e + f*x]),x]
```

```
[Out] (f*((a + I*b)*f*x*(2*c + d*x) - 2*b*(c + d*x)*Log[1 - ((a + I*b)*E^((2*I)*(
e + f*x)))/(a - I*b)]) + I*b*d*PolyLog[2, ((a + I*b)*E^((2*I)*(e + f*x)))/(
a - I*b)])/(2*(a^2 + b^2)*f^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(112) = 224.
time = 0.85, size = 445, normalized size = 3.53

method	result

risch	$\frac{d\,x^2}{2ib+2a}+\frac{cx}{ib+a}-\frac{2bc\ln\left(\mathrm{e}^{i(fx+e)}\right)}{f(ib+a)(ib-a)}+\frac{bc\ln\left(a\,\mathrm{e}^{2i(fx+e)}+ib\,\mathrm{e}^{2i(fx+e)}-a+ib\right)}{f(ib+a)(ib-a)}-\frac{bd\ln\left(1-\frac{(ib+a)\mathrm{e}^{2i(fx+e)}}{-ib+a}\right)x}{f(ib+a)(-ib+a)}-\frac{bd\ln\left(1-\frac{(ib+a)\mathrm{e}}{-}\right.}{f^2(ib+a)(-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*cot(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(I*b+a)*d*x^2+1/(I*b+a)*c*x-2/f*b/(I*b+a)*c/(I*b-a)*ln(exp(I*(f*x+e)))+
1/f*b/(I*b+a)*c/(I*b-a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)-1
/f*b/(I*b+a)*d/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*x-1/f^2*b/(I*
b+a)*d/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*e+I*b/(I*b+a)*d/(a-I*
b)*x^2+2*I/f*b/(I*b+a)*d/(a-I*b)*e*x+I/f^2*b/(I*b+a)*d/(a-I*b)*e^2+1/2*I/f^
2*b/(I*b+a)*d/(a-I*b)*polylog(2,(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))+2/f^2*b/(
I*b+a)*d*e/(I*b-a)*ln(exp(I*(f*x+e)))-1/f^2*b/(I*b+a)*d*e/(I*b-a)*ln(a*exp(
2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)
```

<u>Maxima</u> [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(107) = 214$.
time = 0.62, size = 428, normalized size = 3.40

$$\frac{(a+ib)df^2x^2+2(a+ib)cf^2x-2i\,bdfx\arctan\left(-\frac{2ab\cos(2fx+2e)+(a^2-b^2)\sin(2fx+2e)}{a^2+b^2},\frac{2ab\sin(2fx+2e)+a^2+b^2-(a^2-b^2)\cos(2fx+2e)}{a^2+b^2}\right)-bdfx\log\left(\frac{(a^2+b^2)\cos(2fx+2e)^2+4ab\sin(2fx+2e)+(a^2+b^2)\sin(2fx+2e)^2+a^2+b^2-2(a^2-b^2)\cos(2fx+2e)}{a^2+b^2}\right)-2i\,bcf\arctan\left(b\cos(2fx+2e)+a\sin(2fx+2e)+b,a\cos(2fx+2e)-b\sin(2fx+2e)-a\right)-bcf\log\left((a^2+b^2)\cos(2fx+2e)^2+4ab\sin(2fx+2e)+(a^2+b^2)\sin(2fx+2e)^2+a^2+b^2-2(a^2-b^2)\cos(2fx+2e)\right)+i\,bd\mathrm{Li}_2\left(-\frac{\left(-i\,ae^{(2ie)}+be^{(2ie)}\right)e^{(2ifx)}}{ia+b}\right)}{2(a^2+b^2)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*((a + I*b)*d*f^2*x^2 + 2*(a + I*b)*c*f^2*x - 2*I*b*d*f*x*arctan2(-(2*a*
b*cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f*x + 2*e))/(a^2 + b^2), (2*a*b*sin(
2*f*x + 2*e) + a^2 + b^2 - (a^2 - b^2)*cos(2*f*x + 2*e))/(a^2 + b^2)) - b*d
*f*x*log(((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 +
b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e))/(a^2
+ b^2)) - 2*I*b*c*f*arctan2(b*cos(2*f*x + 2*e) + a*sin(2*f*x + 2*e) + b, a*
cos(2*f*x + 2*e) - b*sin(2*f*x + 2*e) - a) - b*c*f*log((a^2 + b^2)*cos(2*f*
x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2
+ b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e)) + I*b*d*dilog(-(-I*a*e^(2*I*e) + b*
e^(2*I*e))*e^(2*I*f*x)/(I*a + b)))/((a^2 + b^2)*f^2)
```

<u>Fricas</u> [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(107) = 214$.
time = 3.80, size = 491, normalized size = 3.90

$$\frac{2adf^2x^2+4acf^2x+i\,bd\mathrm{Li}_2\left(-\frac{a^2+b^2-(a^2+2i\,ab-b^2)\cos(2fx+2e)+(-i\,a^2+2ab+i\,b^2)\sin(2fx+2e)}{a^2+b^2}+1\right)-i\,bd\mathrm{Li}_2\left(-\frac{a^2+b^2-(a^2-2i\,ab-b^2)\cos(2fx+2e)+(i\,a^2+2ab-i\,b^2)\sin(2fx+2e)}{a^2+b^2}+1\right)-2(bcf-bde)\log\left(\frac{1}{2}a^2+i\,ab-\frac{1}{2}b^2-\frac{1}{2}(a^2+b^2)\cos(2fx+2e)+\frac{1}{2}(i\,a^2+i\,b^2)\sin(2fx+2e)\right)-2(bcf-bde)\log\left(-\frac{1}{2}a^2+i\,ab+\frac{1}{2}b^2+\frac{1}{2}(a^2+b^2)\cos(2fx+2e)+\frac{1}{2}(i\,a^2+i\,b^2)\sin(2fx+2e)\right)-2(bdfx+bde)\log\left(\frac{a^2+b^2-(a^2+2i\,ab-b^2)\cos(2fx+2e)+(-i\,a^2+2ab+i\,b^2)\sin(2fx+2e)}{a^2+b^2}\right)-2(bdfx+bde)\log\left(\frac{a^2+b^2-(a^2-2i\,ab-b^2)\cos(2fx+2e)+(i\,a^2+2ab-i\,b^2)\sin(2fx+2e)}{a^2+b^2}\right)}{4(a^2+b^2)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d*f^2*x^2 + 4*a*c*f^2*x + I*b*d*dilog(-(a^2 + b^2 - (a^2 + 2*I*a*b
 - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2
+ b^2) + 1) - I*b*d*dilog(-(a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2
*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) - 2*(b*c*f
 - b*d*e)*log(1/2*a^2 + I*a*b - 1/2*b^2 - 1/2*(a^2 + b^2)*cos(2*f*x + 2*e)
+ 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 2*(b*c*f - b*d*e)*log(-1/2*a^2 +
I*a*b + 1/2*b^2 + 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*si
n(2*f*x + 2*e)) - 2*(b*d*f*x + b*d*e)*log((a^2 + b^2 - (a^2 + 2*I*a*b - b^2
)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)
) - 2*(b*d*f*x + b*d*e)*log((a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x +
2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)))/((a^2 + b^2)
*f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{a+b\cot(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e)),x)
```

```
[Out] Integral((c + d*x)/(a + b*cot(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*cot(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x}{a+b\cot(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*cot(e + f*x)),x)
```

```
[Out] int((c + d*x)/(a + b*cot(e + f*x)), x)
```

3.55 $\int \frac{1}{(c+dx)(a+b\cot(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+b\cot(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*cot(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\cot(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Cot[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + b*Cot[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\cot(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+b\cot(e+fx))}\,dx$$

Mathematica [A]
time = 1.68, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b\cot(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Cot[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + b*Cot[e + f*x])), x]
```

Maple [A]
time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\,(a+b\cot\,(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*cot(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a+b*cot(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] (2*(a^2*b + b^3)*d*integrate(-(2*a*b*cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f
*x + 2*e))/((a^4 + 2*a^2*b^2 + b^4)*d*x + ((a^4 + 2*a^2*b^2 + b^4)*d*x + (a
^4 + 2*a^2*b^2 + b^4)*c)*cos(2*f*x + 2*e)^2 + ((a^4 + 2*a^2*b^2 + b^4)*d*x
+ (a^4 + 2*a^2*b^2 + b^4)*c)*sin(2*f*x + 2*e)^2 + (a^4 + 2*a^2*b^2 + b^4)*c
 - 2*((a^4 - b^4)*d*x + (a^4 - b^4)*c)*cos(2*f*x + 2*e) + 4*((a^3*b + a*b^3
)*d*x + (a^3*b + a*b^3)*c)*sin(2*f*x + 2*e)), x) + a*log(d*x + c))/((a^2 +
b^2)*d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c + (b*d*x + b*c)*cot(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\cot(e + fx))(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e)),x)
```

```
[Out] Integral(1/((a + b*cot(e + f*x))*(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*cot(f*x + e) + a)), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\cot \left(e+f\,x\right)\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*cot(e + f*x))*(c + d*x)),x)
```

```
[Out] int(1/((a + b*cot(e + f*x))*(c + d*x)), x)
```

3.56 $\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b\cot(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+b*cot(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Cot[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Cot[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+b\cot(e+fx))}\,dx$$

Mathematica [A]
time = 3.54, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Cot[e + f*x])),x]
[Out] Integrate[1/((c + d*x)^2*(a + b*Cot[e + f*x])), x]
```

Maple [A]
time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx+c\right)^{2}\left(a+b\cot\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*cot(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*cot(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] (2*((a^2*b + b^3)*d^2*x + (a^2*b + b^3)*c*d)*integrate(-(2*a*b*cos(2*f*x +
2*e) + (a^2 - b^2)*sin(2*f*x + 2*e))/((a^4 + 2*a^2*b^2 + b^4)*d^2*x^2 + 2*(
a^4 + 2*a^2*b^2 + b^4)*c*d*x + (a^4 + 2*a^2*b^2 + b^4)*c^2 + ((a^4 + 2*a^2*
b^2 + b^4)*d^2*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*c*d*x + (a^4 + 2*a^2*b^2 + b
^4)*c^2)*cos(2*f*x + 2*e)^2 + ((a^4 + 2*a^2*b^2 + b^4)*d^2*x^2 + 2*(a^4 + 2
*a^2*b^2 + b^4)*c*d*x + (a^4 + 2*a^2*b^2 + b^4)*c^2)*sin(2*f*x + 2*e)^2 - 2
*((a^4 - b^4)*d^2*x^2 + 2*(a^4 - b^4)*c*d*x + (a^4 - b^4)*c^2)*cos(2*f*x +
2*e) + 4*((a^3*b + a*b^3)*d^2*x^2 + 2*(a^3*b + a*b^3)*c*d*x + (a^3*b + a*b^
3)*c^2)*sin(2*f*x + 2*e)), x) - a)/((a^2 + b^2)*d^2*x + (a^2 + b^2)*c*d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)
*cot(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\cot{(e + fx)})\,(c + dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*cot(f*x+e)),x)
```

```
[Out] Integral(1/((a + b*cot(e + f*x))*(c + d*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*cot(f*x + e) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\cot\left(e+f\,x\right)\right)\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*cot(e + f*x))*(c + d*x)^2),x)
```

```
[Out] int(1/((a + b*cot(e + f*x))*(c + d*x)^2), x)
```

3.57 $\int \frac{(c+dx)^3}{(a+b\cot(e+fx))^2}\,dx$

Optimal. Leaf size=839

$$-\frac{2ib^2(c+dx)^3}{\left(a^2+b^2\right)^2 f}-\frac{2b^2(c+dx)^3}{(a-ib)(a+ib)^2\left(ia+b-(ia-b)e^{2ie+2ifx}\right) f}+\frac{(c+dx)^4}{4(a+ib)^2 d}-\frac{b(c+dx)^4}{(a+ib)^2(ia+b)d}-\frac{b^2(c+dx)}{\left(a^2+b^2\right)^2}$$

```
[Out] -3*I*b^2*d^2*(d*x+c)*polylog(2,(a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2
)^2/f^3-2*b^2*(d*x+c)^3/(a-I*b)/(a+I*b)^2/(I*a+b-(I*a-b)*exp(2*I*e+2*I*f*x)
)/f+1/4*(d*x+c)^4/(a+I*b)^2/d-b*(d*x+c)^4/(a+I*b)^2/(I*a+b)/d-b^2*(d*x+c)^4
/(a^2+b^2)^2/d+3*b^2*d*(d*x+c)^2*ln(1-(a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(
a^2+b^2)^2/f^2-2*b*(d*x+c)^3*ln(1-(a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a-I*
b)/(a+I*b)^2/f-2*I*b^2*(d*x+c)^3/(a^2+b^2)^2/f-2*I*b^2*(d*x+c)^3*ln(1-(a+I*
b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f-3*b*d*(d*x+c)^2*polylog(2,(a+I
*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a+I*b)^2/(I*a+b)/f^2-3*b^2*d*(d*x+c)^2*pol
ylog(2,(a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^2+3/2*b^2*d^3*poly
log(3,(a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^4-3*b*d^2*(d*x+c)*p
olylog(3,(a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a-I*b)/(a+I*b)^2/f^3-3*I*b^2*
d^2*(d*x+c)*polylog(3,(a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^3+3
/2*b*d^3*polylog(4,(a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a+I*b)^2/(I*a+b)/f^
4+3/2*b^2*d^3*polylog(4,(a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^4
```

Rubi [A]
time = 1.38, antiderivative size = 839, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3815, 2216, 2215, 2221, 2611, 6744, 2320, 6724, 2222}

$$-\frac{b(c+dx)^4}{(a+ib)^2(ia+b)d}+\frac{(c+dx)^4}{4(a+ib)^2d}-\frac{b^2(c+dx)^4}{\left(a^2+b^2\right)^2 d}-\frac{2b\log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)^3}{(a-ib)(a+ib)^2 f}-\frac{2ib^2\log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)^3}{\left(a^2+b^2\right)^2 f}-\frac{2b^2(c+dx)^3}{(a-ib)(a+ib)^2\left(ia-(ia-b)e^{2ie+2ifx}+b\right)f}-\frac{2ib^2(c+dx)^3}{\left(a^2+b^2\right)^2 f}+\frac{3b^2d\log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)^2}{\left(a^2+b^2\right)^2 f^2}-\frac{3bd\,\mathrm{Li}_2\left(\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)^2}{(a+ib)^2(ia+b)f^2}-\frac{3b^2d\,\mathrm{Li}_2\left(\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)^2}{\left(a^2+b^2\right)^2 f^2}-\frac{3ib^2d^2\mathrm{Li}_2\left(\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)}{\left(a^2+b^2\right)^2 f^3}-\frac{3bd^2\mathrm{Li}_3\left(\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)}{(a-ib)(a+ib)^2f^3}-\frac{3ib^2d^2\mathrm{Li}_3\left(\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)}{\left(a^2+b^2\right)^2 f^3}+\frac{3b^2d^3\mathrm{Li}_3\left(\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{2\left(a^2+b^2\right)^2 f^4}+\frac{3bd^3\mathrm{Li}_4\left(\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{2(a+ib)^2(ia+b)f^4}+\frac{3b^2d^3\mathrm{Li}_4\left(\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{2\left(a^2+b^2\right)^2 f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Cot[e + f*x])^2,x]
```

```
[Out] ((-2*I)*b^2*(c + d*x)^3)/((a^2 + b^2)^2*f) - (2*b^2*(c + d*x)^3)/((a - I*b)
*(a + I*b)^2*(I*a + b - (I*a - b)*E^((2*I)*e + (2*I)*f*x))*f) + (c + d*x)^4
/(4*(a + I*b)^2*d) - (b*(c + d*x)^4)/((a + I*b)^2*(I*a + b)*d) - (b^2*(c +
d*x)^4)/((a^2 + b^2)^2*d) + (3*b^2*d*(c + d*x)^2*Log[1 - ((a + I*b)*E^((2*I
)*e + (2*I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^2) - (2*b*(c + d*x)^3*Log[1
- ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a - I*b)*(a + I*b)^2*f)
 - ((2*I)*b^2*(c + d*x)^3*Log[1 - ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a -
I*b)])/((a^2 + b^2)^2*f) - ((3*I)*b^2*d^2*(c + d*x)*PolyLog[2, ((a + I*b)*E
^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^3) - (3*b*d*(c + d*x)^
2*PolyLog[2, ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a + I*b)^2*(
I*a + b)*f^2) - (3*b^2*d*(c + d*x)^2*PolyLog[2, ((a + I*b)*E^((2*I)*e + (2*
I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^2) + (3*b^2*d^3*PolyLog[3, ((a + I*b)
```

```
*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/(2*(a^2 + b^2)^2*f^4) - (3*b*d^2*(c +
 d*x)*PolyLog[3, ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a - I*b)
*(a + I*b)^2*f^3) - ((3*I)*b^2*d^2*(c + d*x)*PolyLog[3, ((a + I*b)*E^((2*I)
*e + (2*I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^3) + (3*b*d^3*PolyLog[4, ((a
+ I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/(2*(a + I*b)^2*(I*a + b)*f^4) +
 (3*b^2*d^3*PolyLog[4, ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/(2*(
a^2 + b^2)^2*f^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3815

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - 2*I*(b/(a^2 +
 b^2 + (a - I*b)^2*E^(2*I*(e + f*x)))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+b\cot(e+fx))^2}\,dx = \int \left(\frac{(c+dx)^3}{(a+ib)^2} - \frac{4b^2(c+dx)^3}{(ia-b)^2\left(ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}\right)^2} + \frac{}{(a+ib)^2\,(-}\right.$$

$$= \frac{(c+dx)^4}{4(a+ib)^2d} + \frac{(4b)\int \frac{(c+dx)^3}{-ia\left(1-\frac{ib}{a}\right)+ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}dx}{(a+ib)^2} - \frac{(4b^2)\int \frac{(c+dx)^3}{\left(ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e\right.}}{(ia-b)^2}$$

$$= \frac{(c+dx)^4}{4(a+ib)^2d} - \frac{b(c+dx)^4}{(a+ib)^2(ia+b)d} + \frac{(4b^2)\int \frac{(c+dx)^3}{ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}dx}{(a+ib)^2(ia+b)} + \frac{(4b)}{}$$

$$= -\frac{2b^2(c+dx)^3}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a+ib)^2d} - \frac{b(c+dx}{(a+ib)^2(ia}$$

$$= -\frac{2ib^2(c+dx)^3}{\left(a^2+b^2\right)^2f} - \frac{2b^2(c+dx)^3}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a+ib)^2}$$

$$= -\frac{2ib^2(c+dx)^3}{\left(a^2+b^2\right)^2f} - \frac{2b^2(c+dx)^3}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a+ib)^2}$$

$$= -\frac{2ib^2(c+dx)^3}{\left(a^2+b^2\right)^2f} - \frac{2b^2(c+dx)^3}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a+ib)^2}$$

$$= -\frac{2ib^2(c+dx)^3}{\left(a^2+b^2\right)^2f} - \frac{2b^2(c+dx)^3}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a+ib)^2}$$

$$= -\frac{2ib^2(c+dx)^3}{\left(a^2+b^2\right)^2f} - \frac{2b^2(c+dx)^3}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a+ib)^2}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2879 vs. 2(839) = 1678.
time = 10.69, size = 2879, normalized size = 3.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3/(a + b*Cot[e + f*x])^2,x]
```

```
[Out] (b*((-12*I)*a*b*c^2*d*E^((2*I)*e)*f^3*x + 12*b^2*c^2*d*E^((2*I)*e)*f^3*x +
(8*I)*a^2*c^3*E^((2*I)*e)*f^4*x - 8*a*b*c^3*E^((2*I)*e)*f^4*x - (12*I)*a*b*
```

c*d^2*E^((2*I)*e)*f^3*x^2 + 12*b^2*c*d^2*E^((2*I)*e)*f^3*x^2 + (12*I)*a^2*c
^2*d*E^((2*I)*e)*f^4*x^2 - 12*a*b*c^2*d*E^((2*I)*e)*f^4*x^2 - (4*I)*a*b*d^3
*E^((2*I)*e)*f^3*x^3 + 4*b^2*d^3*E^((2*I)*e)*f^3*x^3 + (8*I)*a^2*c*d^2*E^((
2*I)*e)*f^4*x^3 - 8*a*b*c*d^2*E^((2*I)*e)*f^4*x^3 + (2*I)*a^2*d^3*E^((2*I)*
e)*f^4*x^4 - 2*a*b*d^3*E^((2*I)*e)*f^4*x^4 - 6*a*b*c^2*d*f^2*Log[a - I*b -
(a + I*b)*E^((2*I)*(e + f*x))] + (6*I)*b^2*c^2*d*f^2*Log[a - I*b - (a + I*b
)*E^((2*I)*(e + f*x))] + 6*a*b*c^2*d*E^((2*I)*e)*f^2*Log[a - I*b - (a + I*b
)*E^((2*I)*(e + f*x))] + (6*I)*b^2*c^2*d*E^((2*I)*e)*f^2*Log[a - I*b - (a +
 I*b)*E^((2*I)*(e + f*x))] + 4*a^2*c^3*f^3*Log[a - I*b - (a + I*b)*E^((2*I)
*(e + f*x))] - (4*I)*a*b*c^3*f^3*Log[a - I*b - (a + I*b)*E^((2*I)*(e + f*x)
)] - 4*a^2*c^3*E^((2*I)*e)*f^3*Log[a - I*b - (a + I*b)*E^((2*I)*(e + f*x))]
 - (4*I)*a*b*c^3*E^((2*I)*e)*f^3*Log[a - I*b - (a + I*b)*E^((2*I)*(e + f*x)
)] - 12*a*b*c*d^2*f^2*x*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)]
+ (12*I)*b^2*c*d^2*f^2*x*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)]
 + 12*a*b*c*d^2*E^((2*I)*e)*f^2*x*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(
a - I*b)] + (12*I)*b^2*c*d^2*E^((2*I)*e)*f^2*x*Log[1 - ((a + I*b)*E^((2*I)*
(e + f*x)))/(a - I*b)] + 12*a^2*c^2*d*f^3*x*Log[1 - ((a + I*b)*E^((2*I)*(e
+ f*x)))/(a - I*b)] - (12*I)*a*b*c^2*d*f^3*x*Log[1 - ((a + I*b)*E^((2*I)*(e
 + f*x)))/(a - I*b)] - 12*a^2*c^2*d*E^((2*I)*e)*f^3*x*Log[1 - ((a + I*b)*E^
((2*I)*(e + f*x)))/(a - I*b)] - (12*I)*a*b*c^2*d*E^((2*I)*e)*f^3*x*Log[1 -
((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] - 6*a*b*d^3*f^2*x^2*Log[1 - ((a
+ I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] + (6*I)*b^2*d^3*f^2*x^2*Log[1 - ((a
+ I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] + 6*a*b*d^3*E^((2*I)*e)*f^2*x^2*Log[
1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] + (6*I)*b^2*d^3*E^((2*I)*e)*
f^2*x^2*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] + 12*a^2*c*d^2*f
^3*x^2*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] - (12*I)*a*b*c*d^
2*f^3*x^2*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] - 12*a^2*c*d^2
*E^((2*I)*e)*f^3*x^2*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] - (
12*I)*a*b*c*d^2*E^((2*I)*e)*f^3*x^2*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))
/(a - I*b)] + 4*a^2*d^3*f^3*x^3*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a
- I*b)] - (4*I)*a*b*d^3*f^3*x^3*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a
- I*b)] - 4*a^2*d^3*E^((2*I)*e)*f^3*x^3*Log[1 - ((a + I*b)*E^((2*I)*(e + f*
x)))/(a - I*b)] - (4*I)*a*b*d^3*E^((2*I)*e)*f^3*x^3*Log[1 - ((a + I*b)*E^((
2*I)*(e + f*x)))/(a - I*b)] + (6*I)*d*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((
2*I)*e)))*f*(c + d*x)*(-(b*d) + a*f*(c + d*x))*PolyLog[2, ((a + I*b)*E^((2*
I)*(e + f*x)))/(a - I*b)] - 3*d^2*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)
e)))(-(b*d) + 2*a*f*(c + d*x))*PolyLog[3, ((a + I*b)*E^((2*I)*(e + f*x)))
/(a - I*b)] + (3*I)*a^2*d^3*PolyLog[4, ((a + I*b)*E^((2*I)*(e + f*x)))/(a -
 I*b)] + 3*a*b*d^3*PolyLog[4, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)] -
(3*I)*a^2*d^3*E^((2*I)*e)*PolyLog[4, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I
*b)] + 3*a*b*d^3*E^((2*I)*e)*PolyLog[4, ((a + I*b)*E^((2*I)*(e + f*x)))/(a
- I*b)]))/(2*(a^2 + b^2)^2*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*f
^4) + (3*x^2*(-(a*c^2*d) - I*b*c^2*d + a*c^2*d*Cos[2*e] - I*b*c^2*d*Cos[2*e
] + I*a*c^2*d*Sin[2*e] + b*c^2*d*Sin[2*e]))/(2*(a - I*b)*(a + I*b)*(-a + I*
b + a*Cos[2*e] + I*b*Cos[2*e] + I*a*Sin[2*e] - b*Sin[2*e])) + (x^3*(-(a*c*d

```
^2) - I*b*c*d^2 + a*c*d^2*Cos[2*e] - I*b*c*d^2*Cos[2*e] + I*a*c*d^2*Sin[2*e
] + b*c*d^2*Sin[2*e]))/((a - I*b)*(a + I*b)*(-a + I*b + a*Cos[2*e] + I*b*Co
s[2*e] + I*a*Sin[2*e] - b*Sin[2*e])) + (x^4*(-(a*d^3) - I*b*d^3 + a*d^3*Cos
[2*e] - I*b*d^3*Cos[2*e] + I*a*d^3*Sin[2*e] + b*d^3*Sin[2*e]))/(4*(a - I*b)
*(a + I*b)*(-a + I*b + a*Cos[2*e] + I*b*Cos[2*e] + I*a*Sin[2*e] - b*Sin[2*e
])) + x*(c^3/(a^2 - (2*I)*a*b - b^2 + a^2*Cos[4*e] + (2*I)*a*b*Cos[4*e] - b
^2*Cos[4*e] + I*a^2*Sin[4*e] - 2*a*b*Sin[4*e] - I*b^2*Sin[4*e]) + (((-I)*a
- b - I*a*Cos[2*e] + b*Cos[2*e] + a*Sin[2*e] + I*b*Sin[2*e])*(4*a*b*c^3*Cos
[2*e] + (4*I)*a*b*c^3*Sin[2*e]))/((a - I*b)*(a + I*b)*(-a + I*b + a*Cos[2*e
] + I*b*Cos[2*e] + I*a*Sin[2*e] - b*Sin[2*e])*(a^2 - (2*I)*a*b - b^2 + a^2*
Cos[4*e] + (2*I)*a*b*Cos[4*e] - b^2*Cos[4*e] + I*a^2*Sin[4*e] - 2*a*b*Sin[4
*e] - I*b^2*Sin[4*e])) + (c^3*Cos[4*e] + I*c^3*Sin[4*e])/(a^2 - (2*I)*a*b -
 b^2 + a^2*Cos[4*e] + (2*I)*a*b*Cos[4*e] - b^2*Cos[4*e] + I*a^2*Sin[4*e] -
2*a*b*Sin[4*e] - I*b^2*Sin[4*e])) + (b^2*c^3*Sin[f*x] + 3*b^2*c^2*d*x*Sin[f
*x] + 3*b^2*c*d^2*x^2*Sin[f*x] + b^2*d^3*x^3*Sin[f*x])/((a - I*b)*(a + I*b)
*f*(b*Cos[e] + a*Sin[e])*(b*Cos[e + f*x] + a*Sin[e + f*x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3469 vs. $2(754) = 1508$.
time = 1.36, size = 3470, normalized size = 4.14

method	result	size
risch	Expression too large to display	3470

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*cot(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 6*I/(b-I*a)^2/f^2/(b+I*a)*b*a^2*c^2*d*e/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e
))+I*b*exp(2*I*(f*x+e))-a+I*b)-6*I/(b-I*a)^2/f^3/(b+I*a)*b^2*c*d^2*e/(I*b-a
)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)*a-6*I/(b-I*a)^2
/f^3/(b+I*a)*b*a^2*c*d^2*e^2/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(
2*I*(f*x+e))-a+I*b)-3/2/(b-I*a)^2/f^4/(b+I*a)*b*a*d^3/(a-I*b)*polylog(4,(I*
b+a)*exp(2*I*(f*x+e))/(a-I*b))+3/(b-I*a)^2/f^4/(b+I*a)*b*a*d^3/(a-I*b)*e^4+
6/(b-I*a)^2/f^3/(b+I*a)*b^2*d^3*e^2/(a-I*b)*x-6/(b-I*a)^2/f/(b+I*a)*b^2*c*d
^2/(a-I*b)*x^2-6/(b-I*a)^2/f^3/(b+I*a)*b^2*c*d^2/(a-I*b)*e^2-3/(b-I*a)^2/f^
3/(b+I*a)*b^2*c*d^2/(a-I*b)*polylog(2,(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))-3/(
b-I*a)^2/f^3/(b+I*a)*b^2*d^3/(a-I*b)*polylog(2,(I*b+a)*exp(2*I*(f*x+e))/(a-
I*b))*x+6/(b-I*a)^2/(b+I*a)*b*a*c^2*d/(a-I*b)*x^2+4/(b-I*a)^2/(b+I*a)*b*a*c
*d^2/(a-I*b)*x^3-3/2*I/(b-I*a)^2/f^4/(b+I*a)*b^2*d^3/(a-I*b)*polylog(3,(I*b
+a)*exp(2*I*(f*x+e))/(a-I*b))+2*I*b^2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/(
b+I*a)/f/(b-I*a)^2/(b*exp(2*I*(f*x+e))-I*a*exp(2*I*(f*x+e))+b+I*a)+1/4*d^3/
(2*I*a*b+a^2-b^2)*x^4-6*I/(b-I*a)^2/f^4/(b+I*a)*b^2*d^3*e^2/(I*b-a)*ln(exp(
I*(f*x+e)))+4*I/(b-I*a)^2/f/(b+I*a)*b*a*c^3/(I*b-a)*ln(exp(I*(f*x+e)))-6*I/
(b-I*a)^2/f^2/(b+I*a)*b^2*c^2*d/(I*b-a)*ln(exp(I*(f*x+e)))+2*I/(b-I*a)^2/f^
4/(b+I*a)*b*a*d^3/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*e^3+6/(b-I
```

```
*a)^2/f^2/(b+I*a)*b*a*c*d^2/(a-I*b)*polylog(2,(I*b+a)*exp(2*I*(f*x+e))/(a-I
*b))*x-6/(b-I*a)^2/f^2/(b+I*a)*b^2*a*c^2*d*e/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(
f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)-3*I/(b-I*a)^2/f^2/(b+I*a)*b^2*d^3/(a-I*
b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*x^2+3*I/(b-I*a)^2/f^4/(b+I*a)*b^2
*d^3/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*e^2-2*I/(b-I*a)^2/f/(b+
I*a)*b*a^2*c^3/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a
+I*b)+4/(b-I*a)^2/f^3/(b+I*a)*b*a*d^3/(a-I*b)*e^3*x+2/(b-I*a)^2/f/(b+I*a)*b
^2*a*c^3/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)+
6/(b-I*a)^2/f^3/(b+I*a)*b^3*c*d^2*e/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I
*b*exp(2*I*(f*x+e))-a+I*b)+12*I/(b-I*a)^2/f^3/(b+I*a)*b^2*c*d^2*e/(I*b-a)*l
n(exp(I*(f*x+e)))+12*I/(b-I*a)^2/f^3/(b+I*a)*b*a*c*d^2*e^2/(I*b-a)*ln(exp(I
*(f*x+e)))+2*I/(b-I*a)^2/f^4/(b+I*a)*b*a^2*d^3*e^3/(I*b-a)/(I*b+a)*ln(a*exp
(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)+3*I/(b-I*a)^2/f^2/(b+I*a)*b^2*c^2
*d/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)*a-12*I
/(b-I*a)^2/f^2/(b+I*a)*b*a*c^2*d*e/(I*b-a)*ln(exp(I*(f*x+e)))+6*I/(b-I*a)^2
/f^2/(b+I*a)*b*a*c^2*d/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*e+3*I
/(b-I*a)^2/f^4/(b+I*a)*b^2*d^3*e^2/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*
b*exp(2*I*(f*x+e))-a+I*b)*a-6*I/(b-I*a)^2/f^3/(b+I*a)*b*a*c*d^2/(a-I*b)*ln(
1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*e^2+6/(b-I*a)^2/f^3/(b+I*a)*b^2*a*c*d^2
*e^2/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)+6*I/
(b-I*a)^2/f/(b+I*a)*b*a*c*d^2/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b)
)*x^2+6*I/(b-I*a)^2/f/(b+I*a)*b*a*c^2*d/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e
))/(a-I*b))*x+1/(b-I*a)^2/(b+I*a)*b*a*d^3/(a-I*b)*x^4-2/(b-I*a)^2/f/(b+I*a)
*b^2*d^3/(a-I*b)*x^3+4/(b-I*a)^2/f^4/(b+I*a)*b^2*d^3*e^3/(a-I*b)+d^2/(2*I*a
*b+a^2-b^2)*c*x^3+3/2*d/(2*I*a*b+a^2-b^2)*c^2*x^2+1/(2*I*a*b+a^2-b^2)*c^3*x
+1/4/d/(2*I*a*b+a^2-b^2)*c^4+3/(b-I*a)^2/f^2/(b+I*a)*b*a*d^3/(a-I*b)*polylo
g(2,(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*x^2-12/(b-I*a)^2/f^2/(b+I*a)*b*a*c*d^
2/(a-I*b)*e^2*x-2/(b-I*a)^2/f^4/(b+I*a)*b^2*a*d^3*e^3/(I*b-a)/(I*b+a)*ln(a*
exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)-6*I/(b-I*a)^2/f^2/(b+I*a)*b^2*
c*d^2/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*x+3*I/(b-I*a)^2/f^3/(b
+I*a)*b*a*d^3/(a-I*b)*polylog(3,(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*x+2*I/(b-
I*a)^2/f/(b+I*a)*b*a*d^3/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*x^3
-4*I/(b-I*a)^2/f^4/(b+I*a)*b*a*d^3*e^3/(I*b-a)*ln(exp(I*(f*x+e)))+3*I/(b-I*
a)^2/f^3/(b+I*a)*b*a*c*d^2/(a-I*b)*polylog(3,(I*b+a)*exp(2*I*(f*x+e))/(a-I*
b))-6*I/(b-I*a)^2/f^3/(b+I*a)*b^2*c*d^2/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e
))/(a-I*b))*e+12/(b-I*a)^2/f/(b+I*a)*b*a*c^2*d/(a-I*b)*e*x+3/(b-I*a)^2/f^2/
(b+I*a)*b*a*c^2*d/(a-I*b)*polylog(2,(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))-3/(b-
I*a)^2/f^4/(b+I*a)*b^3*d^3*e^2/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*ex
p(2*I*(f*x+e))-a+I*b)-3/(b-I*a)^2/f^2/(b+I*a)*b^3*c^2*d/(I*b-a)/(I*b+a)*ln(
a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)-12/(b-I*a)^2/f^2/(b+I*a)*b^2
*c*d^2/(a-I*b)*e*x-8/(b-I*a)^2/f^3/(b+I*a)*b*a*c*d^2/(a-I*b)*e^3+6/(b-I*a)^
2/f^2/(b+I*a)*b*a*c^2*d/(a-I*b)*e^2
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4518 vs. $2(699) = 1398$.

time = 2.75, size = 4518, normalized size = 5.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(36*(b^2/((a^4 + a^2*b^2)*f*tan(f*x + e) + (a^3*b + a*b^3)*f) + 2*a*b*
log(a*tan(f*x + e) + b)/((a^4 + 2*a^2*b^2 + b^4)*f) - a*b*log(tan(f*x + e)^
2 + 1)/((a^4 + 2*a^2*b^2 + b^4)*f) - (a^2 - b^2)*(f*x + e)/((a^4 + 2*a^2*b^
2 + b^4)*f))*c^2*d*e - 12*(2*a*b*log(a*tan(f*x + e) + b)/(a^4 + 2*a^2*b^2 +
 b^4) - a*b*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*(
f*x + e)/(a^4 + 2*a^2*b^2 + b^4) + b^2/(a^3*b + a*b^3 + (a^4 + a^2*b^2)*tan
(f*x + e)))*c^3 - (3*(a^3 + I*a^2*b + a*b^2 + I*b^3)*(f*x + e)^4*d^3 + 72*(
I*a*b^2*e^2 + b^3*e^2)*c*d^2*f + 12*((a^3 + I*a^2*b + a*b^2 + I*b^3)*c*d^2*
f - (a^3*e + I*a^2*b*e + a*b^2*e + I*b^3*e)*d^3)*(f*x + e)^3 + 24*(-I*a*b^2
*e^3 - b^3*e^3)*d^3 + 18*((a^3 + I*a^2*b + a*b^2 + I*b^3)*c^2*d*f^2 - 2*(a^
3*e + I*a^2*b*e + a*b^2*e + I*b^3*e)*c*d^2*f + (a^3*e^2 + I*a^2*b*e^2 + a*b
^2*e^2 + I*b^3*e^2)*d^3)*(f*x + e)^2 + 12*(3*(a^3*e^2 + I*a^2*b*e^2 + a*b^2
*e^2 + I*b^3*e^2)*c*d^2*f - (a^3*e^3 + I*a^2*b*e^3 + a*b^2*e^3 + I*b^3*e^3)
*d^3)*(f*x + e) + 12*(3*(I*a*b^2 + b^3)*c^2*d*f^2 - 6*(a*b^2*(e^2 + I*e) +
I*a^2*b*e^2 + b^3*e)*c*d^2*f + (a*b^2*(2*e^3 + 3*I*e^2) + 2*I*a^2*b*e^3 + 3
*b^3*e^2)*d^3 + (3*(-I*a*b^2 + b^3)*c^2*d*f^2 - 6*(a*b^2*(e^2 - I*e) - I*a^
2*b*e^2 + b^3*e)*c*d^2*f + (a*b^2*(2*e^3 - 3*I*e^2) - 2*I*a^2*b*e^3 + 3*b^3
*e^2)*d^3)*cos(2*f*x + 2*e) + (3*(a*b^2 + I*b^3)*c^2*d*f^2 + 6*(a*b^2*(-I*e
^2 - e) - a^2*b*e^2 - I*b^3*e)*c*d^2*f + (a*b^2*(2*I*e^3 + 3*e^2) + 2*a^2*b
*e^3 + 3*I*b^3*e^2)*d^3)*sin(2*f*x + 2*e))*arctan2(b*cos(2*f*x + 2*e) + a*s
in(2*f*x + 2*e) + b, a*cos(2*f*x + 2*e) - b*sin(2*f*x + 2*e) - a) + 4*(8*(-
I*a^2*b - a*b^2)*(f*x + e)^3*d^3 + 9*(2*(-I*a^2*b - a*b^2)*c*d^2*f + (a*b^2
*(2*e + I) + 2*I*a^2*b*e + b^3)*d^3)*(f*x + e)^2 + 18*((-I*a^2*b - a*b^2)*c
^2*d*f^2 + (a*b^2*(2*e + I) + 2*I*a^2*b*e + b^3)*c*d^2*f - (a*b^2*(e^2 + I*
e) + I*a^2*b*e^2 + b^3*e)*d^3)*(f*x + e) + (8*(I*a^2*b - a*b^2)*(f*x + e)^3
*d^3 + 9*(2*(I*a^2*b - a*b^2)*c*d^2*f + (a*b^2*(2*e - I) - 2*I*a^2*b*e + b^
3)*d^3)*(f*x + e)^2 + 18*((I*a^2*b - a*b^2)*c^2*d*f^2 + (a*b^2*(2*e - I) -
2*I*a^2*b*e + b^3)*c*d^2*f - (a*b^2*(e^2 - I*e) - I*a^2*b*e^2 + b^3*e)*d^3)
*(f*x + e))*cos(2*f*x + 2*e) - (8*(a^2*b + I*a*b^2)*(f*x + e)^3*d^3 + 9*(2*
(a^2*b + I*a*b^2)*c*d^2*f - (a*b^2*(2*I*e + 1) + 2*a^2*b*e + I*b^3)*d^3)*(f
*x + e)^2 + 18*((a^2*b + I*a*b^2)*c^2*d*f^2 - (a*b^2*(2*I*e + 1) + 2*a^2*b*
e + I*b^3)*c*d^2*f - (a*b^2*(-I*e^2 - e) - a^2*b*e^2 - I*b^3*e)*d^3)*(f*x +
 e))*sin(2*f*x + 2*e))*arctan2(-(2*a*b*cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2
*f*x + 2*e))/(a^2 + b^2), (2*a*b*sin(2*f*x + 2*e) + a^2 + b^2 - (a^2 - b^2)
*cos(2*f*x + 2*e))/(a^2 + b^2)) - 3*((a^3 + 3*I*a^2*b - 3*a*b^2 - I*b^3)*(f
*x + e)^4*d^3 + 4*((a^3 + 3*I*a^2*b - 3*a*b^2 - I*b^3)*c*d^2*f + (a*b^2*(3*
e - 2*I) - b^3*(-I*e - 2) - a^3*e - 3*I*a^2*b*e)*d^3)*(f*x + e)^3 + 6*((a^3
 + 3*I*a^2*b - 3*a*b^2 - I*b^3)*c^2*d*f^2 + 2*(a*b^2*(3*e - 2*I) - b^3*(-I*
```

```
e - 2) - a^3*e - 3*I*a^2*b*e)*c*d^2*f - (a*b^2*(3*e^2 - 4*I*e) + b^3*(I*e^2
 + 4*e) - a^3*e^2 - 3*I*a^2*b*e^2)*d^3)*(f*x + e)^2 - 4*(6*(I*a*b^2 - b^3)*
c^2*d*f^2 + 3*(a*b^2*(3*e^2 - 4*I*e) + b^3*(I*e^2 + 4*e) - a^3*e^2 - 3*I*a^
2*b*e^2)*c*d^2*f - (3*a*b^2*(e^3 - 2*I*e^2) - b^3*(-I*e^3 - 6*e^2) - a^3*e^
3 - 3*I*a^2*b*e^3)*d^3)*(f*x + e))*cos(2*f*x + 2*e) + 12*(4*(I*a^2*b + a*b^
2)*(f*x + e)^2*d^3 + 3*(I*a^2*b + a*b^2)*c^2*d*f^2 - 3*(a*b^2*(2*e + I) + 2
*I*a^2*b*e + b^3)*c*d^2*f + 3*(a*b^2*(e^2 + I*e) + I*a^2*b*e^2 + b^3*e)*d^3
 + 3*(2*(I*a^2*b + a*b^2)*c*d^2*f - (a*b^2*(2*e + I) + 2*I*a^2*b*e + b^3)*d
^3)*(f*x + e) + (4*(-I*a^2*b + a*b^2)*(f*x + e)^2*d^3 + 3*(-I*a^2*b + a*b^2
)*c^2*d*f^2 - 3*(a*b^2*(2*e - I) - 2*I*a^2*b*e + b^3)*c*d^2*f + 3*(a*b^2*(e
^2 - I*e) - I*a^2*b*e^2 + b^3*e)*d^3 + 3*(2*(-I*a^2*b + a*b^2)*c*d^2*f - (a
*b^2*(2*e - I) - 2*I*a^2*b*e + b^3)*d^3)*(f*x + e))*cos(2*f*x + 2*e) + (4*(
a^2*b + I*a*b^2)*(f*x + e)^2*d^3 + 3*(a^2*b + I*a*b^2)*c^2*d*f^2 + 3*(a*b^2
*(-2*I*e - 1) - 2*a^2*b*e - I*b^3)*c*d^2*f + 3*(a*b^2*(I*e^2 + e) + a^2*b*e
^2 + I*b^3*e)*d^3 + 3*(2*(a^2*b + I*a*b^2)*c*d^2*f + (a*b^2*(-2*I*e - 1) -
2*a^2*b*e - I*b^3)*d^3)*(f*x + e))*sin(2*f*x + 2*e))*dilog((I*a - b)*e^(2*I
*f*x + 2*I*e)/(I*a + b)) + 6*(3*(a*b^2 - I*b^3)*c^2*d*f^2 + 6*(a*b^2*(I*e^2
 - e) - a^2*b*e^2 + I*b^3*e)*c*d^2*f + (a*b^2*(-2*I*e^3 + 3*e^2) + 2*a^2*b*
e^3 - 3*I*b^3*e^2)*d^3 - (3*(a*b^2 + I*b^3)*c^2*d*f^2 - 6*(a*b^2*(I*e^2 + e
) + a^2*b*e^2 + I*b^3*e)*c*d^2*f - (a*b^2*(-2*I*e^3 - 3*e^2) - 2*a^2*b*e^3
- 3*I*b^3*e^2)*d^3)*cos(2*f*x + 2*e) + (3*(-I*a*b^2 + b^3)*c^2*d*f^2 - 6*(a
*b^2*(e^2 - I*e) - I*a^2*b*e^2 + b^3*e)*c*d^2*f + (a*b^2*(2*e^3 - 3*I*e^2)
- 2*I*a^2*b*e^3 + 3*b^3*e^2)*d^3)*sin(2*f*x + 2*e))*log((a^2 + b^2)*cos(2*f
*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2
 + b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e)) - 2*(8*(a^2*b - I*a*b^2)*(f*x + e)
^3*d^3 + 9*(2*(a^2*b - I*a*b^2)*c*d^2*f - (a*b^2*(-2*I*e + 1) + 2*a^2*b*e -
 I*b^3)*d^3)*(f*x + e)^2 + 18*((a^2*b - I*a*b^2)*c^2*d*f^2 - (a*b^2*(-2*I*e
 + 1) + 2*a^2*b*e - I*b^3)*c*d^2*f - (a*b^2*(I*e^2 - e) - a^2*b*e^2 + I*b^3
*e)*d^3)*(f*x + e) - (8*(a^2*b + I*a*b^2)*(f*x ...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3413 vs. $2(699) = 1398$.
time = 3.23, size = 3413, normalized size = 4.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a^2*b - b^3)*d^3*f^4*x^4 - 4*a*b^2*c^3*f^3 - 4*(a*b^2*d^3*f^3 - (a^2*
b - b^3)*c*d^2*f^4)*x^3 - 6*(2*a*b^2*c*d^2*f^3 - (a^2*b - b^3)*c^2*d*f^4)*x
^2 - 4*(3*a*b^2*c^2*d*f^3 - (a^2*b - b^3)*c^3*f^4)*x + ((a^2*b - b^3)*d^3*f
^4*x^4 - 4*a*b^2*c^3*f^3 - 4*(a*b^2*d^3*f^3 - (a^2*b - b^3)*c*d^2*f^4)*x^3
- 6*(2*a*b^2*c*d^2*f^3 - (a^2*b - b^3)*c^2*d*f^4)*x^2 - 4*(3*a*b^2*c^2*d*f^
3 - (a^2*b - b^3)*c^3*f^4)*x)*cos(2*f*x + 2*e) - 6*(-I*a*b^2*d^3*f^2*x^2 -
```

```
I*a*b^2*c^2*d*f^2 + I*b^3*c*d^2*f - I*(2*a*b^2*c*d^2*f^2 - b^3*d^3*f)*x + (
-I*a*b^2*d^3*f^2*x^2 - I*a*b^2*c^2*d*f^2 + I*b^3*c*d^2*f - I*(2*a*b^2*c*d^2
*f^2 - b^3*d^3*f)*x)*cos(2*f*x + 2*e) + (-I*a^2*b*d^3*f^2*x^2 - I*a^2*b*c^2
*d*f^2 + I*a*b^2*c*d^2*f - I*(2*a^2*b*c*d^2*f^2 - a*b^2*d^3*f)*x)*sin(2*f*x
 + 2*e))*dilog(-(a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a
^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) - 6*(I*a*b^2*d^3*f^2
*x^2 + I*a*b^2*c^2*d*f^2 - I*b^3*c*d^2*f + I*(2*a*b^2*c*d^2*f^2 - b^3*d^3*f
)*x + (I*a*b^2*d^3*f^2*x^2 + I*a*b^2*c^2*d*f^2 - I*b^3*c*d^2*f + I*(2*a*b^2
*c*d^2*f^2 - b^3*d^3*f)*x)*cos(2*f*x + 2*e) + (I*a^2*b*d^3*f^2*x^2 + I*a^2*
b*c^2*d*f^2 - I*a*b^2*c*d^2*f + I*(2*a^2*b*c*d^2*f^2 - a*b^2*d^3*f)*x)*sin(
2*f*x + 2*e))*dilog(-(a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) +
(I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) - 2*(2*a*b^2*c^3
*f^3 - 3*b^3*c^2*d*f^2 - 2*a*b^2*d^3*e^3 + (2*a*b^2*c^3*f^3 - 3*b^3*c^2*d*f
^2 - 2*a*b^2*d^3*e^3 + 3*(2*a*b^2*c*d^2*f - b^3*d^3)*e^2 - 6*(a*b^2*c^2*d*f
^2 - b^3*c*d^2*f)*e)*cos(2*f*x + 2*e) + 3*(2*a*b^2*c*d^2*f - b^3*d^3)*e^2 -
 6*(a*b^2*c^2*d*f^2 - b^3*c*d^2*f)*e + (2*a^2*b*c^3*f^3 - 3*a*b^2*c^2*d*f^2
 - 2*a^2*b*d^3*e^3 + 3*(2*a^2*b*c*d^2*f - a*b^2*d^3)*e^2 - 6*(a^2*b*c^2*d*f
^2 - a*b^2*c*d^2*f)*e)*sin(2*f*x + 2*e))*log(1/2*a^2 + I*a*b - 1/2*b^2 - 1/
2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 2*
(2*a*b^2*c^3*f^3 - 3*b^3*c^2*d*f^2 - 2*a*b^2*d^3*e^3 + (2*a*b^2*c^3*f^3 - 3
*b^3*c^2*d*f^2 - 2*a*b^2*d^3*e^3 + 3*(2*a*b^2*c*d^2*f - b^3*d^3)*e^2 - 6*(a
*b^2*c^2*d*f^2 - b^3*c*d^2*f)*e)*cos(2*f*x + 2*e) + 3*(2*a*b^2*c*d^2*f - b^
3*d^3)*e^2 - 6*(a*b^2*c^2*d*f^2 - b^3*c*d^2*f)*e + (2*a^2*b*c^3*f^3 - 3*a*b
^2*c^2*d*f^2 - 2*a^2*b*d^3*e^3 + 3*(2*a^2*b*c*d^2*f - a*b^2*d^3)*e^2 - 6*(a
^2*b*c^2*d*f^2 - a*b^2*c*d^2*f)*e)*sin(2*f*x + 2*e))*log(-1/2*a^2 + I*a*b +
 1/2*b^2 + 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x
 + 2*e)) - 2*(2*a*b^2*d^3*f^3*x^3 + 2*a*b^2*d^3*e^3 + 3*(2*a*b^2*c*d^2*f^3
- b^3*d^3*f^2)*x^2 + 6*(a*b^2*c^2*d*f^3 - b^3*c*d^2*f^2)*x + (2*a*b^2*d^3*f
^3*x^3 + 2*a*b^2*d^3*e^3 + 3*(2*a*b^2*c*d^2*f^3 - b^3*d^3*f^2)*x^2 + 6*(a*b
^2*c^2*d*f^3 - b^3*c*d^2*f^2)*x - 3*(2*a*b^2*c*d^2*f - b^3*d^3)*e^2 + 6*(a*
b^2*c^2*d*f^2 - b^3*c*d^2*f)*e)*cos(2*f*x + 2*e) - 3*(2*a*b^2*c*d^2*f - b^3
*d^3)*e^2 + 6*(a*b^2*c^2*d*f^2 - b^3*c*d^2*f)*e + (2*a^2*b*d^3*f^3*x^3 + 2*
a^2*b*d^3*e^3 + 3*(2*a^2*b*c*d^2*f^3 - a*b^2*d^3*f^2)*x^2 + 6*(a^2*b*c^2*d*
f^3 - a*b^2*c*d^2*f^2)*x - 3*(2*a^2*b*c*d^2*f - a*b^2*d^3)*e^2 + 6*(a^2*b*c
^2*d*f^2 - a*b^2*c*d^2*f)*e)*sin(2*f*x + 2*e))*log((a^2 + b^2 - (a^2 + 2*I*
a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a
^2 + b^2)) - 2*(2*a*b^2*d^3*f^3*x^3 + 2*a*b^2*d^3*e^3 + 3*(2*a*b^2*c*d^2*f^
3 - b^3*d^3*f^2)*x^2 + 6*(a*b^2*c^2*d*f^3 - b^3*c*d^2*f^2)*x + (2*a*b^2*d^3
*f^3*x^3 + 2*a*b^2*d^3*e^3 + 3*(2*a*b^2*c*d^2*f^3 - b^3*d^3*f^2)*x^2 + 6*(a
*b^2*c^2*d*f^3 - b^3*c*d^2*f^2)*x - 3*(2*a*b^2*c*d^2*f - b^3*d^3)*e^2 + 6*(
a*b^2*c^2*d*f^2 - b^3*c*d^2*f)*e)*cos(2*f*x + 2*e) - 3*(2*a*b^2*c*d^2*f - b
^3*d^3)*e^2 + 6*(a*b^2*c^2*d*f^2 - b^3*c*d^2*f)*e + (2*a^2*b*d^3*f^3*x^3 +
2*a^2*b*d^3*e^3 + 3*(2*a^2*b*c*d^2*f^3 - a*b^2*d^3*f^2)*x^2 + 6*(a^2*b*c^2*
d*f^3 - a*b^2*c*d^2*f^2)*x - 3*(2*a^2*b*c*d^2*f - a*b^2*d^3)*e^2 + 6*(a^2*b
*c^2*d*f^2 - a*b^2*c*d^2*f)*e)*sin(2*f*x + 2*e))*log((a^2 + b^2 - (a^2 - 2*
```

```
I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(
a^2 + b^2)) - 3*(I*a*b^2*d^3*cos(2*f*x + 2*e) + I*a^2*b*d^3*sin(2*f*x + 2*e
) + I*a*b^2*d^3)*polylog(4, ((a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^
2 - 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 3*(-I*a*b^2*d^3*cos(2*f
*x + 2*e) - I*a^2*b*d^3*sin(2*f*x + 2*e) - I*a*b^2*d^3)*polylog(4, ((a^2 -
2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 - 2*a*b + I*b^2)*sin(2*f*x + 2*e)
)/(a^2 + b^2)) - 3*(2*a*b^2*d^3*f*x + 2*a*b^2*c*d^2*f - b^3*d^3 + (2*a*b^2*
d^3*f*x + 2*a*b^2*c*d^2*f - b^3*d^3)*cos(2*f*x + 2*e) + (2*a^2*b*d^3*f*x +
2*a^2*b*c*d^2*f - a*b^2*d^3)*sin(2*f*x + 2*e))*polylog(3, ((a^2 + 2*I*a*b -
 b^2)*cos(2*f*x + 2*e) + (I*a^2 - 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b
^2)) - 3*(2*a*b^2*d^3*f*x + 2*a*b^2*c*d^2*f - b^3*d^3 + (2*a*b^2*d^3*f*x +
2*a*b^2*c*d^2*f - b^3*d^3)*cos(2*f*x + 2*e) + (2*a^2*b*d^3*f*x + 2*a^2*b*c*
d^2*f - a*b^2*d^3)*sin(2*f*x + 2*e))*polylog(3, ((a^2 - 2*I*a*b - b^2)*cos(
2*f*x + 2*e) + (-I*a^2 - 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) + ((
a^3 - a*b^2)*d^3*f^4*x^4 + 4*b^3*c^3*f^3 + 4*(b^3*d^3*f^3 + (a^3 - a*b^2)*c
*d^2*f^4)*x^3 + 6*(2*b^3*c*d^2*f^3 + (a^3 - a*b...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+b\cot(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*cot(f*x+e))**2,x)
[Out] Integral((c + d*x)**3/(a + b*cot(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cot(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*x + c)^3/(b*cot(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{(a+b\cot(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*cot(e + f*x))^2,x)
[Out] int((c + d*x)^3/(a + b*cot(e + f*x))^2, x)
```

3.58 $\int \frac{(c+dx)^2}{(a+b\cot(e+fx))^2}\,dx$

Optimal. Leaf size=650

$$-\frac{2ib^2(c+dx)^2}{\left(a^2+b^2\right)^2 f}-\frac{2b^2(c+dx)^2}{(a-ib)(a+ib)^2\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f}+\frac{(c+dx)^3}{3(a+ib)^2d}-\frac{4b(c+dx)^3}{3(a+ib)^2(ia+b)d}-\frac{4b^2(c+}{3\left(a^2+\right.}$$

```
[Out] -2*I*b^2*(d*x+c)^2/(a^2+b^2)^2/f-2*b^2*(d*x+c)^2/(a-I*b)/(a+I*b)^2/(I*a+b-(
I*a-b)*exp(2*I*e+2*I*f*x))/f+1/3*(d*x+c)^3/(a+I*b)^2/d-4/3*b*(d*x+c)^3/(a+I
*b)^2/(I*a+b)/d-4/3*b^2*(d*x+c)^3/(a^2+b^2)^2/d+2*b^2*d*(d*x+c)*ln(1-(a+I*b
)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^2-2*b*(d*x+c)^2*ln(1-(a+I*b)*ex
p(2*I*e+2*I*f*x)/(a-I*b))/(a-I*b)/(a+I*b)^2/f-2*I*b^2*(d*x+c)^2*ln(1-(a+I*b
)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f-I*b^2*d^2*polylog(2,(a+I*b)*exp
(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^3-2*b*d*(d*x+c)*polylog(2,(a+I*b)*ex
p(2*I*e+2*I*f*x)/(a-I*b))/(a+I*b)^2/(I*a+b)/f^2-2*b^2*d*(d*x+c)*polylog(2,(
a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^2-b*d^2*polylog(3,(a+I*b)*
exp(2*I*e+2*I*f*x)/(a-I*b))/(a-I*b)/(a+I*b)^2/f^3-I*b^2*d^2*polylog(3,(a+I*
b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^3
```

Rubi [A]
time = 1.06, antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3815, 2216, 2215, 2221, 2611, 2320, 6724, 2222, 2317, 2438}

$$-\frac{2b^2d(c+dx)\text{Li}_2\left(\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^2\left(a^2+b^2\right)^2}+\frac{2b^2d(c+dx)\log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^2\left(a^2+b^2\right)^2}-\frac{2ib^2(c+dx)^2\log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f\left(a^2+b^2\right)^2}-\frac{2ib^2(c+dx)^2}{f\left(a^2+b^2\right)^2}-\frac{4b^2(c+dx)^3}{3d\left(a^2+b^2\right)^2}-\frac{ib^2d^2\text{Li}_2\left(\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^3\left(a^2+b^2\right)^2}-\frac{ib^2d^2\text{Li}_3\left(\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^3\left(a^2+b^2\right)^2}-\frac{2b^2(c+dx)^2}{f(a-ib)(a+ib)^2\left(-(-b+ia)e^{2ie+2ifx}+ia+b\right)}-\frac{2bd(c+dx)\text{Li}_2\left(\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^2(a+ib)^2(b+ia)}-\frac{2b(c+dx)^2\log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f(a-ib)(a+ib)^2}-\frac{4b(c+dx)^3}{3d(a+ib)^2(b+ia)}+\frac{(c+dx)^3}{3d(a+ib)^2}-\frac{bd^2\text{Li}_3\left(\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^3(a-ib)(a+ib)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Cot[e + f*x])^2,x]
```

```
[Out] ((-2*I)*b^2*(c + d*x)^2)/((a^2 + b^2)^2*f) - (2*b^2*(c + d*x)^2)/((a - I*b)
*(a + I*b)^2*(I*a + b - (I*a - b)*E^((2*I)*e + (2*I)*f*x))*f) + (c + d*x)^3
/(3*(a + I*b)^2*d) - (4*b*(c + d*x)^3)/(3*(a + I*b)^2*(I*a + b)*d) - (4*b^2
*(c + d*x)^3)/(3*(a^2 + b^2)^2*d) + (2*b^2*d*(c + d*x)*Log[1 - ((a + I*b)*E
^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^2) - (2*b*(c + d*x)^2*
Log[1 - ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a - I*b)*(a + I*b
)^2*f) - ((2*I)*b^2*(c + d*x)^2*Log[1 - ((a + I*b)*E^((2*I)*e + (2*I)*f*x))
/(a - I*b)])/((a^2 + b^2)^2*f) - (I*b^2*d^2*PolyLog[2, ((a + I*b)*E^((2*I)*
e + (2*I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^3) - (2*b*d*(c + d*x)*PolyLog[
2, ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a + I*b)^2*(I*a + b)*f
^2) - (2*b^2*d*(c + d*x)*PolyLog[2, ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a
- I*b)])/((a^2 + b^2)^2*f^2) - (b*d^2*PolyLog[3, ((a + I*b)*E^((2*I)*e + (2
*I)*f*x))/(a - I*b)])/((a - I*b)*(a + I*b)^2*f^3) - (I*b^2*d^2*PolyLog[3, (
(a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3815

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - 2*I*(b/(a^2 +
 b^2 + (a - I*b)^2*E^(2*I*(e + f*x)))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+b\cot(e+fx))^2}\,dx = \int \left(\frac{(c+dx)^2}{(a+ib)^2} - \frac{4b^2(c+dx)^2}{(ia-b)^2\left(ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}\right)^2} + \frac{\cdots}{(a+ib)^2\left(-i\cdots\right.}\right.$$

$$= \frac{(c+dx)^3}{3(a+ib)^2d} + \frac{(4b)\int \frac{(c+dx)^2}{-ia\left(1-\frac{ib}{a}\right)+ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}\,dx}{(a+ib)^2} - \frac{(4b^2)\int \frac{(c+dx)^2}{\left(ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2i\cdots}\right.}}{(ia-b)^2}\cdots$$

$$= \frac{(c+dx)^3}{3(a+ib)^2d} - \frac{4b(c+dx)^3}{3(a+ib)^2(ia+b)d} + \frac{(4b^2)\int \frac{(c+dx)^2}{ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}\,dx}{(a+ib)^2(ia+b)} + \frac{(4b)\cdots}{\cdots}$$

$$= -\frac{2b^2(c+dx)^2}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^3}{3(a+ib)^2d} - \frac{4b(c+dx\cdots}{3(a+ib)^2(ia\cdots}$$

$$= -\frac{2ib^2(c+dx)^2}{\left(a^2+b^2\right)^2f} - \frac{2b^2(c+dx)^2}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^3}{3(a+ib)^2d}\cdots$$

$$= -\frac{2ib^2(c+dx)^2}{\left(a^2+b^2\right)^2f} - \frac{2b^2(c+dx)^2}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^3}{3(a+ib)^2d}\cdots$$

$$= -\frac{2ib^2(c+dx)^2}{\left(a^2+b^2\right)^2f} - \frac{2b^2(c+dx)^2}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^3}{3(a+ib)^2d}\cdots$$

$$= -\frac{2ib^2(c+dx)^2}{\left(a^2+b^2\right)^2f} - \frac{2b^2(c+dx)^2}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^3}{3(a+ib)^2d}\cdots$$

Mathematica [A]
time = 8.07, size = 554, normalized size = 0.85

$$\frac{\frac{2b\left(2f\left(3ic\left(a\left(-1+e^{2ie}\right)+ib\left(1+e^{2ie}\right)\right)(-bd+acf)\log\left(a-ib-(a+ib)e^{2i(e+fx)}\right)+x\left((a+ib)e^{2ie}f\left(-3bd(2c+dx)+2af\left(3c^2+3cdx+d^2x^2\right)\right)+3id\left(a\left(-1+e^{2ie}\right)+ib\left(1+e^{2ie}\right)\right)(-bd+af(2c+dx))\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)\right)\right)+3d\left(a\left(-1+e^{2ie}\right)+ib\left(1+e^{2ie}\right)\right)(-bd+2af(c+dx))\text{PolyLog}\left(2,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)+3iad^2\left(a\left(-1+e^{2ie}\right)+ib\left(1+e^{2ie}\right)\right)\text{PolyLog}\left(3,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)\right)}{\left(a^2+b^2\right)^2\left(-ia\left(-1+e^{2ie}\right)+b\left(1+e^{2ie}\right)\right)} - \frac{f^2\left(-\left(\left(a^2-b^2\right)fx\left(3c^2+3cdx+d^2x^2\right)\cos(fx)\right)+\left(a^2+b^2\right)fx\left(3c^2+3cdx+d^2x^2\right)\cos(2e+fx)+2b\left(-3b(c+dx)^2+afx\left(3c^2+3cdx+d^2x^2\right)\right)\sin(fx)\right)}{(a-ib)(a+ib)(b\cos(e)+a\sin(e))(b\cos(e+fx)+a\sin(e+fx))}}{6f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Cot[e + f*x])^2,x]
```

```
[Out] ((2*b*(2*f*((3*I)*c*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(-(b*d)
+ a*c*f)*Log[a - I*b - (a + I*b)*E^((2*I)*(e + f*x))] + x*((a + I*b)*E^((2*
I)*e)*f*(-3*b*d*(2*c + d*x) + 2*a*f*(3*c^2 + 3*c*d*x + d^2*x^2)) + (3*I)*d*
(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(-(b*d) + a*f*(2*c + d*x))*L
og[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)])) + 3*d*(a*(-1 + E^((2*I)
```

```
*e)) + I*b*(1 + E^((2*I)*e)))*(-(b*d) + 2*a*f*(c + d*x))*PolyLog[2, ((a + I
*b)*E^((2*I)*(e + f*x)))/(a - I*b)] + (3*I)*a*d^2*(a*(-1 + E^((2*I)*e)) + I
*b*(1 + E^((2*I)*e)))*PolyLog[3, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)]
))/((a^2 + b^2)^2*((-I)*a*(-1 + E^((2*I)*e)) + b*(1 + E^((2*I)*e)))) - (f^2
*(-((a^2 - b^2)*f*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cos[f*x]) + (a^2 + b^2)*f*x
*(3*c^2 + 3*c*d*x + d^2*x^2)*Cos[2*e + f*x] + 2*b*(-3*b*(c + d*x)^2 + a*f*x
*(3*c^2 + 3*c*d*x + d^2*x^2))*Sin[f*x]))/((a - I*b)*(a + I*b)*(b*Cos[e] + a
*Sin[e])*(b*Cos[e + f*x] + a*Sin[e + f*x]))))/(6*f^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2160 vs. $2(584) = 1168$.
time = 1.26, size = 2161, normalized size = 3.32

method	result	size
risch	Expression too large to display	2161

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*cot(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/(b-I*a)^2/f/(b+I*a)*b^2*d^2/(a-I*b)*x^2-2/(b-I*a)^2/f^3/(b+I*a)*b^2*d^2/
(a-I*b)*e^2-1/(b-I*a)^2/f^3/(b+I*a)*b^2*d^2/(a-I*b)*polylog(2,(I*b+a)*exp(2
*I*(f*x+e))/(a-I*b))+4/3/(b-I*a)^2/(b+I*a)*b*a*d^2/(a-I*b)*x^3-8*I/(b-I*a)^
2/f^2/(b+I*a)*b*a*c*d*e/(I*b-a)*ln(exp(I*(f*x+e)))+4*I/(b-I*a)^2/f^2/(b+I*a
)*b*a*c*d/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*e+2*I/(b-I*a)^2/f^
2/(b+I*a)*b^2*c*d/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e)
)-a+I*b)*a-2*I/(b-I*a)^2/f^3/(b+I*a)*b*a^2*d^2*e^2/(I*b-a)/(I*b+a)*ln(a*exp
(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)+4*I/(b-I*a)^2/f/(b+I*a)*b*a*c*d/(
a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*x-2*I/(b-I*a)^2/f^3/(b+I*a)*b
^2*d^2*e/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)*
a-4/(b-I*a)^2/f^2/(b+I*a)*b^2*d^2/(a-I*b)*e*x-8/3/(b-I*a)^2/f^3/(b+I*a)*b*a
*d^2/(a-I*b)*e^3+4/(b-I*a)^2/(b+I*a)*b*a*c*d/(a-I*b)*x^2+2*I*b^2*(d^2*x^2+2
*c*d*x+c^2)/(b+I*a)/f/(b-I*a)^2/(b*exp(2*I*(f*x+e))-I*a*exp(2*I*(f*x+e))+b+
I*a)-4/(b-I*a)^2/f^2/(b+I*a)*b^2*a*c*d*e/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+
e))+I*b*exp(2*I*(f*x+e))-a+I*b)+4*I/(b-I*a)^2/f^2/(b+I*a)*b*a^2*c*d*e/(I*b-
a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)+I/(b-I*a)^2/f^
3/(b+I*a)*b*a*d^2/(a-I*b)*polylog(3,(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))-2/(b-
I*a)^2/f^2/(b+I*a)*b^3*c*d/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*
I*(f*x+e))-a+I*b)+2/(b-I*a)^2/f^3/(b+I*a)*b^3*d^2*e/(I*b-a)/(I*b+a)*ln(a*ex
p(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)+2/(b-I*a)^2/f/(b+I*a)*b^2*a*c^2/
(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)-2*I/(b-I*
a)^2/f^3/(b+I*a)*b*a*d^2*e^2/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))
+4*I/(b-I*a)^2/f/(b+I*a)*b*a*c^2/(I*b-a)*ln(exp(I*(f*x+e)))+4*I/(b-I*a)^2/f
^3/(b+I*a)*b^2*d^2*e/(I*b-a)*ln(exp(I*(f*x+e)))+2/(b-I*a)^2/f^2/(b+I*a)*b*a
*c*d/(a-I*b)*polylog(2,(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))+2/(b-I*a)^2/f^2/(b
+I*a)*b*a*d^2/(a-I*b)*polylog(2,(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*x-4*I/(b-
```

```
I*a)^2/f^2/(b+I*a)*b^2*c*d/(I*b-a)*ln(exp(I*(f*x+e)))+d/(2*I*a*b+a^2-b^2)*c
*x^2+1/3*d^2/(2*I*a*b+a^2-b^2)*x^3+1/(2*I*a*b+a^2-b^2)*c^2*x+1/3/d/(2*I*a*b
+a^2-b^2)*c^3+8/(b-I*a)^2/f/(b+I*a)*b*a*c*d/(a-I*b)*e*x+4*I/(b-I*a)^2/f^3/(
b+I*a)*b*a*d^2*e^2/(I*b-a)*ln(exp(I*(f*x+e)))+2*I/(b-I*a)^2/f/(b+I*a)*b*a*d
^2/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*x^2-2*I/(b-I*a)^2/f/(b+I*
a)*b*a^2*c^2/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I
*b)-2*I/(b-I*a)^2/f^2/(b+I*a)*b^2*d^2/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))
/(a-I*b))*x-2*I/(b-I*a)^2/f^3/(b+I*a)*b^2*d^2/(a-I*b)*ln(1-(I*b+a)*exp(2*I*
(f*x+e))/(a-I*b))*e+2/(b-I*a)^2/f^3/(b+I*a)*b^2*a*d^2*e^2/(I*b-a)/(I*b+a)*l
n(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)+4/(b-I*a)^2/f^2/(b+I*a)*b*
a*c*d/(a-I*b)*e^2-4/(b-I*a)^2/f^2/(b+I*a)*b*a*d^2/(a-I*b)*e^2*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2560 vs. $2(540) = 1080$.
time = 1.29, size = 2560, normalized size = 3.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(6*(b^2/((a^4 + a^2*b^2)*f*tan(f*x + e) + (a^3*b + a*b^3)*f) + 2*a*b*lo
g(a*tan(f*x + e) + b)/((a^4 + 2*a^2*b^2 + b^4)*f) - a*b*log(tan(f*x + e)^2
+ 1)/((a^4 + 2*a^2*b^2 + b^4)*f) - (a^2 - b^2)*(f*x + e)/((a^4 + 2*a^2*b^2
+ b^4)*f))*c*d*e - 3*(2*a*b*log(a*tan(f*x + e) + b)/(a^4 + 2*a^2*b^2 + b^4)
 - a*b*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*(f*x +
 e)/(a^4 + 2*a^2*b^2 + b^4) + b^2/(a^3*b + a*b^3 + (a^4 + a^2*b^2)*tan(f*x
+ e)))*c^2 - ((a^3 + I*a^2*b + a*b^2 + I*b^3)*(f*x + e)^3*d^2 + 3*(a^3*e^2
+ I*a^2*b*e^2 + a*b^2*e^2 + I*b^3*e^2)*(f*x + e)*d^2 + 3*((a^3 + I*a^2*b +
a*b^2 + I*b^3)*c*d*f - (a^3*e + I*a^2*b*e + a*b^2*e + I*b^3*e)*d^2)*(f*x +
e)^2 + 6*(I*a*b^2*e^2 + b^3*e^2)*d^2 + 6*((I*a*b^2 + b^3)*c*d*f - (a*b^2*(e
^2 + I*e) + I*a^2*b*e^2 + b^3*e)*d^2 + ((-I*a*b^2 + b^3)*c*d*f - (a*b^2*(e^
2 - I*e) - I*a^2*b*e^2 + b^3*e)*d^2)*cos(2*f*x + 2*e) + ((a*b^2 + I*b^3)*c*
d*f + (a*b^2*(-I*e^2 - e) - a^2*b*e^2 - I*b^3*e)*d^2)*sin(2*f*x + 2*e))*arc
tan2(b*cos(2*f*x + 2*e) + a*sin(2*f*x + 2*e) + b, a*cos(2*f*x + 2*e) - b*si
n(2*f*x + 2*e) - a) + 6*((-I*a^2*b - a*b^2)*(f*x + e)^2*d^2 + (2*(-I*a^2*b
- a*b^2)*c*d*f + (a*b^2*(2*e + I) + 2*I*a^2*b*e + b^3)*d^2)*(f*x + e) + ((I
*a^2*b - a*b^2)*(f*x + e)^2*d^2 + (2*(I*a^2*b - a*b^2)*c*d*f + (a*b^2*(2*e
- I) - 2*I*a^2*b*e + b^3)*d^2)*(f*x + e))*cos(2*f*x + 2*e) - ((a^2*b + I*a*
b^2)*(f*x + e)^2*d^2 + (2*(a^2*b + I*a*b^2)*c*d*f - (a*b^2*(2*I*e + 1) + 2*
a^2*b*e + I*b^3)*d^2)*(f*x + e))*sin(2*f*x + 2*e))*arctan2(-(2*a*b*cos(2*f*
x + 2*e) + (a^2 - b^2)*sin(2*f*x + 2*e))/(a^2 + b^2), (2*a*b*sin(2*f*x + 2*
e) + a^2 + b^2 - (a^2 - b^2)*cos(2*f*x + 2*e))/(a^2 + b^2)) - ((a^3 + 3*I*a
^2*b - 3*a*b^2 - I*b^3)*(f*x + e)^3*d^2 + 3*((a^3 + 3*I*a^2*b - 3*a*b^2 - I
*b^3)*c*d*f + (a*b^2*(3*e - 2*I) - b^3*(-I*e - 2) - a^3*e - 3*I*a^2*b*e)*d^
```

```
2)*(f*x + e)^2 - 3*(4*(I*a*b^2 - b^3)*c*d*f + (a*b^2*(3*e^2 - 4*I*e) + b^3*
(I*e^2 + 4*e) - a^3*e^2 - 3*I*a^2*b*e^2)*d^2)*(f*x + e))*cos(2*f*x + 2*e) +
 3*(2*(I*a^2*b + a*b^2)*(f*x + e)*d^2 + 2*(I*a^2*b + a*b^2)*c*d*f - (a*b^2*
(2*e + I) + 2*I*a^2*b*e + b^3)*d^2 + (2*(-I*a^2*b + a*b^2)*(f*x + e)*d^2 +
2*(-I*a^2*b + a*b^2)*c*d*f - (a*b^2*(2*e - I) - 2*I*a^2*b*e + b^3)*d^2)*cos
(2*f*x + 2*e) + (2*(a^2*b + I*a*b^2)*(f*x + e)*d^2 + 2*(a^2*b + I*a*b^2)*c*
d*f + (a*b^2*(-2*I*e - 1) - 2*a^2*b*e - I*b^3)*d^2)*sin(2*f*x + 2*e))*dilog
((I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) + 3*((a*b^2 - I*b^3)*c*d*f + (a*b
^2*(I*e^2 - e) - a^2*b*e^2 + I*b^3*e)*d^2 - ((a*b^2 + I*b^3)*c*d*f - (a*b^2
*(I*e^2 + e) + a^2*b*e^2 + I*b^3*e)*d^2)*cos(2*f*x + 2*e) + ((-I*a*b^2 + b^
3)*c*d*f - (a*b^2*(e^2 - I*e) - I*a^2*b*e^2 + b^3*e)*d^2)*sin(2*f*x + 2*e))
*log((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*
sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e)) - 3*((a^2*
b - I*a*b^2)*(f*x + e)^2*d^2 + (2*(a^2*b - I*a*b^2)*c*d*f - (a*b^2*(-2*I*e
+ 1) + 2*a^2*b*e - I*b^3)*d^2)*(f*x + e) - ((a^2*b + I*a*b^2)*(f*x + e)^2*d
^2 + (2*(a^2*b + I*a*b^2)*c*d*f + (a*b^2*(-2*I*e - 1) - 2*a^2*b*e - I*b^3)*
d^2)*(f*x + e))*cos(2*f*x + 2*e) - ((I*a^2*b - a*b^2)*(f*x + e)^2*d^2 + (2*
(I*a^2*b - a*b^2)*c*d*f + (a*b^2*(2*e - I) - 2*I*a^2*b*e + b^3)*d^2)*(f*x +
 e))*sin(2*f*x + 2*e))*log(((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*
x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2
*f*x + 2*e))/(a^2 + b^2)) + 3*((a^2*b + I*a*b^2)*d^2*cos(2*f*x + 2*e) + (I*
a^2*b - a*b^2)*d^2*sin(2*f*x + 2*e) - (a^2*b - I*a*b^2)*d^2)*polylog(3, (I*
a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) - ((I*a^3 - 3*a^2*b - 3*I*a*b^2 + b^3
)*(f*x + e)^3*d^2 - 3*((-I*a^3 + 3*a^2*b + 3*I*a*b^2 - b^3)*c*d*f + (b^3*(e
 - 2*I) + a*b^2*(-3*I*e - 2) + I*a^3*e - 3*a^2*b*e)*d^2)*(f*x + e)^2 + 3*(4
*(a*b^2 + I*b^3)*c*d*f + (b^3*(e^2 - 4*I*e) - a*b^2*(3*I*e^2 + 4*e) + I*a^3
*e^2 - 3*a^2*b*e^2)*d^2)*(f*x + e))*sin(2*f*x + 2*e))/((a^5 + I*a^4*b + 2*a
^3*b^2 + 2*I*a^2*b^3 + a*b^4 + I*b^5)*f^2*cos(2*f*x + 2*e) + (I*a^5 - a^4*b
 + 2*I*a^3*b^2 - 2*a^2*b^3 + I*a*b^4 - b^5)*f^2*sin(2*f*x + 2*e) - (a^5 - I
*a^4*b + 2*a^3*b^2 - 2*I*a^2*b^3 + a*b^4 - I*b^5)*f^2))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2092 vs. $2(540) = 1080$.
time = 4.32, size = 2092, normalized size = 3.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(a^2*b - b^3)*d^2*f^3*x^3 - 6*a*b^2*c^2*f^2 - 6*(a*b^2*d^2*f^2 - (a^
2*b - b^3)*c*d*f^3)*x^2 - 6*(2*a*b^2*c*d*f^2 - (a^2*b - b^3)*c^2*f^3)*x + 2
*((a^2*b - b^3)*d^2*f^3*x^3 - 3*a*b^2*c^2*f^2 - 3*(a*b^2*d^2*f^2 - (a^2*b -
 b^3)*c*d*f^3)*x^2 - 3*(2*a*b^2*c*d*f^2 - (a^2*b - b^3)*c^2*f^3)*x)*cos(2*f
*x + 2*e) - 3*(-2*I*a*b^2*d^2*f*x - 2*I*a*b^2*c*d*f + I*b^3*d^2 + (-2*I*a*b
```

```
^2*d^2*f*x - 2*I*a*b^2*c*d*f + I*b^3*d^2)*cos(2*f*x + 2*e) + (-2*I*a^2*b*d^
2*f*x - 2*I*a^2*b*c*d*f + I*a*b^2*d^2)*sin(2*f*x + 2*e))*dilog(-(a^2 + b^2
- (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f
*x + 2*e))/(a^2 + b^2) + 1) - 3*(2*I*a*b^2*d^2*f*x + 2*I*a*b^2*c*d*f - I*b^
3*d^2 + (2*I*a*b^2*d^2*f*x + 2*I*a*b^2*c*d*f - I*b^3*d^2)*cos(2*f*x + 2*e)
+ (2*I*a^2*b*d^2*f*x + 2*I*a^2*b*c*d*f - I*a*b^2*d^2)*sin(2*f*x + 2*e))*dil
og(-(a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a*b -
I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) - 6*(a*b^2*c^2*f^2 - b^3*c*d*f +
a*b^2*d^2*e^2 + (a*b^2*c^2*f^2 - b^3*c*d*f + a*b^2*d^2*e^2 - (2*a*b^2*c*d*f
 - b^3*d^2)*e)*cos(2*f*x + 2*e) - (2*a*b^2*c*d*f - b^3*d^2)*e + (a^2*b*c^2*
f^2 - a*b^2*c*d*f + a^2*b*d^2*e^2 - (2*a^2*b*c*d*f - a*b^2*d^2)*e)*sin(2*f*
x + 2*e))*log(1/2*a^2 + I*a*b - 1/2*b^2 - 1/2*(a^2 + b^2)*cos(2*f*x + 2*e)
+ 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 6*(a*b^2*c^2*f^2 - b^3*c*d*f + a*
b^2*d^2*e^2 + (a*b^2*c^2*f^2 - b^3*c*d*f + a*b^2*d^2*e^2 - (2*a*b^2*c*d*f -
 b^3*d^2)*e)*cos(2*f*x + 2*e) - (2*a*b^2*c*d*f - b^3*d^2)*e + (a^2*b*c^2*f^
2 - a*b^2*c*d*f + a^2*b*d^2*e^2 - (2*a^2*b*c*d*f - a*b^2*d^2)*e)*sin(2*f*x
+ 2*e))*log(-1/2*a^2 + I*a*b + 1/2*b^2 + 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) +
 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 6*(a*b^2*d^2*f^2*x^2 - a*b^2*d^2*e
^2 + (2*a*b^2*c*d*f^2 - b^3*d^2*f)*x + (a*b^2*d^2*f^2*x^2 - a*b^2*d^2*e^2 +
 (2*a*b^2*c*d*f^2 - b^3*d^2*f)*x + (2*a*b^2*c*d*f - b^3*d^2)*e)*cos(2*f*x +
 2*e) + (2*a*b^2*c*d*f - b^3*d^2)*e + (a^2*b*d^2*f^2*x^2 - a^2*b*d^2*e^2 +
(2*a^2*b*c*d*f^2 - a*b^2*d^2*f)*x + (2*a^2*b*c*d*f - a*b^2*d^2)*e)*sin(2*f*
x + 2*e))*log((a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2
 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 6*(a*b^2*d^2*f^2*x^2 - a
*b^2*d^2*e^2 + (2*a*b^2*c*d*f^2 - b^3*d^2*f)*x + (a*b^2*d^2*f^2*x^2 - a*b^2
*d^2*e^2 + (2*a*b^2*c*d*f^2 - b^3*d^2*f)*x + (2*a*b^2*c*d*f - b^3*d^2)*e)*c
os(2*f*x + 2*e) + (2*a*b^2*c*d*f - b^3*d^2)*e + (a^2*b*d^2*f^2*x^2 - a^2*b*
d^2*e^2 + (2*a^2*b*c*d*f^2 - a*b^2*d^2*f)*x + (2*a^2*b*c*d*f - a*b^2*d^2)*e
)*sin(2*f*x + 2*e))*log((a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e)
 + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 3*(a*b^2*d^2*co
s(2*f*x + 2*e) + a^2*b*d^2*sin(2*f*x + 2*e) + a*b^2*d^2)*polylog(3, ((a^2 +
 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 - 2*a*b - I*b^2)*sin(2*f*x + 2*e)
)/(a^2 + b^2)) - 3*(a*b^2*d^2*cos(2*f*x + 2*e) + a^2*b*d^2*sin(2*f*x + 2*e)
 + a*b^2*d^2)*polylog(3, ((a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2
- 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) + 2*((a^3 - a*b^2)*d^2*f^3*
x^3 + 3*b^3*c^2*f^2 + 3*(b^3*d^2*f^2 + (a^3 - a*b^2)*c*d*f^3)*x^2 + 3*(2*b^
3*c*d*f^2 + (a^3 - a*b^2)*c^2*f^3)*x)*sin(2*f*x + 2*e))/((a^4*b + 2*a^2*b^3
 + b^5)*f^3*cos(2*f*x + 2*e) + (a^5 + 2*a^3*b^2 + a*b^4)*f^3*sin(2*f*x + 2*
e) + (a^4*b + 2*a^2*b^3 + b^5)*f^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{(a+b\cot(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*cot(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**2/(a + b*cot(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*cot(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^2}{(a+b\cot(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*cot(e + f*x))^2,x)
```

```
[Out] int((c + d*x)^2/(a + b*cot(e + f*x))^2, x)
```

3.59 $\int \frac{c+dx}{(a+b\cot(e+fx))^2}\,dx$

Optimal. Leaf size=213

$$-\frac{(c+dx)^2}{2\left(a^2+b^2\right)d}+\frac{(bd-2acf-2adfx)^2}{4a(a-ib)^2(a+ib)df^2}+\frac{b(c+dx)}{\left(a^2+b^2\right)f(a+b\cot(e+fx))}+\frac{b(bd-2acf-2adfx)\log\left(1-\frac{(a+ib}{}\right.}{\left(a^2+b^2\right)^2f^2}$$

```
[Out] -1/2*(d*x+c)^2/(a^2+b^2)/d+1/4*(-2*a*d*f*x-2*a*c*f+b*d)^2/a/(a-I*b)^2/(a+I*
b)/d/f^2+b*(d*x+c)/(a^2+b^2)/f/(a+b*cot(f*x+e))+b*(-2*a*d*f*x-2*a*c*f+b*d)*
ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)^2/f^2+I*a*b*d*polylog(2,(a
+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)^2/f^2
```

Rubi [A]
time = 0.21, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3814, 3812, 2221, 2317, 2438}

$$\frac{b(-2acf-2adfx+bd)\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f^2\left(a^2+b^2\right)^2}+\frac{b(c+dx)}{f\left(a^2+b^2\right)(a+b\cot(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2+b^2\right)}+\frac{iabd\text{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f^2\left(a^2+b^2\right)^2}+\frac{(-2acf-2adfx+bd)^2}{4adf^2(a-ib)^2(a+ib)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Cot[e + f*x])^2,x]
```

```
[Out] -1/2*(c + d*x)^2/((a^2 + b^2)*d) + (b*d - 2*a*c*f - 2*a*d*f*x)^2/(4*a*(a -
I*b)^2*(a + I*b)*d*f^2) + (b*(c + d*x))/((a^2 + b^2)*f*(a + b*Cot[e + f*x])
) + (b*(b*d - 2*a*c*f - 2*a*d*f*x)*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/
(a - I*b)])/((a^2 + b^2)^2*f^2) + (I*a*b*d*PolyLog[2, ((a + I*b)*E^((2*I)*(
e + f*x)))/(a - I*b)])/((a^2 + b^2)^2*f^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3812

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist
[2*I*b, Int[(c + d*x)^m*E^(2*I*k*Pi)*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2
 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])), x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3814

```
Int[((c_.) + (d_.)*(x_))/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol
] :> Simp[-(c + d*x)^2/(2*d*(a^2 + b^2)), x] + (Dist[1/(f*(a^2 + b^2)), Int
[(b*d + 2*a*c*f + 2*a*d*f*x)/(a + b*Tan[e + f*x]), x], x] - Simp[b*((c + d*
x)/(f*(a^2 + b^2)*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{c+dx}{(a+b\cot(e+fx))^2}\,dx = -\frac{(c+dx)^2}{2\left(a^2+b^2\right)d}+\frac{b(c+dx)}{\left(a^2+b^2\right)f(a+b\cot(e+fx))}+\frac{\int \frac{-bd+2acf+2adfx}{a+b\cot(e+fx)}\,dx}{\left(a^2+b^2\right)f}$$

$$= -\frac{(c+dx)^2}{2\left(a^2+b^2\right)d}+\frac{(bd-2acf-2adfx)^2}{4a(a-ib)^2(a+ib)df^2}+\frac{b(c+dx)}{\left(a^2+b^2\right)f(a+b\cot(e+fx))}+$$

$$= -\frac{(c+dx)^2}{2\left(a^2+b^2\right)d}+\frac{(bd-2acf-2adfx)^2}{4a(a-ib)^2(a+ib)df^2}+\frac{b(c+dx)}{\left(a^2+b^2\right)f(a+b\cot(e+fx))}+$$

$$= -\frac{(c+dx)^2}{2\left(a^2+b^2\right)d}+\frac{(bd-2acf-2adfx)^2}{4a(a-ib)^2(a+ib)df^2}+\frac{b(c+dx)}{\left(a^2+b^2\right)f(a+b\cot(e+fx))}+$$

$$= -\frac{(c+dx)^2}{2\left(a^2+b^2\right)d}+\frac{(bd-2acf-2adfx)^2}{4a(a-ib)^2(a+ib)df^2}+\frac{b(c+dx)}{\left(a^2+b^2\right)f(a+b\cot(e+fx))}+$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 730 vs. 2(213) = 426.
time = 6.84, size = 730, normalized size = 3.43

$$-\frac{(e+fx)(-2de+2cf+d(e+fx))\csc^2(e+fx)(b\cos(e+fx)+a\sin(e+fx))^2}{2(-ia+b)(ia+b)f^2(a+b\cot(e+fx))^2}+\frac{bd\csc^2(e+fx)(-a(e+fx)+b\log(b\cos(e+fx)+a\sin(e+fx)))(b\cos(e+fx)+a\sin(e+fx))^2}{(-ia+b)(ia+b)\left(a^2+b^2\right)f^2(a+b\cot(e+fx))^2}+\frac{2ade\csc^2(e+fx)(-a(e+fx)+b\log(b\cos(e+fx)+a\sin(e+fx)))(b\cos(e+fx)+a\sin(e+fx))^2}{(-ia+b)(ia+b)\left(a^2+b^2\right)f^2(a+b\cot(e+fx))^2}-\frac{2ac\csc^2(e+fx)(-a(e+fx)+b\log(b\cos(e+fx)+a\sin(e+fx)))(b\cos(e+fx)+a\sin(e+fx))^2}{(-ia+b)(ia+b)\left(a^2+b^2\right)f(a+b\cot(e+fx))^2}+\frac{d\csc^2(e+fx)\left(e^{i\text{ArcTan}\left(\frac{b}{a}\right)}(e+fx)^2+\frac{b\left(i(e+fx)\left(-\pi+2\text{ArcTan}\left(\frac{b}{a}\right)\right)-\pi\log\left(1+e^{-2i(e+fx)}\right)-2\left(e+fx+\text{ArcTan}\left(\frac{b}{a}\right)\right)\log\left(1-e^{2i\left(e+fx+\text{ArcTan}\left(\frac{b}{a}\right)\right)}\right)+\pi\log(\cos(e+fx))+2\text{ArcTan}\left(\frac{b}{a}\right)\log\left(\sin\left(e+fx+\text{ArcTan}\left(\frac{b}{a}\right)\right)\right)+i\,\text{PolyLog}\left(2,e^{2i\left(e+fx+\text{ArcTan}\left(\frac{b}{a}\right)\right)}\right)\right)}{a\sqrt{1+\frac{b^2}{a^2}}}\right)(b\cos(e+fx)+a\sin(e+fx))^2}{(-ia+b)(ia+b)\sqrt{\frac{a^2+b^2}{a^2}}\,f^2(a+b\cot(e+fx))^2}+\frac{\csc^2(e+fx)(b\cos(e+fx)+a\sin(e+fx))(-bde\sin(e+fx)+bcf\sin(e+fx)+bd(e+fx)\sin(e+fx))}{(-ia+b)(ia+b)f^2(a+b\cot(e+fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)/(a + b*Cot[e + f*x])^2,x]
```

```
[Out] -1/2*((e + f*x)*(-2*d*e + 2*c*f + d*(e + f*x))*Csc[e + f*x]^2*(b*Cos[e + f*
x] + a*Sin[e + f*x])^2)/(((-I)*a + b)*(I*a + b)*f^2*(a + b*Cot[e + f*x])^2)
 + (b*d*Csc[e + f*x]^2*(-(a*(e + f*x)) + b*Log[b*Cos[e + f*x] + a*Sin[e + f
*x]])*(b*Cos[e + f*x] + a*Sin[e + f*x])^2)/(((-I)*a + b)*(I*a + b)*(a^2 + b
^2)*f^2*(a + b*Cot[e + f*x])^2) + (2*a*d*e*Csc[e + f*x]^2*(-(a*(e + f*x)) +
 b*Log[b*Cos[e + f*x] + a*Sin[e + f*x]])*(b*Cos[e + f*x] + a*Sin[e + f*x])^
2)/(((-I)*a + b)*(I*a + b)*(a^2 + b^2)*f^2*(a + b*Cot[e + f*x])^2) - (2*a*c
*Csc[e + f*x]^2*(-(a*(e + f*x)) + b*Log[b*Cos[e + f*x] + a*Sin[e + f*x]])*(
b*Cos[e + f*x] + a*Sin[e + f*x])^2)/(((-I)*a + b)*(I*a + b)*(a^2 + b^2)*f*(
a + b*Cot[e + f*x])^2) + (d*Csc[e + f*x]^2*(E^(I*ArcTan[b/a])*(e + f*x)^2 +
 (b*(I*(e + f*x)*(-Pi + 2*ArcTan[b/a]) - Pi*Log[1 + E^((-2*I)*(e + f*x))] -
 2*(e + f*x + ArcTan[b/a])*Log[1 - E^((2*I)*(e + f*x + ArcTan[b/a]))] + Pi*
Log[Cos[e + f*x]] + 2*ArcTan[b/a]*Log[Sin[e + f*x + ArcTan[b/a]]] + I*PolyL
og[2, E^((2*I)*(e + f*x + ArcTan[b/a]))]))/(a*Sqrt[1 + b^2/a^2]))*(b*Cos[e
+ f*x] + a*Sin[e + f*x])^2)/(((-I)*a + b)*(I*a + b)*Sqrt[(a^2 + b^2)/a^2]*f
^2*(a + b*Cot[e + f*x])^2) + (Csc[e + f*x]^2*(b*Cos[e + f*x] + a*Sin[e + f*
x])*(-(b*d*e*Sin[e + f*x]) + b*c*f*Sin[e + f*x] + b*d*(e + f*x)*Sin[e + f*x
]))/(((-I)*a + b)*(I*a + b)*f^2*(a + b*Cot[e + f*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 989 vs. 2(198) = 396.
time = 1.12, size = 990, normalized size = 4.65

method	result
risch	$\frac{d\,x^2}{4iab+2a^2-2b^2}+\frac{cx}{2iab+a^2-b^2}+\frac{2ib\,a^2de\ln\left(a\,e^{2i(fx+e)}+ib\,e^{2i(fx+e)}-a+ib\right)}{(-ia+b)^2f^2(ia+b)(ib-a)(ib+a)}+\frac{2ibad\ln\left(1-\frac{(ib+a)e^{2i(fx+e)}}{-ib+a}\right)e}{(-ia+b)^2f^2(ia+b)(-ib+a)}+\frac{}{(ia+b)f(-ia}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*cot(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(2*I*a*b+a^2-b^2)*d*x^2+1/(2*I*a*b+a^2-b^2)*c*x+2*I/(b-I*a)^2/f^2/(b+I*
a)*b*a^2*d*e/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I
*b)+2*I/(b-I*a)^2/f^2/(b+I*a)*b*a*d/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(
a-I*b))*e+2*I*b^2*(d*x+c)/(b+I*a)/f/(b-I*a)^2/(b*exp(2*I*(f*x+e))-I*a*exp(2
*I*(f*x+e))+b+I*a)-1/(b-I*a)^2/f^2/(b+I*a)*b^3*d/(I*b-a)/(I*b+a)*ln(a*exp(2
*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)+I/(b-I*a)^2/f^2/(b+I*a)*b^2*d/(I*b-
a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))-a+I*b)*a+2*I/(b-I*a)^
2/f/(b+I*a)*b*a*d/(a-I*b)*ln(1-(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))*x+2/(b-I*a
)^2/f/(b+I*a)*b^2*a*c/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*
x+e))-a+I*b)-2*I/(b-I*a)^2/f^2/(b+I*a)*b^2*d/(I*b-a)*ln(exp(I*(f*x+e)))-4*I
/(b-I*a)^2/f^2/(b+I*a)*b*a*d*e/(I*b-a)*ln(exp(I*(f*x+e)))-2/(b-I*a)^2/f^2/(
```

```
b+I*a)*b^2*a*d*e/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e))+I*b*exp(2*I*(f*x+e))
-a+I*b)-2*I/(b-I*a)^2/f/(b+I*a)*b*a^2*c/(I*b-a)/(I*b+a)*ln(a*exp(2*I*(f*x+e
))+I*b*exp(2*I*(f*x+e))-a+I*b)+4*I/(b-I*a)^2/f/(b+I*a)*b*a*c/(I*b-a)*ln(exp
(I*(f*x+e)))+2/(b-I*a)^2/(b+I*a)*b*a*d/(a-I*b)*x^2+4/(b-I*a)^2/f/(b+I*a)*b*
a*d/(a-I*b)*e*x+2/(b-I*a)^2/f^2/(b+I*a)*b*a*d/(a-I*b)*e^2+1/(b-I*a)^2/f^2/(
b+I*a)*b*a*d/(a-I*b)*polylog(2,(I*b+a)*exp(2*I*(f*x+e))/(a-I*b))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1207 vs. $2(196) = 392$.
time = 0.87, size = 1207, normalized size = 5.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/2*((a^3 + I*a^2*b + a*b^2 + I*b^3)*d*f^2*x^2 + 2*(a^3 + I*a^2*b + a*b^2
+ I*b^3)*c*f^2*x + 4*(I*a*b^2 + b^3)*c*f + 2*(2*(-I*a^2*b - a*b^2)*c*f + (I
*a*b^2 + b^3)*d + (2*(I*a^2*b - a*b^2)*c*f + (-I*a*b^2 + b^3)*d)*cos(2*f*x
+ 2*e) - (2*(a^2*b + I*a*b^2)*c*f - (a*b^2 + I*b^3)*d)*sin(2*f*x + 2*e))*ar
ctan2(b*cos(2*f*x + 2*e) + a*sin(2*f*x + 2*e) + b, a*cos(2*f*x + 2*e) - b*s
in(2*f*x + 2*e) - a) + 4*((I*a^2*b - a*b^2)*d*f*x*cos(2*f*x + 2*e) - (a^2*b
 + I*a*b^2)*d*f*x*sin(2*f*x + 2*e) + (-I*a^2*b - a*b^2)*d*f*x)*arctan2(-(2*
a*b*cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f*x + 2*e))/(a^2 + b^2), (2*a*b*si
n(2*f*x + 2*e) + a^2 + b^2 - (a^2 - b^2)*cos(2*f*x + 2*e))/(a^2 + b^2)) - (
(a^3 + 3*I*a^2*b - 3*a*b^2 - I*b^3)*d*f^2*x^2 + 2*((a^3 + 3*I*a^2*b - 3*a*b
^2 - I*b^3)*c*f^2 - 2*(I*a*b^2 - b^3)*d*f)*x)*cos(2*f*x + 2*e) + 2*((-I*a^2
*b + a*b^2)*d*cos(2*f*x + 2*e) + (a^2*b + I*a*b^2)*d*sin(2*f*x + 2*e) + (I*
a^2*b + a*b^2)*d)*dilog(-(-I*a*e^(2*I*e) + b*e^(2*I*e))*e^(2*I*f*x)/(I*a +
b)) - (2*(a^2*b - I*a*b^2)*c*f - (a*b^2 - I*b^3)*d - (2*(a^2*b + I*a*b^2)*c
*f - (a*b^2 + I*b^3)*d)*cos(2*f*x + 2*e) - (2*(I*a^2*b - a*b^2)*c*f - (I*a*
b^2 - b^3)*d)*sin(2*f*x + 2*e))*log((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*
sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^
2)*cos(2*f*x + 2*e)) + 2*((a^2*b + I*a*b^2)*d*f*x*cos(2*f*x + 2*e) + (I*a^2
*b - a*b^2)*d*f*x*sin(2*f*x + 2*e) - (a^2*b - I*a*b^2)*d*f*x)*log(((a^2 + b
^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2
*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e))/(a^2 + b^2)) - ((I*a^3
- 3*a^2*b - 3*I*a*b^2 + b^3)*d*f^2*x^2 - 2*((-I*a^3 + 3*a^2*b + 3*I*a*b^2 -
 b^3)*c*f^2 - 2*(a*b^2 + I*b^3)*d*f)*x)*sin(2*f*x + 2*e))/((a^5 + I*a^4*b +
 2*a^3*b^2 + 2*I*a^2*b^3 + a*b^4 + I*b^5)*f^2*cos(2*f*x + 2*e) + (I*a^5 - a
^4*b + 2*I*a^3*b^2 - 2*a^2*b^3 + I*a*b^4 - b^5)*f^2*sin(2*f*x + 2*e) - (a^5
 - I*a^4*b + 2*a^3*b^2 - 2*I*a^2*b^3 + a*b^4 - I*b^5)*f^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1101 vs. $2(196) = 392$.

time = 3.49, size = 1101, normalized size = 5.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*b - b^3)*d*f^2*x^2 - 2*a*b^2*c*f - 2*(a*b^2*d*f - (a^2*b - b^3)*c
*f^2)*x + ((a^2*b - b^3)*d*f^2*x^2 - 2*a*b^2*c*f - 2*(a*b^2*d*f - (a^2*b -
b^3)*c*f^2)*x)*cos(2*f*x + 2*e) + (I*a*b^2*d*cos(2*f*x + 2*e) + I*a^2*b*d*s
in(2*f*x + 2*e) + I*a*b^2*d)*dilog(-(a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(
2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1)
+ (-I*a*b^2*d*cos(2*f*x + 2*e) - I*a^2*b*d*sin(2*f*x + 2*e) - I*a*b^2*d)*di
log(-(a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a*b -
 I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) - (2*a*b^2*c*f - 2*a*b^2*d*e - b
^3*d + (2*a*b^2*c*f - 2*a*b^2*d*e - b^3*d)*cos(2*f*x + 2*e) + (2*a^2*b*c*f
- 2*a^2*b*d*e - a*b^2*d)*sin(2*f*x + 2*e))*log(1/2*a^2 + I*a*b - 1/2*b^2 -
1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) -
(2*a*b^2*c*f - 2*a*b^2*d*e - b^3*d + (2*a*b^2*c*f - 2*a*b^2*d*e - b^3*d)*co
s(2*f*x + 2*e) + (2*a^2*b*c*f - 2*a^2*b*d*e - a*b^2*d)*sin(2*f*x + 2*e))*lo
g(-1/2*a^2 + I*a*b + 1/2*b^2 + 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^
2 + I*b^2)*sin(2*f*x + 2*e)) - 2*(a*b^2*d*f*x + a*b^2*d*e + (a*b^2*d*f*x +
a*b^2*d*e)*cos(2*f*x + 2*e) + (a^2*b*d*f*x + a^2*b*d*e)*sin(2*f*x + 2*e))*l
og((a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b +
I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 2*(a*b^2*d*f*x + a*b^2*d*e + (a*b^2
*d*f*x + a*b^2*d*e)*cos(2*f*x + 2*e) + (a^2*b*d*f*x + a^2*b*d*e)*sin(2*f*x
+ 2*e))*log((a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 +
2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) + ((a^3 - a*b^2)*d*f^2*x^2 +
2*b^3*c*f + 2*(b^3*d*f + (a^3 - a*b^2)*c*f^2)*x)*sin(2*f*x + 2*e))/((a^4*b
+ 2*a^2*b^3 + b^5)*f^2*cos(2*f*x + 2*e) + (a^5 + 2*a^3*b^2 + a*b^4)*f^2*sin
(2*f*x + 2*e) + (a^4*b + 2*a^2*b^3 + b^5)*f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx}{\left(a + b\cot\left(e + fx\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)/(a + b*cot(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*cot(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c+d\,x}{{\left(a+b\,\mathrm{cot}\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*cot(e + f*x))^2,x)
```

```
[Out] int((c + d*x)/(a + b*cot(e + f*x))^2, x)
```

3.60 $\int \frac{1}{(c+dx)(a+b\cot(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{1}{(c+dx)(a+b\cot(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*cot(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\cot(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Cot[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + b*Cot[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\cot(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+b\cot(e+fx))^2}\,dx$$

Mathematica [A]
time = 16.45, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b\cot(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Cot[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + b*Cot[e + f*x])^2), x]
```

Maple [A]
time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(a+b\cot\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*cot(f*x+e))^2,x)
[Out] int(1/(d*x+c)/(a+b*cot(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="maxima")
[Out] (((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*cos(2*f*x + 2*e)^2*log(d*x + c) + ((
a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*log(d*x + c)*sin(2*f*x + 2*e)^2 - 2*(2*
a*b^3*d + ((a^4 - 2*a^2*b^2 + b^4)*d*f*x + (a^4 - 2*a^2*b^2 + b^4)*c*f)*log
(d*x + c))*cos(2*f*x + 2*e) + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*x
+ (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f)*cos(2*f*x + 2*
e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)*c*d*f)*sin(2*f*x + 2*e)^2 - 2*((a^6 + a^4*b^2 - a^2*b^4 - b^
6)*d^2*f*x + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d*f)*cos(2*f*x + 2*e) + 4*((
a^5*b + 2*a^3*b^3 + a*b^5)*d^2*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*d*f)*sin
(2*f*x + 2*e))*integrate(-2*(2*(2*a^2*b^2*d*f*x + 2*a^2*b^2*c*f + a*b^3*d)*
cos(2*f*x + 2*e) + (2*(a^3*b - a*b^3)*d*f*x + 2*(a^3*b - a*b^3)*c*f + (a^2*
b^2 - b^4)*d)*sin(2*f*x + 2*e))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*
x^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f*x + (a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)*c^2*f + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*x^2 + 2*(
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*c^2*f)*cos(2*f*x + 2*e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*
f*x^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f*x + (a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*c^2*f)*sin(2*f*x + 2*e)^2 - 2*((a^6 + a^4*b^2 - a^2*b^4 -
b^6)*d^2*f*x^2 + 2*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d*f*x + (a^6 + a^4*b^2
 - a^2*b^4 - b^6)*c^2*f)*cos(2*f*x + 2*e) + 4*((a^5*b + 2*a^3*b^3 + a*b^5)*
d^2*f*x^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c*d*f*x + (a^5*b + 2*a^3*b^3 + a*
b^5)*c^2*f)*sin(2*f*x + 2*e)), x) + ((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*l
og(d*x + c) - 2*((a^2*b^2 - b^4)*d - 2*((a^3*b - a*b^3)*d*f*x + (a^3*b - a*
b^3)*c*f)*log(d*x + c))*sin(2*f*x + 2*e))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f + ((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f)*co
s(2*f*x + 2*e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*x + (a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f)*sin(2*f*x + 2*e)^2 - 2*((a^6 + a^4*b^2 -
a^2*b^4 - b^6)*d^2*f*x + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d*f)*cos(2*f*x +
 2*e) + 4*((a^5*b + 2*a^3*b^3 + a*b^5)*d^2*f*x + (a^5*b + 2*a^3*b^3 + a*b^5
)*c*d*f)*sin(2*f*x + 2*e))
```

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*cot(f*x + e)^2 + 2*(a*b*d*x
 + a*b*c)*cot(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\cot(e + fx))^2 (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e))**2,x)
```

```
[Out] Integral(1/((a + b*cot(e + f*x))**2*(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*cot(f*x + e) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a + b\cot(e + f\,x))^2\ (c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*cot(e + f*x))^2*(c + d*x)),x)
```

```
[Out] int(1/((a + b*cot(e + f*x))^2*(c + d*x)), x)
```

3.61 $\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b\cot(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Cot[e + f*x])^2),x]
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Cot[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))^2}\,dx=\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))^2}\,dx$$

Mathematica [A]
time = 15.52, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Cot[e + f*x])^2),x]
[Out] Integrate[1/((c + d*x)^2*(a + b*Cot[e + f*x])^2), x]
```

Maple [A]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\,(a+b\cot(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f + ((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*
f)*cos(2*f*x + 2*e)^2 + ((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*sin(2*f*x + 2
*e)^2 + 2*(2*a*b^3*d - (a^4 - 2*a^2*b^2 + b^4)*d*f*x - (a^4 - 2*a^2*b^2 + b
^4)*c*f)*cos(2*f*x + 2*e) - ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^2
+ 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*c^2*d*f + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^2 + 2*(
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*c^2*d*f)*cos(2*f*x + 2*e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*
d^3*f*x^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x + (a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)*c^2*d*f)*sin(2*f*x + 2*e)^2 - 2*((a^6 + a^4*b^2 - a^
2*b^4 - b^6)*d^3*f*x^2 + 2*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d^2*f*x + (a^6
 + a^4*b^2 - a^2*b^4 - b^6)*c^2*d*f)*cos(2*f*x + 2*e) + 4*((a^5*b + 2*a^3*b
^3 + a*b^5)*d^3*f*x^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c*d^2*f*x + (a^5*b +
2*a^3*b^3 + a*b^5)*c^2*d*f)*sin(2*f*x + 2*e))*integrate(-4*(2*(a^2*b^2*d*f*
x + a^2*b^2*c*f + a*b^3*d)*cos(2*f*x + 2*e) + ((a^3*b - a*b^3)*d*f*x + (a^3
*b - a*b^3)*c*f + (a^2*b^2 - b^4)*d)*sin(2*f*x + 2*e))/((a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*d^3*f*x^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*
x^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d*f*x + (a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*c^3*f + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^3 + 3
*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x^2 + 3*(a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)*c^2*d*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3*f)*cos(2*
f*x + 2*e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^3 + 3*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*c^2*d*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3*f)*sin(2*f*x + 2*e)
^2 - 2*((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d^3*f*x^3 + 3*(a^6 + a^4*b^2 - a^2*
b^4 - b^6)*c*d^2*f*x^2 + 3*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c^2*d*f*x + (a^6
 + a^4*b^2 - a^2*b^4 - b^6)*c^3*f)*cos(2*f*x + 2*e) + 4*((a^5*b + 2*a^3*b^3
 + a*b^5)*d^3*f*x^3 + 3*(a^5*b + 2*a^3*b^3 + a*b^5)*c*d^2*f*x^2 + 3*(a^5*b
+ 2*a^3*b^3 + a*b^5)*c^2*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c^3*f)*sin(2*f
*x + 2*e)), x) + 2*(2*(a^3*b - a*b^3)*d*f*x + 2*(a^3*b - a*b^3)*c*f + (a^2*
b^2 - b^4)*d)*sin(2*f*x + 2*e))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*
x^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x + (a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*c^2*d*f + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^2 +
 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2
```

```
*b^4 + b^6)*c^2*d*f)*cos(2*f*x + 2*e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*d^3*f*x^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x + (a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d*f)*sin(2*f*x + 2*e)^2 - 2*((a^6 + a^4*b^2
- a^2*b^4 - b^6)*d^3*f*x^2 + 2*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d^2*f*x +
(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c^2*d*f)*cos(2*f*x + 2*e) + 4*((a^5*b + 2*a
^3*b^3 + a*b^5)*d^3*f*x^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c*d^2*f*x + (a^5*
b + 2*a^3*b^3 + a*b^5)*c^2*d*f)*sin(2*f*x + 2*e))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*cot(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*cot(f
*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\cot{(e+fx)})^2\,(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*cot(f*x+e))**2,x)
[Out] Integral(1/((a + b*cot(e + f*x))**2*(c + d*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="giac")
[Out] integrate(1/((d*x + c)^2*(b*cot(f*x + e) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{cot}\left(e+f\,x\right)\right)}^2\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*cot(e + f*x))^2*(c + d*x)^2),x)
[Out] int(1/((a + b*cot(e + f*x))^2*(c + d*x)^2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
       if expr.args[1] == Rational(1,2):
          return True
       else:
          return False
    else:
       return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
          asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
          asinh,acosh,atanh,acoth,asech,acsch
       ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
           fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
           gamma,loggamma,digamma,zeta,polylog,LambertW,
           elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
       if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
         return True
       else:
         return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.op
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```